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EOQ Model for Deteriorating Item with Expiration Dates Under Advance-Cash-Credit Payment Policy

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ABSTRACT

Today, in the era of competition and market globalization, there is pressure on companies to increase sales and revenue, so managers try to maximize the company's profit by using maximum cash flow. In today's business transactions, the company asks the manufacturer for payment as its sales increase through an advance-cash-credit (ACC) payment plan. This research presents a selling price-dependent demand inventory model with deteriorating items with expiration dates under an advanced cash credit payment policy. The main objective of the proposed model for the retailer is to maximize profit per unit of time and determine the optimal cycle time and order quantity. Numerical examples have been studied to illustrate the value of the proposed model. Sensitivity analysis is used to investigate the effects of parameter changes brought on by market uncertainty.

1. Introduction

The means of inventory is a stock of usable products that can be used in the future. All resources can be used to minimize investment. It is very important to estimate future demand because a shortage of products on the market can collapse any business or market system. To avoid such discrepancies in business, it is important to keep in mind the available status of raw materials or the demand status of finished goods. To develop a sustainable inventory model, it is important to consider the natural environment as perishable products such as food, dairy products, meat, vegetables, etc. are regularly emitted into the environment. The spoilage rate of a product increases over time and reaches 100% by the time it is finished. As the expiry date of the product approaches, its freshness decreases or it can be said that the product is no longer as fresh as it was at the time of beginning of production. Due to the above the demand for the product also gradually decreases and finally the demand becomes zero on the expiry date. By looking at the expiration date on the product, it can be easily identified how fresh the product is or when it will expire. From this, it may be inferred that the product's expiration date influences the customer's decision to buy it. Expiration dates are a critical factor in inventory modelling for consumable products such as packaged foods and drinks, medicine, etc., as these products become ineffective after expiration. If such products are not sold before their expiration date, retailers will suffer a loss of their investment. It is true that competition has also increased in this rapidly growing market. Attracting customers and converting them into buyers is a very challenging task for any businessman. In this competition, various marketing strategies are being created and

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adopted in the market to increase sales and convert customers into real customers. Among these marketing strategies, the idea of post-payment or pre-payment is being adopted by most of the business organizations nowadays to increase the demand and profits. In the post-payment concept, the supplier gives the buyer a fixed time after supply to settle the account without interest. But there may be risks from the buyer's side, such as cancelling the order, not making payment on time, or switching to another firm. On the other hand, in the prepayment concept, The Buyer shall make partial or full payment of the total purchase price to the Supplier prior to supply. This type of trading behaviour reduces the risk of order cancellation. Apart from the supplier, the customer also gets the benefit of its business dealings in the form of additional discounts or in some other form. The field of inventory management has received considerable attention from researchers, particularly in the context of deteriorating items. To study the dynamic nature of such items, a variety of inventory models have been developed, incorporating diverse payment mechanisms, including advance, cash, and credit payments.

1.1 Literature Review

EOQ Model for Deteriorating Items

Since its inception in 1915, the square roots formula has been the basis of economic order quantity (EOQ) calculations, primarily assuming constant demand rates. Over time, researchers have explored mathematical models for inventory management, including scenarios with fixed or variable demand and cases of deterioration, such as in fashion goods, as notably examined by Whitin (1958). Ghare and Schrader (1963) further extended these models to account for perishable goods, while Shah and Jaiswal (3) introduced an order-level inventory model. Aggarwal (1977) later refined this model, rectifying previous errors. However, these early models often assumed a constant rate of spoilage. Subsequent research has delved into more complex scenarios. For instance, Kumar *et al.* (2011) investigated an inventory model for deteriorating items with quadratic demand and permissible payment delays. Taleizadeh *et al.* (2013) analyzed EOQ models with multiple partial prepayments under various conditions. Taleizadeh (2014) extended this work to incorporate deteriorating item EOQ models with multiple partial prepayments and shortages. Mahata and De (2017) developed lot-sizing guidelines for a store selling depreciating items to clients with credit risk. Singh and Kumar (2018) presented an inventory model for deteriorating items with demand sensitive to the selling price and credit period. Mishra *et al.* (2022) explored a carbon tax and cap policy using an inventory model for decaying products based on carbon emissions, considering the entire supply chain. Recent contributions. Include Kumar *et al.* (2023), who proposed a fuzzy inventory model for perishable goods where demand is taken as exponential and preservation technology. Mondal *et al.* (2023) introduced an inventory model for perishable goods where the demand is continuous linear function of time with considering trade credit. Yang (2023) developed an inventory model with fluctuating demand for deteriorating items. An economic ordering quantity model with variable holding costs, trade credit, with demand function is multivariate was presented by Yadav *et al.* (2023) for deteriorating products. By combining partial backordering, social responsibility, and environmental impact into a fuzzy based EOQ model for decaying products, Kumar *et al.* (2023) solved sustainability challenges. Sharma *et al.* (2024) established a two-warehouse green inventory system that took into account energy usage, carbon emissions, demand is the function of time, and variable carrying costs.

Inventory Problems with Advance-Cash-Credit Payment.

