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Sustainable Vendor-Buyer Inventory Model for Disaster Relief Deteriorating Items under Inflation with Reduction in GHG Emission

Mandeep Singh[1], Ravish Kumar Yadav[2], SR Singh[3], Dipti Singh[4]
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ABSTRACT

In today's world, GHG emission is a crucial problem worldwide. GHG emission is the cause of global warming, which increases the occurrence and frequency of natural disasters. This paper considers a 2-echelon supply chain system consisting of one vendor and one disaster-affected buyer and develops a sustainable vendor-buyer inventory model for disaster relief deteriorating items like medicines, food, etc., under inflation with a technical corporation on investment to reduce GHG emissions. The proposed study considers time-dependent deterioration, inflation's effects, and GHG emissions, which are critical factors that affect inventory management decisions. This suggested model aims to minimize the inventory system's overall cost while reducing GHG emissions and ensuring sustainable development. A mathematical framework of the model is developed, and to demonstrate its effectiveness numerical example is provided with the help of MATHEMATICA 12.0. The result shows that the proposed model can achieve a total cost significant reduction in GHG emissions. The numerical results show that the cap-and-trade policy yields the most total profit compared to the carbon tax policy. Sensitivity analysis is carried out to check the sensitive and non-sensitive parameters. The findings of this study have significant implications for organizations committed to sustainability in their inventory management decisions.

1. Introduction

Global warming is the main problem for the world, and it is continuously increasing due to GHG emissions. As a result of global warming, in the climate of the atmosphere, the growing summer season, the decreasing cold weather, the melting of ice rocks, the rise of temperature, the change in air circulation patterns, Bin weather rainfall, the ozone layer has holes, heavy storm events, cyclones, droughts, floods, and similar effects. More frequent and intense droughts, hurricanes, heat waves, sea level rise, glacier melting, and ocean warming can directly harm human life, destroy living places, and wreak havoc on people's livelihoods and communities. Natural disasters are increasing due to global warming. Reduction in carbon emissions can reduce the consequences of global warming.

Sustainability is the ability to satisfy current needs without jeopardizing the ability of future generations to meet their own needs. Social, environmental, and balancing economic factors create a livable and equitable world for all humans while preserving natural resources and ecosystems.

Today's, inflation is typical, particularly in emerging countries such as India. So, we have considered inflation to make our proposed model more realistic. In simple terms, inflation refers to the rise in

Corresponding author : Mandeep Singh, Department of Mathematics, Hindu College Moradabad.
Email: mandeepsinghbsr@gmail.com
[2]Department of Mathematics, Hindu College Moradabad.
[3,4]Department of Mathematics, Chaudhary Charan Singh University Meerut.

prices of goods and services. For inventory managers, inflation is a crucial factor to consider while managing their inventory. Time is a critical aspect of business, and inflation is closely related to time. Moreover, inflation also affects the demand for specific products. So, inflation plays a significant role in designing any inventory model. Neglecting inflation while developing inventory models can lead to misleading results. Thus, incorporating inflation in the model is necessary to ensure accurate decision-making.

In this paper, we developed a vendor-buyer inventory model for disaster relief. They jointly invest in the carbon emission reduction technique. The effect of inflation is taken into consideration by both the vendor and buyer. Deterioration is taken as time-dependent. The buyer generates carbon emissions during operational activities like ordering, inventory holding, transportation, and procurement. Similarly, the vendor contributes to the carbon emissions through operational activities such as material procurement, setup, production, and inventory holding. To reduce carbon emissions, carbon tax and carbon cap-and-trade policies are used.

2. Literature Review

Inventory models can be designed with sustainability in mind by considering the environmental impact of inventory management practices. This includes reducing waste and minimizing the use of energy and raw materials type resources. One approach to creating sustainable inventory models is through the use of green inventory management practices. This involves reducing waste and inefficiency in the supply chain, using sustainable packaging materials, and implementing sustainable transportation practices. Kelle and Silver's (1989) article was the first of its kind to create a comprehensive forecast system for organizations to use to predict potential reusable products. With a cap-and-trade regulation, the decision behavior and the coordination mechanisms are analyzed for a 2-echelon sustainable supply chain by Xu *et al.* (2016). Downstream manufacturing processes are largely responsible for carbon emissions in a make-to-order setting, and two decision variables((a) selling price and (b) the sustainability level) influence the market demand of the supply chain. A sustainable second S.C. is created by Ahmed *et al.* (2018) to reduce carbon emissions for second-generation biofuels. Lee and Tang (2018) have shown that research on sustainability has grown significantly since (1995). According to Wang *et al.* (2019), green growth is becoming a key method for the sustainable functioning of the world’s economy and society. To get green growth, countries around the world are creating novel methodologies. Two key technologies that help business leaders make decisions are inventory and pricing. Environmental protection and resource sustainability are becoming more urgently needed due to the environment’s rising pollution levels. Mishra *et al.* (2020) developed a policy for the distribution of sustainable assets in a rapidly declining system with a need for non-price-based, nonlinear-stock-based, reducing natural carbon emissions and order costs. Saxena and Sarkar (2020) devised a reverse logistic supply chain model for manufacturing and remanufacturing when considering defective items. This model is for single vendors and single buyers with the consideration of linearly dependent demand on time. A joint supply chain network model is investigated by Wang *et al.* (2022) , which incorporates inventory, location, and the third-party logistics provider decision in a 3-level supply chain including manufacturers, DCs (distribution centers), and retailers. Data sharing value in two-echelon fresh items S.C. with manufacturer and retailer is analyzed by Katzenberg *et al.* (2023). There are price-sensitive and stochastic demands. The operation of S.C. is done with a simple linear wholesale price contract, and decision-making is decentralized. By Utama *et al.* (2023), a sustainable production-inventory model is created. They consider probabilistic demand, multi-material, and quality degradation in this. Buzacott, in 1975, created an inventory model for EOQs that takes the time value of money into account. Singh and Singh (2011) presented an integrated production inventory model in which they assume exponential demand rate, demand-dependent production rate with inflation, imperfect production process, and multiple deliveries. For two warehouse systems, an inventory model is provided by Singh *et al*. (2013) for goods of imperfect quality that consider learning and inflation. In their 2016 article, Kumar and Kumar addressed the effect of system parameters on an inflationary environment. In general, all items experience a decline in quality over time, albeit at varying rates. Traditional inventory models assume that the objects stored can continue to fulfil their purpose indefinitely without losing value. Ghare and Schrader (1963) developed an inventory model with deterioration. For one management with two shops, an inventory model for deteriorating items is examined by Singh *et al.* (2010), which shows shortages and stock-based demand under inflation. In an imperfect production system, an EPQ model is developed by Moon and Sarkar (2011) with inflation. Jawla and Singh (2016) provide a reverse logistic inventory model. The preservation technology investment in a learning and inflationary environment is taken into account in this model for imperfect production processes. Tayal *et al.* (2016) provide a production inventory model for items that degrade while considering erratic demand, inflation, and production system reliability. To determine an economic order quantity, Rabta proposed a degrading inventory model with trade credit that depends on order size and conditions and has no backlogs (2020). Pan *et al.* (2020) developed a sustainable integrated inventory model with technical investment in carbon reduction policies. The primary purpose of this manuscript is to investigate the results of green investing technologies to reduce emissions by cap-and-trade and its regulations. Tiwari (2022) also addressed other issues in his study of managing carbon emissions and producing deterioration simultaneously in a greenhouse farm. This essay compares and contrasts price-dependent demand functions that are linear and nonlinear.

The proposed model addresses several challenges that arise in the management of inventory systems. Firstly, it considers the impact of inflation, which can significantly affect the cost of holding/ secondly, it takes into account the deterioration of items, which is an important consideration for many inventory systems dealing with perishable goods.

The model considers several challenges that arise in managing inventory systems, including the impact of inflation on inventory costs, the deterioration of items, and the need for sustainable practices to reduce GHG emissions. The proposed sustainable investment strategy involves the vendor and buyer sharing the investment to reduce GHG emissions, which incentivizes both parties to adopt sustainable practices and reduces the supply chain's carbon footprint.

3. Notations and Assumptions

The following are the notations and assumptions of this research paper.

3.1 Notations:

P Rate of production for a vendor (Thousand Units)

D Rate of demand for buyers' (Thousand Units)

A Ordering cost per cycle length for the buyer ($ Per order)

$\hat{A}$ For the buyer, the fixed carbon emissions per order ($ Per order)

S Setup cost per production cycle of the vendor. ($ per setup)

$\hat{S}$ For vendor fixed carbon emissions per setup. ($ per setup)

c Per unit product cost of vendor. ($)

$\hat{C}$ Carbon emissions amount per unit related with the manufacturing of vendor. ($ per unit)

v Per unit selling price of vendor. ($)

$\hat{v}$ The buyer's related carbon emissions amount per unit purchased. ($ per unit)

p Buyer's per-unit selling price. ($)

θ Rate of product's deterioration. $0 \leq \theta \leq 1$

r Rate of inflation. $0 \leq r \leq 1$

h_b Cost of holding for the buyer per unit of time ($ per unit per unit time)

$\hat{h}_b$ Amount of carbon emissions per unit of inventory held by the buyer over some time

h_v Cost of holding for vendor per unit per time. ($ per unit per unit time)

$\hat{h}_v$ For the vendor, per unit of time carbon emissions amount. ($ per unit per unit time)

C_T Per shipment Buyer's fixed shipping cost ($)

$\hat{C}_T$ Fixed carbon emissions Amount per shipment for the buyer ($)

C_t Per unit Variable shipping cost for the buyer. ($)

$\hat{C}_t$ Associated carbon emissions amount per unit shipped for the buyer. ($)

C Per-unit tax rate for carbon emissions. ($)

ω_b Amount of the buyer's carbon emissions per unit of time (unit)

ω_v Amount of vendor's carbon emissions per unit of time. (unit)

$m(\xi)$ The proportion of reduced carbon emissions, as a function of ξ

T_p Length of the first production and shipping quantity from a vendor to the buyer (days)

T_v Vendor's production cycle length. (days)

n Shipment's number from vendor to buyer.

Decision Variables

ξ Investing in technology for reducing carbon emissions. ($)

q q = Q/n, shipped quantity on each occasion from a vendor to the buyer. (unit)

Q Buyer's order quantity. (unit)

T_b A decision variable, buyer's replenishment cycle length. (days)

T_s A decision variable, vendor's period of production length. (days)

3.2 Assumptions:

(1) The proposed model considers a single vendor and a single buyer.