In the aspect of inventory management, Zhang (1996) explored optimal advance payment schemes, highlighting their dependency on factors such as demand rates, prepayment costs, and holding costs. Teng *et al*. (2016) used an EOQ model that allowed for partial backordering and lost sales under partial prepayment to address the steady increase in deterioration rates with the expiration date. Taleizadeh (2014) presented an advance-cash payment model for evaporating items with partial backordering, building on earlier research by Taleizadeh *et al.* (2013) and implemented in a real petrol station case study. Zhang *et al.* (2014) examined the best ordering practices under different prepayment scenarios by examining inventory models that included purchase and prepayment discounts. An EOQ model with hybrid partial payments was created by Lashgari *et al.* (2016), taking into account the possibilities of full backordering, partial backordering, and no shortage. Wu *et al.* (2018) investigated advance-cash-credit payment plans in which the demand was linked to selling prices. They examined the friction of no shortage and ideal cycle times while utilizing different payment due dates. An inventory model that incorporates the best lot-sizing and pricing techniques with upstream and downstream trade credit concerns was developed by Feng and Chan (2019). Advance cash credit payment methods were included into the exponential price-dependent demand (EPQ) model by Tsao *et al.* (2019). Li *et al.* (2019) examined advance-cash-credit payment schemes with declining demand while examining inventory models for profit maximization. Research on inventory models for deteriorating objects that included advance payment systems and preservation technologies was done by Khan *et al.* (2020) and Chaudhary *et al.* (2020). A two-warehouse inventory model with advanced payment procedures was created by Mashud *et al.* in 2020. whereas Khakzad and Gholamian (2020) presented an inventory model that took into account the impact of inspection times during replenishment periods on average rates of deterioration. While Nath *et al.* (2021) concentrated on profit maximization for degrading items under pre-payment policies, Yang, H.-L. (2021) examined an integrated inventory model utilizing advance-cash-credit payment systems. De (2021) looked into how advance payment with discount options affected inventory models for degrading items. Duary *et al.* (2022) provided inventory models with advance and delay in payments, accounting for capacity limitations and partially backlogged shortages. Jain *et al.* (2022) examined advance payment policies for decaying commodities where the demand is the function of price. In order to minimize the costs in EOQ models for degrading items where the demand is the function of time and partial backlog, Rahman, M.S. *et al.* (2022) suggested an advance payment strategy. The best inventory strategies for time-varying deteriorating products and non-instantaneous deteriorating items, respectively, were investigated by Das *et al.* (2023) and Choudhury *et al.* (2024), taking into account a number of variables such trade credit, selling price, and demand functions.

Inventory Problems with Expiration Dates

An EOQ model designed for continually degrading products with finite lifespans was proposed by Wang *et al.* (2014). For products with maximal lifespans, Yang (2020) presented a supplier-retailer-customer chain model that considers demand rates and expiration dates. Liao *et al.* (2020) considered faulty quality and allowable payment delays while establishing an ideal ordering policy inside an EOQ framework for non-instantaneously deteriorating commodities. An EOQ model under two-level partial trade credit for time-varying degrading items with expiration dates was covered by Mahata *et al.* (2020). In a two-warehouse environment, Gupta *et al.* (2020) investigated a cost-minimization approach to retailer ordering practices, which included partial backlogging and allowable payment delays for time-varying degrading commodities. In a two-level trade credit arrangement, Mahato *et al.* (2021) addressed the best inventory management strategy for deteriorating items with expiration dates and dynamic demand. For deteriorating items with time-sensitive demand, cash discount limitations, and acceptable payment delays, Tripathi (2021) presented a novel EOQ structure. While Supakar *et al.* (2023) used different particle swarm optimization variants to analyze a trade-credit EOQ model for deteriorating items with inventory-dependent demand and expiration dates, Liao *et al.* (2021) developed an inventory model for non-instantaneous deteriorating items under a hybrid payment policy. Using Grey Wolf Optimizer, Alrasheedi (2023) proposed a credit policy technique for green items with expiration date-dependent deterioration. A restricted storage capacity inventory model was developed by Yang (2023) to account for deferred cash flow, downstream partial trade credit transactions, and decaying products with variable demand. A two-warehouse inventory model with trade credit policies and time-varying holding costs under quantity discounts was described by Momena *et al.* in 2023.

1.2 Structure of Study

In this study, we looked at an inventory model with an advance cash-credit (ACC) payment policy, in which the spoiling rate of items rises with time and reaches 100% at the expiration date. The novelty of this paper is proved by introducing the Advance-Cash-Credit Payment Policy with expiration date of the product and proving that the EOQ model attains optimal value. The remainder of the paper is arranged as follows after the introduction: Assumptions and notation are described in Section 2. Section 3 gives the formulation of the model and all calculations related to the profit function. Two numerical examples are taken and given in Section 4 to verify the model. Section 5 gives a sensitivity analysis, which shows the impact of changing the values of various variables on the total profit and decision variables. Finally, Section 6 offers the model's conclusion and suggestions for the direction of future research.