(2) P is finite and exceeds the rate of demand. P > D.

(3) Deterioration rate is time-dependent.

(4) Inflation's effect is considered.

(5) For a substantial quantity of Q units of a commodity, an order is placed by the buyer, who requests that the vendor divide it into n shipments, each containing q units of the items. The buyer covers all shipping costs. Followed by [13].

(6) The buyer generates carbon emissions during operational activities like ordering, inventory holding, transportation, and procurement. Similarly, the vendor contributes to the carbon emissions through operational activities such as material procurement, setup, production, and inventory holding. Followed by [13].

(7) Investing in technology could help reduce carbon emissions, and $m(\xi)(0 < m(\xi) < 1)$ is the reduced carbon emission rate, where $m(\xi)$ is considered as an increasing function of ξ. Followed by [13].

(8) ξ and its associated benefits are divided between the vendor and buyer. Significantly, the buyer contributes a portion of capital investment, denoted by α, and the vendor contributes the remaining proportion 1-α, in which $0 \le \alpha \le 1$. Followed by [13].

(9) Carbon tax and carbon cap-and-trade policy is used.

4. Mathematical Formulation for the model

Mathematical formulation of the model is given in section 4.1 and 4.2.

4.1 Mathematical modeling for buyer.

During the replenishment cycle, the inventory level of the buyer changes at time t as a result of the joint effects of demand and product deterioration. The following differential equation presents the inventory level of the buyers.

$$\frac{dI(t)}{dt} + \theta.t.I(t) = -D, 0 \le t \le T_b. \tag{1}$$

From(1)with boundary condition $I(T_b) = 0$,

$$I(t) = e^{\frac{-\theta t^2}{2}} D\left[(T_b - t) + \frac{\theta}{6}\left({T_b}^3 - t^3\right)\right], 0 \le t \le T_b. \tag{2}$$

And the buyer's cycle length can be determined from (2) by using the condition, q = I(0) as follows

$$T_b = \frac{1}{(2D\theta^2)^{\frac{1}{3}}}\left\{\left[6q\theta + 2\sqrt{9q^2\theta^2 + 8\theta D^2}\right]^{\frac{1}{3}} + \left[6q\theta - 2\sqrt{9q^2\theta^2 + 8\theta D^2}\right]^{\frac{1}{3}}\right\} \tag{3}$$

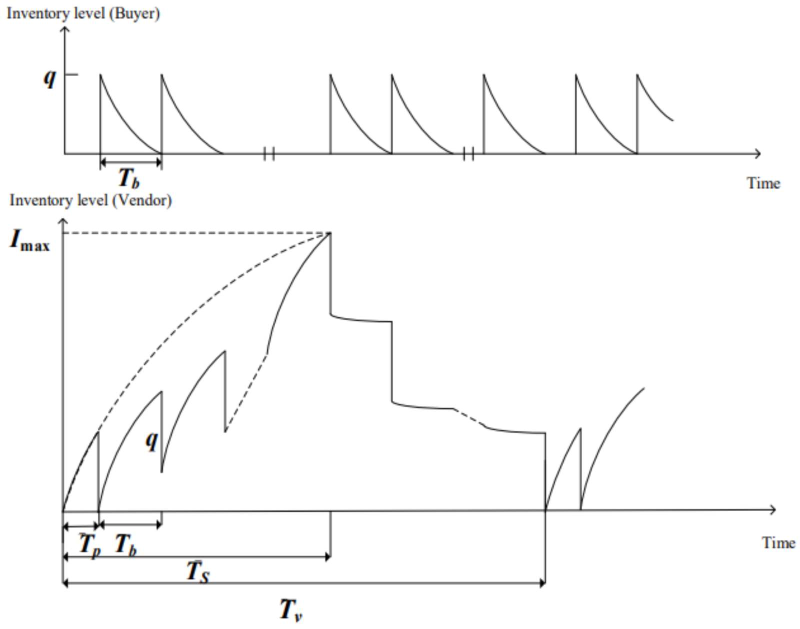

Fig .1 : Represents the vendor buyer inventory level.

The cost for buyer is given by

(a) Sale revenue of the buyer per replenishment cycle is

$$pDT_b e^{-rt} = pe^{-rt} D \frac{1}{(2D\theta^2)^{\frac{1}{3}}} \left\{ \left[6q\theta + 2\sqrt{9q^2\theta^2 + 8\theta D^2}\right]^{\frac{1}{3}} + \left[6q\theta - 2\sqrt{9q^2\theta^2 + 8\theta D^2}\right]^{\frac{1}{3}} \right\}$$

(b) Ae^{-rt} is the ordering cost of the buyer per replenishment cycle.

(c) $vq$is the purchasing cost of the buyer per replenishment cycle.

(d) $(C_T + C_t q)e^{-rt}$is the transportation cost of the buyer per replenishment cycle including fixed and variable cost.

(e) Holding cost of the buyer per replenishment cycle is given by

$$h_b \int_0^{T_b} e^{-rt} I(t)dt = h_b \left[\frac{DT_b^{\,2}}{2} - \frac{rDT_b^{\,3}}{6} + \frac{\theta DT_b^{\,4}}{12} - \frac{rD\theta T_b^{\,5}}{20} - \frac{\theta^2 DT_b^{\,6}}{72}\right] \tag{4}$$

Where $T_b = \frac{1}{(2D\theta^2)^{\frac{1}{3}}} \left\{ \left[6q\theta + 2\sqrt{9q^2\theta^2 + 8\theta D^2}\right]^{\frac{1}{3}} + \left[6q\theta - 2\sqrt{9q^2\theta^2 + 8\theta D^2}\right]^{\frac{1}{3}} \right\}$

(f) Since there are jointly investment to reduce the carbon emission undertaken by the vendor and buyer and α (0 ≤α< 1) is the fraction of the investment of the buyer, therefore αξ is the investment by the buyer in the carbon emission reduction technologies per replenishment cycle.

Sales revenue, purchase costs, ordering costs, shipping costs, holding costs, and carbon emission technology investment all go towards the total profit of the buyer per cycle length.

The total profit of the buyer per unit of time,$TP_b(q,\xi)$ is:

$$TP_b(q,\xi) = \frac{1}{T_b}\left\{pe^{-rT_b}DT_b - \left[Ae^{-rT_b} + vqe^{-rT_b} + (C_T + C_t q)e^{-rT_b} + h_b\left(\frac{DT_b^{\,2}}{2} - \frac{rDT_b^{\,3}}{6} + \frac{D\theta T_b^{\,4}}{12} - \frac{rD\theta T_b^{\,5}}{20} - \frac{D\theta^2 T_b^{\,6}}{72}\right) + \alpha\xi\right]\right\} \rightarrow (5)$$

Where $T_b = \frac{1}{(2D\theta^2)^{\frac{1}{3}}} \left\{ \left[6q\theta + 2\sqrt{9q^2\theta^2 + 8\theta D^2}\right]^{\frac{1}{3}} + \left[6q\theta - 2\sqrt{9q^2\theta^2 + 8\theta D^2}\right]^{\frac{1}{3}} \right\}$ (5)

The buyer's Carbon emission per replenishment cycle, $E_b(q,\xi)$ is

$$E_b(q,\xi) = [1 - m(\xi)]\left\{\hat{h}_b\left(\frac{DT_b}{2} - \frac{rDT_b^{\,2}}{6} + \frac{D\theta T_b^{\,3}}{12} - \frac{rD\theta T_b^{\,4}}{20} - \frac{D\theta^2 T_b^{\,5}}{72}\right) + \frac{1}{T_b}\left(\hat{A}e^{-rT_b} + \hat{v}qe^{-rT_b} + (\hat{C}_T + \hat{C}_t q)e^{-rT_b}\right)\right\} \tag{6}$$

Where $T_b = \frac{1}{(2D\theta^2)^{\frac{1}{3}}} \left\{ \left(6q\theta + 2\sqrt{9q^2\theta^2 + 8\theta D^2}\right)^{\frac{1}{3}} + \left(6q\theta - 2\sqrt{9q^2\theta^2 + 8\theta D^2}\right)^{\frac{1}{3}} \right\}$

4.2. Model formulation for vendor.

The vendor's inventory level changes due to production rate and deterioration of the items during the time interval [0, T_s] and is presented by the following differential equation:

$$\frac{dI_P(t)}{dt} + \theta.t.I_p(t) = P, 0 \le t \le T_s. \tag{7}$$

Solve (7) with boundary condition $I_p(0) = 0$, we have

$$I_p(t) = Pe^{-\frac{\theta t^2}{2}}\left(t + \frac{\theta t^3}{6}\right), 0 \le t \le T_s\,. \tag{8}$$

From Figure 2, $I_p(T_p) = q$, which means that

$$T_p = \left(\frac{1}{P\theta^{\frac{1}{2}}}\right)\left\{\left(q^3\theta^{\frac{3}{2}} + P\sqrt{8P^4 + 6q^4\theta^2 - 12q^2P^2\theta}\right)^{\frac{1}{3}} + \left(q^3\theta^{\frac{3}{2}} - P\sqrt{8P^4 + 6q^4\theta^2 - 12q^2P^2\theta}\right)^{\frac{1}{3}}\right\} + \frac{q}{P} \tag{9}$$

Except this, Due to the item's deterioration, the inventory level of the vendor decreases during the time interval $[T_S,\ T_v]$and given by the following differential equation:

$$\frac{dI_d(t)}{dt} + \theta.t.I_d(t) = 0, \quad T_s \le t \le T_v \tag{10}$$

Solving (10) with condition $I_d(T_v) = nq$, the vendor's inventory level during the time interval $[T_s, T_v]$ is

$$I_d(t) = nqe^{\frac{\theta(T_v{}^2 - t^2)}{2}}, \quad T_s \le t \le T_v\,. \tag{11}$$

From equation (8), equation (11) and $I_p(T_s) = I_d(T_s)$, the value of T_s is given by

$$T_s = \frac{1}{(P\theta^2)^{\frac{1}{3}}}\left\{\left(3nq\theta\, e^{\frac{\theta T_v{}^2}{2}} + \sqrt{9n^2q^2\theta^2 e^{\theta T_v{}^2} + 8P^2\theta}\right)^{\frac{1}{3}} + \left(3nq\theta\, e^{\frac{\theta T_v{}^2}{2}} - \sqrt{9n^2q^2\theta^2 e^{\theta T_v{}^2} + 8P^2\theta}\right)^{\frac{1}{3}}\right\} \tag{12}$$