2. Preliminaries

To ensure clarity, consistency, and compatibility with existing literature, we will use the following standard notations and assumptions:

2.1 Notations:

a, b Demand parameters
p Selling price per unit in (₹/Unit)
θ Rate of deterioration at time t
m Maximum life time of the item in years
L Time period in year in which advance payment is to be made
M length of credit period offered by supplier to retailer in year
O Ordering cost per order (₹)
c Procurement cost per unit (₹/Unit)
α Part of the purchase price that must be paid in advance of delivery, $0 \leq \alpha \leq 1$
β Payment of a portion of the purchase price at the time of delivery, $0 \leq \beta \leq 1$
γ portion of the purchase price that the supplier gives the retailer on credit for a period of time $[0, M]$, $0 \leq \tau \leq 1$ and$\alpha + \beta + \tau = 1$
h Holding cost per unit per unit time (₹/Unit)
d Deterioration cost per unit per unit time (₹/Unit)
I_e Rate of interest earned by retailer per ₹ per year
I_c Interest rate that the supplier charges each year in ₹
T Length of replenishment cycle time in years $T \leq m$

Q Ordering quantity
r Interest rate that the financial institution charges when $r \leq I_c$
$I(t)$ Inventory level at any time t

2.2 Assumptions

- A single item is considered
- There is single-vendor and single-buyer for this model.
- The demand rate is the function of selling price and defined as $D(p) = a - bp$, where a and b are non-negative constants with $0 < p \leq \frac{a}{b}$ so that the demand must be non-negative.
- The inventory system under study deals with deteriorating items having their particular expiration dates.
- The rate of deterioration is a function of time that increases and is described as $\theta(t) = \frac{1}{1+m-t}$ where m is the maximum life time and $0 \leq t \leq T \leq m$. The item deteriorates entirely as t approaches the maximum lifetime of m, as indicated by the degradation rate $\theta(t)$ tends to 1.
- Replenishment rate is instantaneous and shortages are not allowed.
- The supplier has a special payment policy that requires the retailer to pay a portion of the procurement cost in advance at the time $(t = -L)$ when the retailer places an order, another portion at the time $(t = 0)$ when the supplier delivers the ordered items, and a credit period $[0, M]$ during which the retailer can pay the remaining τ portion of the procurement cost.
- The retailer takes out a loan from a financial institution to pay the advance and cash payments for the purchase's cost. The retailer repays the loan amount plus interest to the financial institution at the end of each repayment cycle.

3. Mathematical Model

This section develops the advance-cash-credit payment scheme as an inventory model for perishable goods. The retailer purchases Q units of any perishable good at time $T = -L$ which he receives at time $t = 0$. As shown in Figure 1, after receiving the consignment, Due to the combined effects of item deterioration and demand, the inventory level gradually drops and reaches zero at $t = T$.

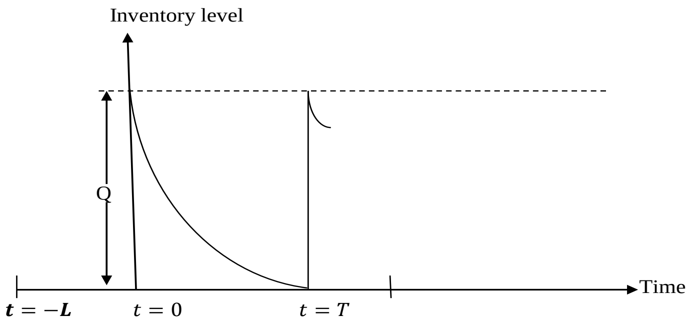


Figure 1: The behavior of inventory levels over time

The differential equations governing the inventory system during the interval $[0.T]$

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -D(p)\ ,0 \leq t \leq t \quad (1)$$

The boundary conditions are $I(0) = Q$ and $I(T) = 0$

Using the boundary condition, the solution of the differential equation (1) is given by

$$I(t) = (a - bp)(1 + m - t)In\left(\frac{1+m-t}{1+m-T}\right), 0 \le t \le t \tag{2}$$

The quantity of items ordered by the retailer in each replenishment cycle is determined by

$$Q = I(0) = (a - \text{bp})\ (1 + m)\log\left(\frac{1+m}{1+m-T}\right) \tag{3}$$

The following are the components in the retailer total profit function

Ordering Cost: The ordering cost associated with each order is

$$\text{OC} = O \tag{4}$$

Sales Revenue: The revenue generated by sales is given by

$$\text{SR} = p\int_0^T D(t)\, dt = p\ (a - bp)\ T \tag{5}$$

Procurement Cost: Retailers incur procurement costs, including purchasing, screening and transporting goods for sale. The formula for calculating these costs:

$$\text{PC} = c\int_0^T I(t)\ \ dt = c(a - bp)\int_0^T (1 + m - t)\ log\left(\frac{1+m-t}{1+m-T}\right) dt$$

$$\text{PC} = \frac{c(a-bp)}{4}\left\{(1 + m)^2[2\, log\left(\frac{1+m}{1+m-T}\right) - (1 + m - T)]\ + (1 + m - T)^3\right\} \tag{6}$$

Procurement costs under the advance-cash-credit payment scheme can be paid as:

(3.1) The amount of advancement, at the time of placing the order is given by

Advanced Payment

$$= \alpha PC = \frac{\alpha c(a-bp)}{4}\left[(1 + m - T)^3 + (1 + m)^2[2\, log\left(\frac{1+m}{1+m-T}\right) - (1 + m - T)]\ \right] \tag{7}$$

(3.2) The amount of cash payment at time of delivery of the goods consignment

Cash Payment

$$= \beta PC = \frac{\beta c(a-bp)}{4}\left[(1 + m - T)^3 + (1 + m)^2[2\, log\left(\frac{1+m}{1+m-T}\right) - (1 + m - T)]\ \right] \tag{8}$$

(3.3) the amount of credit payment at time $t = M$

Credit Payment

$$= \gamma PC = \frac{\gamma c(a-bp)}{4}\left[(1 + m - T)^3 + (1 + m)^2\left[2\, log\left(\frac{1+m}{1+m-T}\right) - (1 + m - T)\ \right]\right] \tag{9}$$

Holding Cost: The holding cost per replenishment cycle is

$$\text{HC} = h\int_0^T I(t)\, dt = h(a - bp)\int_0^T (1 + m - t)\ log\left(\frac{1+m-t}{1+m-T}\right) dt$$

$$\text{HC} = \frac{h(a-bp)}{4}\left[(1+m)^2[2\,log\left(\frac{1+m}{1+m-T}\right) - (1+m-T)] \;+ (1+m-T)^3\right] \tag{10}$$

Deterioration Cost: The deterioration cost per replenishment cycle is

$$\text{DC} = d\int_0^T I(t)\,dt = d(a-bp)\int_0^T (1+m-t)\,log\left(\frac{1+m-t}{1+m-T}\right)dt$$

$$=\frac{d(a-bp)}{4}\left(2(1+m)^2 \log\left(\frac{1+m}{1+m-T}\right) + T(T-2-2m)\right) \tag{11}$$

Loan Interest: Retailers often secure loans from financial institutions to cover advance and cash payments associated with procurement costs. The interest charged by a financial institution is calculated by

$$\text{IC} = [\alpha r(L+T) + \beta rT] \times PC$$

$$\text{IC} = \frac{c(a-bp)\,r(\alpha L + (\alpha+\beta)T)}{4}\left[2(1+m)^2\,log\left(\frac{1+m}{1+m-T}\right) + (1+m-T)^3 - (1+m)^2(1+m-T)\right] \tag{12}$$

The allowed delay period for credit payment by the supplier and the length of the replenishment cycle can lead to following two potential cases:

Case 1: When $\text{T} \leq \text{M} \leq \text{m}$ i.e. the retailer has more time to pay for the goods than it takes to sell them.

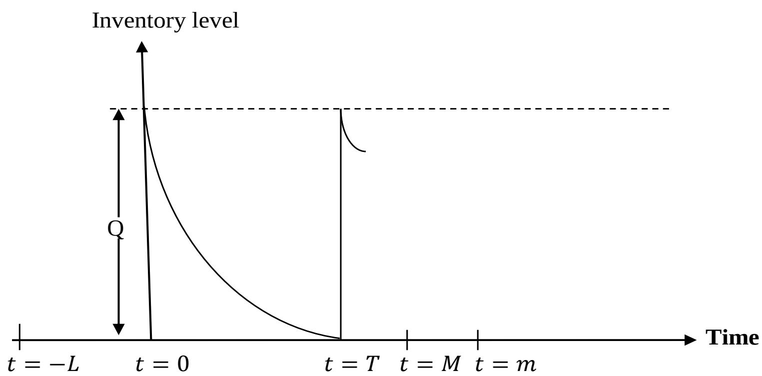


Figure 2: Pictorial representation of the case when $\text{T} \leq \text{M} \leq \text{m}$

The retailer begins selling the products after receiving them from the supplier, and the proceeds to deposited the sales revenue into an interest-bearing account. The retailer earns interest on the sales revenue During the interval between receiving the goods and selling all the stock. The following formula is used to compute interest:

$$\text{IE}_1[0,\text{T}] = pI_e\int_0^T D(p)\,t\,dt = \frac{1}{2}pI_e(a-bp)\,T^2 \tag{13}$$

Total amount in interest bearing account at the time$t = T$ is given by

$$A = \text{SR} + \text{IE}_1[0,\text{M}] = \frac{p\,(a-bp)}{2}\;(2T + I_eT^2) \tag{14}$$

The remaining amount in an interest-bearing account after paying off a loan with interest is calculated as

$U = A - (\alpha + \beta)\mathrm{PC} - \mathrm{IC}$

$$U = \frac{(a-bp)}{4}\left[2p(2T + I_e T^2) - c\{(\alpha+\beta)(1+rT) + \alpha rL\}\left\{2(1+m)^2 \log\left(\frac{1+m}{1+m-T}\right) + (1+m-T)^3 - (1+m)^2(1+m-T)\right\}\right] \quad (15)$$

In the time span $[T.M]$, the retailer also earns the interest on the remaining amount U and pays 'credit payment' to the supplier at time $t = M$. The interest earned on amount U is computed by:

$\mathrm{IE}_1 = UI_e(M - T)$

$$= \frac{I_e(a-bp)(M-T)}{4}\left[2p(2T + I_e T^2) - c\{(\alpha+\beta)(1+rT) + \alpha rL\}\left[2(1+m)^2 \log\left(\frac{1+m}{1+m-T}\right) + (1+m-T)^3 - (1+m)^2(1+m-T)\right]\right] \quad (16)$$

The retailer has sufficient money in interest-bearing account to pay its credit obligations at time $t = M$. Hence, the retailer does not need to incur additional interest charges when making credit payments to the supplier.

$$\mathrm{IC}_1 = 0 \quad (17)$$

Therefore, the total profit earned by the retailer during each replenishment cycle can be expressed as:

$$Z_1 = \frac{1}{T}\{\,\mathrm{SR} - \mathrm{OC} - \mathrm{PC} - \mathrm{HC} - \mathrm{DC} - \mathrm{IC} - \mathrm{IC}_1 + \mathrm{IE}_1\} \quad (18)$$

Case 2: When $M < T \leq m$ i.e. A shorter credit period M than the replenishment cycle time T is provided by the supplier to the retailer for payment.

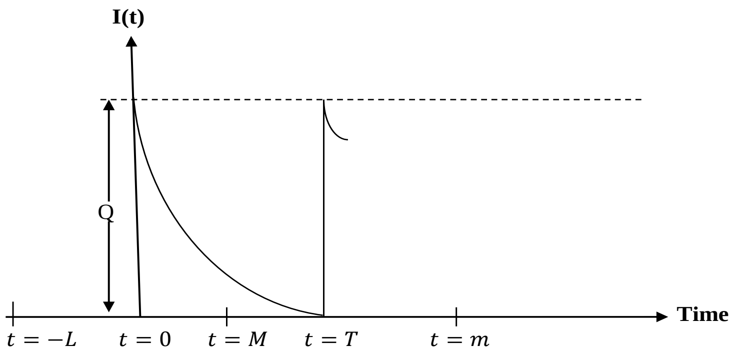


Figure 3: Pictorial representation of the case when $M < T \leq m$

The retailer begins selling the products after receiving them from the supplier, and the proceeds to deposited the sales revenue into an interest-bearing account. Over the period [0,M], the retailer accumulates a total sales revenue. The following formula is used to compute interest:

$$\mathrm{SR}_2[0,\mathrm{M}] = p\int_0^M D(p)\ dt = p(a - bp) \quad (19)$$

The retailer's interest on this revenue can be calculated as follows.

$$\mathrm{IE}_2[0,\mathrm{M}] = pI_e \int_0^M D(p)\ t\, dt = \frac{1}{2} pI_e(a - bp)M^2 \tag{20}$$

The retailer's ability to make a credit payment depends on the balance in his interest-bearing account at the time of the payment. This leads to two potential **sub-cases:**

Sub-Case 2.1: The interest-bearing account at the time of the payment has enough money i.e.

at time $t = M$, $\mathrm{SR}_2[0,\mathrm{M}] + \mathrm{IE}_2[0,\mathrm{M}] \geq \gamma\mathrm{PC}$.

In this case, the retailer will not have to pay any interest to the supplier for credit payment:

$$\mathrm{IC}_{2.1} = 0 \tag{21}$$

After paying the supplier with funds from the interest-bearing account, the retailer starts selling the remaining stock and transfers the sales proceeds back into the same account. The retailer's interest during the time period $[M, T]$ is determined as follows:

$$\mathrm{IE}_{2.1} = \mathrm{I}_e(T - M)\{\mathrm{SR}_2[0,\mathrm{M}] + \mathrm{IE}_2[0,\mathrm{M}] - \gamma\mathrm{PC}\} + \mathrm{pI}_e \int_M^T D(p)\ \mathrm{td}$$

$$\mathrm{IE}_{2.1} = \frac{I_e(a-bp)(T-M)}{4}\left[2p\{2M + I_e(T + M) + I_e M^2\} + -c\gamma\left[2(1 + m)^2 \log\left(\frac{1+m}{1+m-T}\right) + (1 + m - T)^3 - (1 + m)^2(1 + m - T)\right]\right] \tag{22}$$

As a result, the following method can be used to calculate the retailer's overall profit every replenishment cycle.

$$Z_{2.1} = \frac{1}{T}[SR - OC - PC - HC - DC - IC - IC_{2.1} + IE_{2.1}] \tag{23}$$

Sub-Case 2.2: The interest-bearing account at the time of the payment does not have enough money i.e. at time t=m, $\mathrm{SR}_2[0,\mathrm{M}] + \mathrm{IE}_2[0,\mathrm{M}] < \gamma\mathrm{PC}$. The amount that remains outstanding at the moment of settlement is calculated using the formula below:

At the moment of settlement, the outstanding balance that has not been paid is calculated as follows:

$$V = \gamma\mathrm{PC} - \{\mathrm{SR}_2[0,\mathrm{M}] + \mathrm{IE}_2[0,\mathrm{M}]\}$$

$$V = \frac{(a-bp)}{4}\left[c\gamma\left[2(1 + m)^2 \log\left(\frac{1+m}{1+m-T}\right) + (1 + m - T)^3 - (1 + m)^2(1 + m - T)\right] - 2pM(2 + I_e M)\right] \tag{24}$$

Now, the retailer will be allowed to pay the outstanding balance at time $t = T$, but will be charged interest on the unpaid amount from the date of invoice until the balance is paid in full i.e. $[M, T]$. Therefore, the supplier will charge the retailer interest on the unpaid amount as follows

$$\mathrm{IC}_{2.2} = I_c(T - M)V$$

$$= \frac{I_c(a-bp)(T-M)}{4}\left[c\gamma\left[2(1+m)^2 \log\left(\frac{1+m}{1+m-T}\right) + (1+m-T)^3 - (1+m)^2(1+m-T) - 2pM(2+I_eM)\right]\right] \quad (25)$$

After settling the unpaid balance with interest at the time $t = T$, the retailer will sell the remaining stock and deposit the proceeds into an interest-bearing account. During the time period $[M, T]$, he will permit the retailer to earn interest. Consequently, the interest earned can be computed as

$$\mathrm{IE}_{2.2} = pI_e \int_M^T D(p)\ tdt = \frac{1}{2} pI_e(a-bp)\,(T^2 - M^2) \quad (26)$$

As a result, the following method can be used to calculate the retailer's overall profit every replenishment cycle.

$$Z_{2.2} = \frac{1}{T}[SR - OC - PC - HC - DC - IC - IC_{2.2} + IE_{2.2}] \quad (27)$$

4. Numerical Examples

In this section, to validate the purposed model we consider two examples.

Example 1: Consider the values $a = 100$, $b = 0.05$, $O = Rs. 500$ per order, $c = Rs. 5$ / Unit, $p = Rs. 25$, $d = Rs. 4$ /unit / year, $h = Rs. 3$ /unit / year, $I_e = 5\%$ / year, $r = 8\%$ / year $\alpha = 0.03$, $\beta = 0.40$, $m = 3$ year, $M = 0.8$ year and $L = 0.5$ / year

Since the profit function is a higher order polynomial so it is not possible to find the optimum value analytically, we use Mathematica software for its solution. The optimum solution is as follows

$$T^* = 0.2353 \text{ year, } Q^* = 21.2192 \text{ units, and } Z_1^* = \text{Rs } 42014.71$$

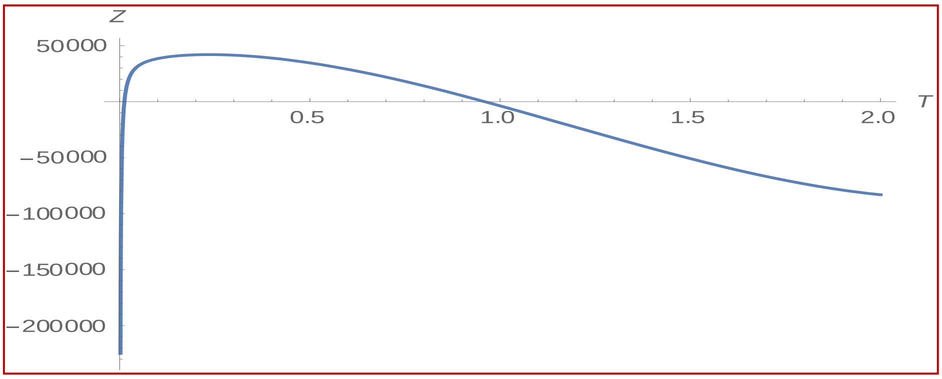


Figure 4: Profit function versus replenishment time for $\boldsymbol{T \leq M \leq m}$

Figure 4 illustrates the behaviour of the profit function Z for case I in relation to the ideal replenishment time T. It is clear from this example that the profit function Z's graph is concave.

Example 2: Consider the values $a = 100$, $b = 0.05$, $O = Rs. 500$ per order, $c = Rs. 5$ / Unit, $p = Rs. 25$, $d = Rs. 4$ /unit/ year, $h = Rs. 3$ /unit /year, $I_e = 5\%$ /year, $r = 8\%$ / year $\alpha = 0.03$, $\beta = 0.40$, $m = 3$ year, $M = 0.6$ year and $L = 0.5$ / year

Since the profit function is a higher order polynomial so it is not possible to find the optimum value analytically, we use Mathematica software for its solution. The optimum solution is as follows

$$T^* = 2.6708 \text{ year}, Q^* = 385.6010 \text{ units, and } Z^*_{2.1} = \text{ Rs } 69572.51$$