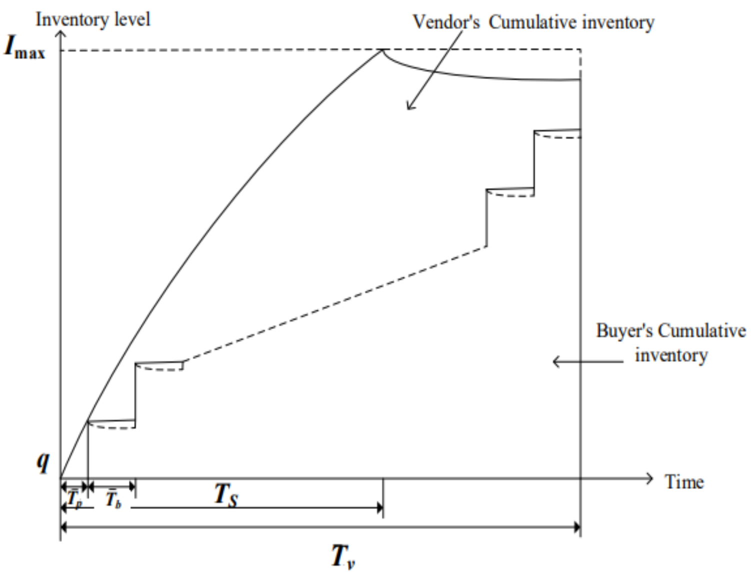

Figure 2. Accumulative inventory of the vendor and buyer.

The following components are involved in the vendor's overall profit per production cycle-

(a) $vQ = vnqe^{-rT_v}$is the sale revenue of the vendor per production cycle.

(b) $Se^{-rT_v}$is the setup cost of the vendor per production cycle.

(c) Production cost of the vendor per production cycle is –

$$cPT_s e^{-rT_v} = \frac{cPe^{-rT_v}}{(P\theta^2)^{\frac{1}{3}}}\left\{\left(3nq\theta\, e^{\frac{\theta T_v^2}{2}} + \sqrt{9n^2q^2\theta^2 e^{\theta T_v^2} + 8P^2\theta}\right)^{\frac{1}{3}} + \left(3nq\theta\, e^{\frac{\theta T_v^2}{2}} - \sqrt{9n^2q^2\theta^2 e^{\theta T_v^2} + 8P^2\theta}\right)^{\frac{1}{3}}\right\}$$

(d) Holding Cost :-

Total holding cost per production Cycle for the vendor is –

$$= h_v\left[\int_0^{T_s} e^{-rt} I_p(t)dt + \int_{T_s}^{T_v} e^{-rt} I_d(t)dt - [qT_b(1+2+3+\ldots\ldots+(n-1))]\right]$$

$$= h_v\left\{nqe^{\frac{\theta T_v^2}{2}}\left(T_v - \frac{rT_v^{\,2}}{2} - \frac{\theta T_v^{\,3}}{6}\right) - \left(nqe^{\frac{\theta T_v^2}{2}}\right)T_s + \left(P + nqr\, e^{\frac{\theta T_v^2}{2}}\right)\frac{T_s^{\,2}}{2} + \left(\frac{-Pr}{3} + \frac{nq\theta\, e^{\frac{\theta T_v^2}{2}}}{6}\right)T_s^{\,3} - \frac{P\theta T_s^{\,4}}{12} - \frac{P\,r\,\theta\, T_s^{\,5}}{30} - \frac{P\theta^2 T_s^{\,6}}{72} - \frac{n(n-1)qT_b}{2}\right\}$$

Where

$$T_s = \frac{1}{(P\theta^2)^{\frac{1}{3}}}\left\{\left(3nq\theta\, e^{\frac{\theta T_v^2}{2}} + \sqrt{9n^2q^2\theta^2 e^{\theta T_v^2} + 8P^2\theta}\right)^{\frac{1}{3}} + \left(3nq\theta\, e^{\frac{\theta T_v^2}{2}} - \sqrt{9n^2q^2\theta^2 e^{\theta T_v^2} + 8P^2\theta}\right)^{\frac{1}{3}}\right\}$$

and $T_b = \frac{1}{(2D\theta^2)^{\frac{1}{3}}}\left\{\left(6q\theta + 2\sqrt{9q^2\theta^2 + 8\theta D^2}\right)^{\frac{1}{2}} + \left(6q\theta - 2\sqrt{9q^2\theta^2 + 8\theta D^2}\right)^{\frac{1}{2}}\right\}$

(e) As the investment is a collaborative effort between the vendor and the buyer (1-α) (where 0 ≤α < 1) is the portion of the vendor's investment .So
(1-α)ξ calculates the vendor's investment for carbon emission reduction technology per production cycle .