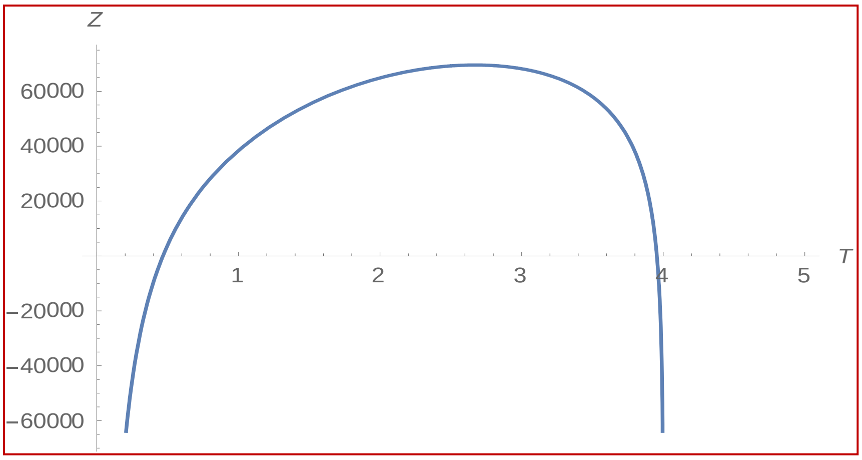


Figure 5: Profit function versus replenishment time for $\boldsymbol{M < T \leq m}$

The graph of the profit function based on Example No. 2 is concave for Case II of the inventory model, and is shown in Figure 5

5. Sensitivity Analysis

Keeping in mind the market uncertainty, a sensitivity study has been done, which shows the impact of decision variables and parameters on total profit. Perform sensitivity analysis to check the variation in the optimal solution by changing all the parameters of numerical example 2 of the above section one by one by -20%, -10%, +10% and +20% and taking the remaining values back to their original values. The results of the sensitivity analysis relative to the parameters taken into account in the model are displayed in Table 1. Which can be managerial insights for better management to avoid losses or increase profits due to changes in value of parameters in business.

Table 1: Sensitivity with respect to used parameters

Parameters	% Change in Parametric values	Parametric values	$\boldsymbol{T^*}$	$\boldsymbol{Q^*}$	$\boldsymbol{Z^*_{2.1}}$
a	-20	80	2.6717	297.6460	53627.44
	-10	90	2.6712	341.6260	61599.97
	0	100	2.6708	385.6010	69572.51
	+10	110	2.6705	429.5820	77545.05
	+20	120	2.6702	473.5420	85517.59
b	-20	0.40	2.67074	396.5910	71565.64
	-10	0.45	2.67079	391.1070	70569.07
	0	0.50	2.67083	385.6010	69572.51
	+10	0.55	2.67087	380.1110	68575.94
	+20	0.60	2.67092	374.6150	67579.37
p	-20	20.00	2.3805	325.5040	55112.05
	-10	22.50	2.5322	355.8980	62232.24

	0	25.00	2.6708	385.6010	69572.51
	+10	27.50	2.7851	413.9650	77068.43
	+20	30.00	2.9049	440.4530	84657.94
α	-20	0.24	2.7227	399.5410	70205.85
	-10	0.27	2.6967	392.4880	69881.45
	0	0.30	2.6708	385.6010	69572.51
	+10	0.33	2.6452	378.9240	69278.71
	+20	0.36	2.6198	372.4230	68999.74
β	-20	0.32	2.7237	399.8150	70193.72
	-10	0.36	2.6971	392.5960	69875.09
	0	0.40	2.6708	385.6010	69572.51
	+10	0.44	2.6449	378.8470	69285.51
	+20	0.48	2.6194	372.3220	69013.64
m	-20	2.40	2.5135	399.9140	60043.65
	-10	2.70	2.6027	393.5120	64579.10
	0	3.00	2.6708	385.6010	69572.51
	+10	3.30	2.7242	377.6990	75143.09
	+20	3.60	2.7691	370.8030	81383.59
M	-20	0.48	2.6102	369.9970	67572.01
	-10	0.54	2.6395	377.4550	68576.29
	0	0.60	2.6708	385.6010	69572.51
	+10	0.66	2.7040	394.4540	70553.69
	+20	0.72	2.7388	403.9810	71513.41
L	-20	0.40	2.6831	388.8550	69734.81
	-10	0.45	2.6769	387.2110	69653.22
	0	0.50	2.6708	385.6010	69572.51
	+10	0.55	2.6647	383.9980	69492.65
	+20	0.60	2.6585	382.3770	69413.67

In this time of rapidly changing market and competition in business, the values of all the parameters do not always remain the same as they were at the beginning of the business. It is natural for the value of parameters to increase or decrease. We see the following results based on Table 1

Retailers' total profit for each replenishment cycle will increase uniformly if the values of parameters $a, p, m,$ and M increase uniformly. Conversely, a consistent decrease in the retailers' total profit each replenishment cycle results from a uniform increase in the values of $b, \alpha, \beta,$ and L.

Order quantity increases uniformly with each replenishment cycle if parameters $a, p,$ and M all experience uniform increases in value. In contrast, a consistent decline in the order amount per replenishment cycle increases if there is a uniform decline in the values of the parameters $b, \alpha, \beta, m,$ and L.

The replenishment cycle's duration will increase uniformly if the values of the parameters $b, p, m,$ and M all rise at the same rate. Conversely, a uniform decrease in the replenishment cycle's duration occurs if the values of parameters $a, \alpha, \beta,$ and L all grow uniformly.