Accordingly, Vendor's Total Profit per Unit of Time $TP_v(T_v, q, n, \xi)$ is

$$= \frac{1}{T_v}\{vnqe^{-rT_v} - Se^{-rT_v} - cPT_s e^{-rT_v} - h_v\left[\int_0^{T_s} e^{-rt} I_p(t)dt + \int_{T_s}^{T_v} e^{-rt} I_d(t)dt - [qT_b(1+2+\ldots.+(n-1)]\right] - (1-\alpha)\xi\}$$

$$= \frac{1}{T_v}\{vnqe^{-rT_v} - Se^{-rT_v} - cPT_s e^{-rT_v} - h_v\left\{nqe^{\frac{\theta T_v^2}{2}}\left(T_v - \frac{rT_v^{\,2}}{2} - \frac{\theta T_v^{\,3}}{6}\right) - nqe^{\frac{\theta T_v^2}{2}}T_s + \left(P + nqr\, e^{\frac{\theta T_v^2}{2}}\right)\frac{T_s^{\,2}}{2} + \left(\frac{-Pr}{3} + \frac{nq\theta\, e^{\frac{\theta T_v^2}{2}}}{6}\right)T_s^{\,3} - \frac{P\theta T_s^{\,4}}{12} - \frac{P\,r\,\theta T_s^{\,5}}{30} - \frac{P\theta^2 T_s^{\,6}}{72} - \frac{n(n-1)qT_b}{2}\right\} - (1-\alpha)\xi\} \quad (13)$$

The Vendor's Carbon Emission Per unit time is

$$E_v\left(T_v, q, n, \xi\right) = \left(\frac{1-m(\xi)}{T_v}\right)\left\{\hat{S} + \hat{c}PT_s + \hat{h}_v\left(nqe^{\frac{\theta T_v^2}{2}}\left(T_v - \frac{rT_v^2}{2} - \frac{\theta T_v^3}{6}\right) - nqe^{\frac{\theta T_v^2}{2}}T_s + \left(P + nqre^{\frac{\theta T_v^2}{2}}\right)\frac{T_s^2}{2} + \left(\frac{-Pr}{3} + \frac{nq\theta\, e^{\frac{\theta T_v^2}{2}}}{6}\right)T_s^3 - \frac{P\theta T_s^4}{12} - \frac{P\,r\,\theta T_s^5}{30} - \frac{P\theta^2 T_s^6}{72} - \frac{n(n-1)qT_b}{2}\right\} - (1-\alpha)\xi\right\} \quad (14)$$

where

$$T_s = \frac{1}{(P\theta^2)^{\frac{1}{3}}}\left\{\left(3nq\theta\, e^{\frac{\theta T_v^2}{2}} + \sqrt{9n^2q^2\theta^2 e^{\theta T_v^2} + 8P^2\theta}\right)^{\frac{1}{3}} + \left(3nq\theta\, e^{\frac{\theta T_v^2}{2}} - \sqrt{9n^2q^2\theta^2 e^{\theta T_v^2} + 8P^2\theta}\right)^{\frac{1}{3}}\right\}$$

and

$$T_b = \frac{1}{(2D\theta^2)^{\frac{1}{3}}}\left\{\left(6q\theta + 2\sqrt{9q^2\theta^2 + 8\theta D^2}\right)^{\frac{1}{2}} + \left(6q\theta - 2\sqrt{9q^2\theta^2 + 8\theta D^2}\right)^{\frac{1}{2}}\right\}$$

4.3 Model under Carbon cap-and-trade policy

If the carbon emissions from the buyer and vendor combined exceed the specified boundary (capped amount), represented by ω_b\$andω_v\$ respectively, any additional emissions beyond the boundary necessitate the purchase of carbon allowances at the market price p_c**.** Thus, the cost of exceeding the boundary is incorporated into the profit calculation. Conversely, if the combined emissions remain below the specified boundary, the surplus allowances can be sold in the market price p_c**,** generating additional profit.

The total profit per unit of time ,JTPCC(q, ξ), under the carbon cap-and-trade policy is

$$\boldsymbol{JTP_{cc} = JTP - p_c[E_b + E_c - \omega_b - \omega_v]}$$

The policy's aims to optimize the order quantity, shipment quantity, and technology investment in order to minimize the carbon emissions within the framework of the carbon cap-and-trade policy .The optimization is driven by the goal of maximizing the joint profit function JTPCC(q, ξ).

4.4 model with Carbon tax policy

The implementation of carbon taxes by external regulatory bodies can incentivize companies to take environmental costs into account. These taxes are typically structured such that enterprises are required to pay a specific amount (in C) for each unit of carbon emissions. As a result, the enhanced model incorporating the carbon tax policy is represented by JTPCT (q, ξ).

$$\boldsymbol{JTP_{CT} = JTP - C[E_b + E_c]}$$

The policy aims to optimize the order quantity, shipment quantity, and technology investment to minimize carbon emissions while maximizing the joint profit function JTPCC (q, ξ). Similar to the carbon cap-and-trade scenario, determining the closed form of q,ξ, and directly assessing their concavity is challenging. Therefore, we conducted a numerical analysis to validate the concavity under the carbon tax regulation.

5. Numerical Illustrations

Example 1: Consider a business situation for buyer in which input parameters are in appropriate units:

$p = 50\$, D = 100 thousand\ , \theta = 0.01, v = 0.2\$, A = 2\$, h_b = 0.5\$, r = 0.03\ i.e., 3\%, C_T = 50\$, C_t = 3\$, \alpha = 0.5, v^{\wedge} = 0.1\$, A' = 1.5\$, h_b^{\wedge} = 0.3\$, C_T^{\wedge} = 30\$, C_t^{\wedge} = 2\$, m(\xi) = 0.4$

Optimal Result

Total profit for buyer $TP_b = 7104.29\$, T_b = 2.16086 days\ , T_p = 20.7038 days, \xi = 100, q = 319.114 units, E_b = 223.543 unit$

Example 2: Consider a business situation for vendor in which input parameters are in appropriate units:

$$v = 20\$\ , c = 1\$\ , D = 100\ thousand, P = 150\ thousand, \theta = 0.01, T_p = 20.7038\ Days, \xi = 100\ \$\ , S = 2\$, h_v = 3\$, r = 0.03\ i.e. 3\%, n = 2, C_T = 50\$,$$

$$q = 227.291 units$$

Optimal Results

Total Profit for Vendor

$TP_v = 21600.6\$, T_v = 25.02552\ days, T_s = 11.4152 days, E_v = 390.206\ unit$

Total profit for Supply Chain

$$\boldsymbol{JTP_{cc}} = TP_b + TP_v = 7104.29 + 21600.6 = 28704.89\$$$

Total Profit under cap-and-Trade policy

$$\boldsymbol{JTP_{cc} = JTP - p_c[E_b + E_c - \omega_b - \omega_v]}$$

$$\mathrm{JTP_{cc}} = 28704.89 - 0.3[223.543 + 390.206 - 5000 - 5000]$$

$$\mathrm{JTP_{cc}} = 31520.7623\$$$

Total profit under Carbon Tax policy

$$\boldsymbol{JTP_{CT} = JTP - C[E_b + E_c]}$$

$$\mathrm{JTP_{CT}} = 28704.89 - 0.1(223.543 + 390.206)$$

$$\mathrm{JTP_{CT}} = 28643.5151\ \$$$

6. Concavity

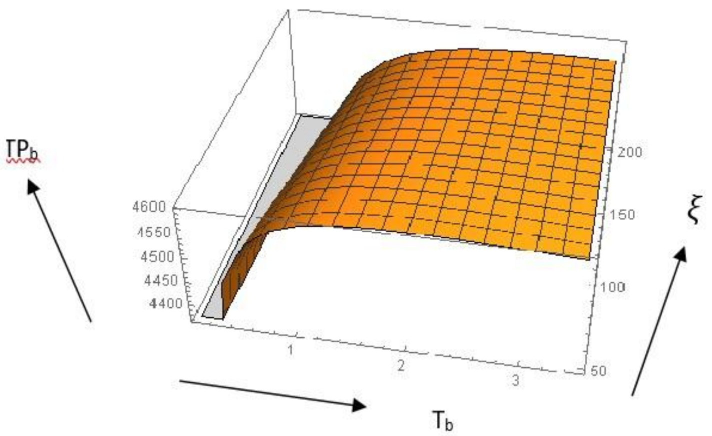

Fig.3: Concavity between Buyer Total Profits TP_b, and ξ.

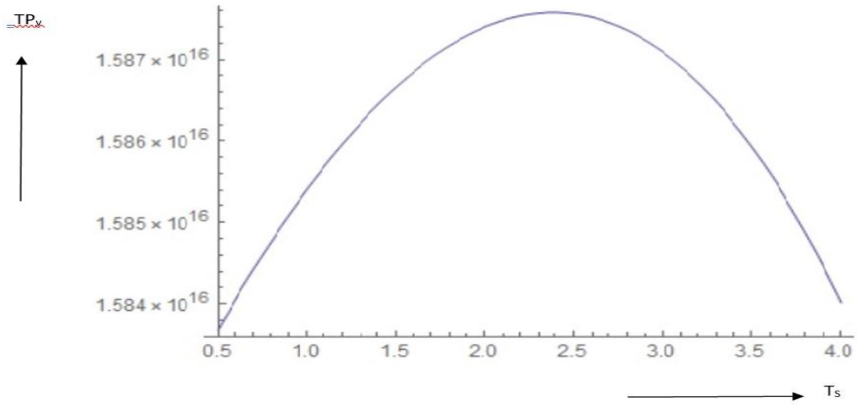

Fig. 4: Concavity between Vendor Total profit and T_S.

7. Sensitivity Analysis

Table 1 provides a sensitivity analysis for the Buyer.

Table 1: Sensitivity analysis of buyer for several parameters.

Parameters	%Change	T_b	q	E_b	TP_b
D	+20%	1.94763	346.162	264.746	8533.08
	+10%	2.04625	332.944	244.183	7818.59
	-10%	-	-	-	-
	-20%	2.4594	289.223	181.979	5676.37
p	+20%	2.16086	319.114	223.543	8604.29
	+10%	2.16086	319.114	223.543	7854.29
	-10%	2.16087	319.115	223.543	6354.29
	-20%	-	-	-	-
θ	+20%	2.22212	326.783	223.366	7105.22
	+10%	2.19044	322.824	223.454	7104.75
	-10%	2.13315	315.67	223.632	7103.84
	-20%	2.10709	312.338	223.722	7103.4
v	+20%	2.16506	319.714	225.335	7101.35
	+10%	-	-	-	-
	-10%	2.15879	318.817	222.647	7105.76
	-20%	2.15672	318.521	221.751	7107.23
r	+20%	2.17534	321.184	223.507	7104.5
	+10%	2.16804	320.141	223.525	7104.4
	-10%	2.15383	318.108	223.561	7104.18
	-20%	2.14691	317.117	223.535	7104.08

A	+20%	2.16543	319.767	223.784	7104.15
	+10%	2.16315	319.441	223.727	7104.22
	-10%	2.15858	318.787	223.614	7104.36
	-20%	2.15629	318.46	223.557	7104.43
h_b	+20%	1.90763	282.689	227.094	7095.3
	+10%	-	-	-	-
	-10%	2.34014	344.657	221.618	7109.27
	-20%	-	-	-	-
α	+20%	2.3101	340.392	230.673	7099.82
	+10%	2.23621	329.875	229.984	7102.02
	-10%	2.08393	308.089	228.605	7106.65
	-20%	2.00524	296.775	227.917	7109.09
C_t	+20%	-	-	-	-
	+10%	2.19243	323.627	247.295	7074.76
	-10%	2.13159	314.923	211.301	7133.83
	-20%	2.10431	311.013	193.314	7163.39
C_T	+20%	-	-	-	-
	+10%	2.20626	325.602	230.524	7102.92
	-10%	2.11491	312.533	228.029	7105.69
	-20%	2..06834	305.85	226.726	7107.13

Sensitivity Analysis for Vendor is given in the table 2.

Table 2: Sensitivity analysis of buyer for several parameters.

Parameter	**%Change /value**	T_s	E_v	TP_v
P	+20%	11.4043	3915.07	21475
	+10%	11.4099	3890.43	21537.8
	-10%	11.4203	3839.87	21663.4
	-20%	11.4252	3814.11	21726.2
θ	+20%	10.6317	-2891.36	51813.7
	+10%	11.0027	1559.91	33715.5
	-10%	11.8751	4862.28	13549.8
	-20%	12.384	5088.3	8242.51
v	+20%	11.4152	3865.31	21673.3
	+10%	11.4152	3865.31	21636.9
	-10%	11.4152	3865.31	21564.3
	-20%	11.4152	3865.31	21528

n	1	11.1353	2892.17	4347.62
	2	11.4152	3865.31	21600.6
	3	11.4432	-6138.4	73921.4
	4	11.4515	-60145.3	225637
	5	11.4547	-271570	658912
r	+20%	10.9525	1855.2	23534.1
	+10%	11.1809	2858.09	22563.5
	-10%	11.6555	4876.8	20645.9
	-20%	11.9018	5892.36	19699.6
h_v	+20%	11.4222	4650.95	25862.2
	+10%	11.419	4258.1	23731.4
	-10%	11.4106	3484.64	19469.8
	-20%	11.4048	3079.8	17339.1

8. Observations

8.1 Observation from the table 1.

(a) When the demand rate increases, the cycle length decreases while the order quantity, carbon emission, and total profit increase.

(b) When the selling price increases, the total profit increases while cycle length and order quantity decrease, and the carbon emission remains the same.

(c) When the deterioration rate increases, the total profit and cycle length slightly increase while order quantity increases and carbon emission slightly decreases.

(d) When buyer purchasing price increases, the total profit decreases while carbon emission, order quantity and cycle length increase.

(e) When the inflation rate increases, the cycle length, order quantity and total profit increase while carbon emission fluctuates.

(f) When the buyer's ordering cost increases, the cycle length, order quantity, and carbon emission increase while total profit slightly decreases.

(g) When the holding cost increases, the carbon emission increases while the cycle length, order quantity and total profit decrease.

(h) When the buyer percentage in the investment to reduce the carbon emission increases, then the carbon emission, cycle length and order quantity increase while the total profit decreases.

(i) When the buyer's shipping cost and variable shipping cost increase, the cycle length, order quantity and carbon emission increase while the total profit decreases.

8.2 Observation from the table 2.

(a) When the production rate increases, the cycle length and the total profit decrease while the carbon emission increases.

(b) When the deterioration rate increases, the carbon emission and cycle length decrease while total profit increases.

(c) When the selling price increases then the cycle length and carbon emission remain same while the total profit increases.
(d) When the inflation rate increases, cycle length and carbon emission decrease while the total profit increases.
(e) When the holding cost increases, then the cycle length, carbon emission and total profit increases

9. Managerial Insight

(a) If the selling price decreases up to 10%, the model will not work.
(b) When the deterioration rate decreases by up to 20%, then carbon emission is minimal.
(c) When the purchasing price increases by 10 %, the model will not work.
(d) Buyer should keep the holding cost minimum because minimum the holding cost higher will be profit and lower will be carbon emission.

10. Conclusion and Future Extension

A sustainable vendor-buyer inventory model for disaster relief deteriorating items under inflation is proposed by this research, including technical cooperation on investment to reduce emissions of GHG. The proposed model effectively reduces the overall cost of the inventory systems to reduce GHG emissions and achieve long-term sustainability. In order to reduce overall costs and the supply chain's carbon footprint, the model aims to incorporate a sustainable investment strategy into the inventory system. The findings reveal that the suggested model can successfully reduce the overall cost of the inventory system while also reducing GHG emissions. Also, under the cap-and-trade policy, we get maximum profit. Sustainable investment strategy is found to be effective in reducing the S.C.'s carbon footprint, and saving resulting from the investment are shared between the buyer and vendor for relief in disaster. From the sensitivity analysis, we observe that inflation positively impacts carbon emissions. This study can be extended further with three echelon supply chain models, trade credit policies, etc.

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