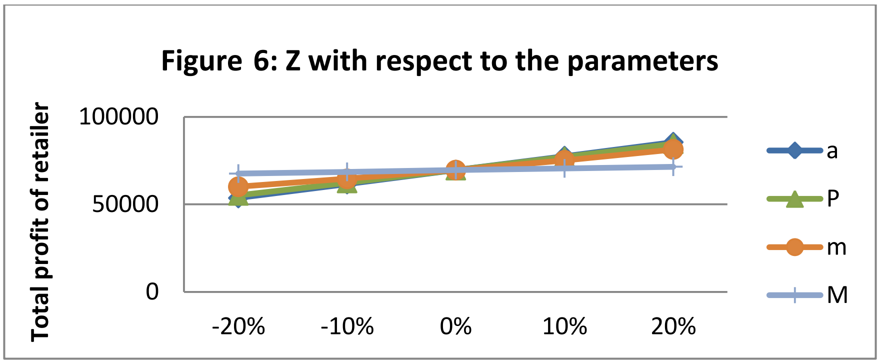


Figure 6: Z with respect to the parameters
Total profit of retailer
100000
50000
0
-20%
-10%
0%
10%
20%
a
P
m
M

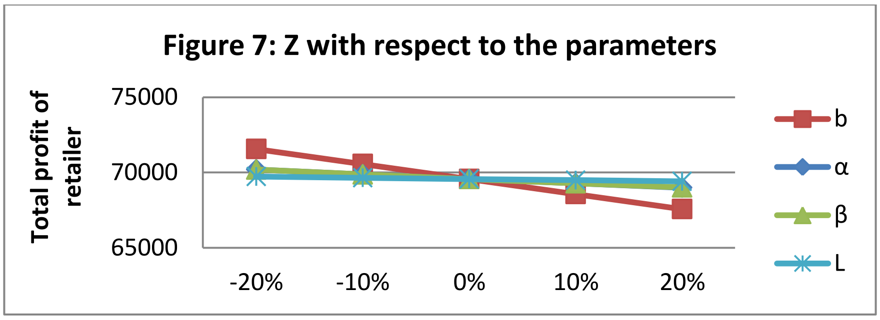


Figure 7: Z with respect to the parameters
Total profit of retailer
75000
70000
65000
-20%
-10%
0%
10%
20%
b
α
β
L

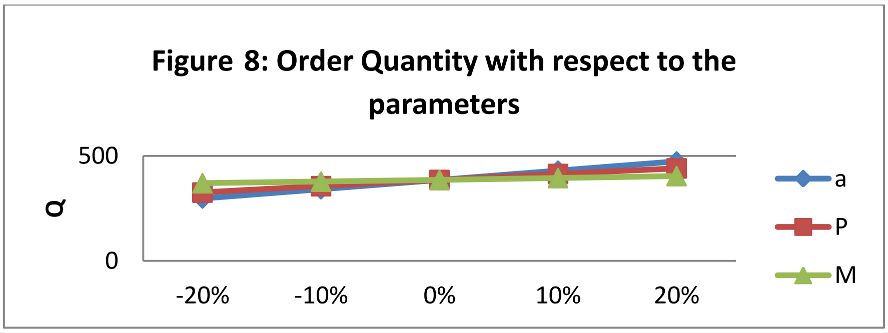


Figure 8: Order Quantity with respect to the parameters
Q
500
0
-20%
-10%
0%
10%
20%
a
P
M

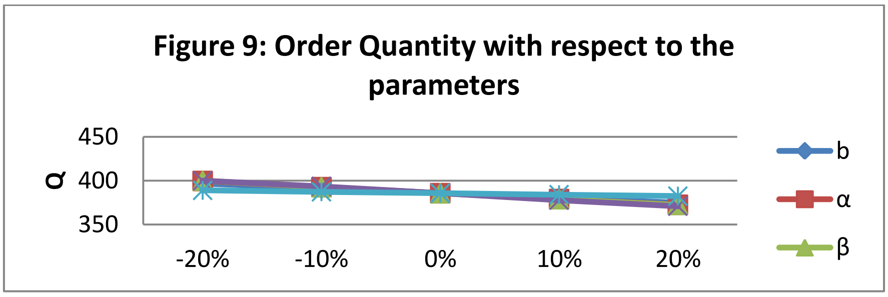
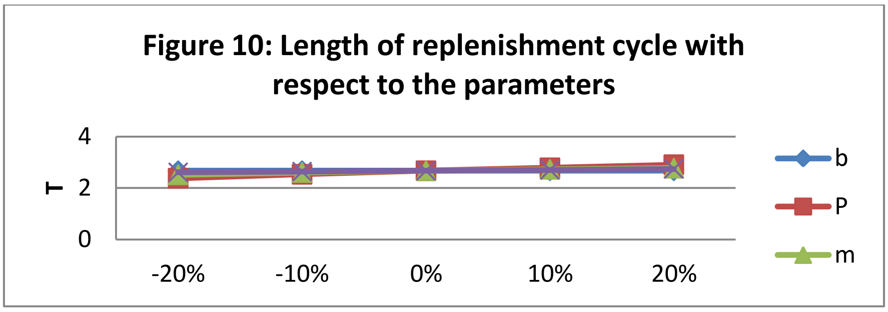
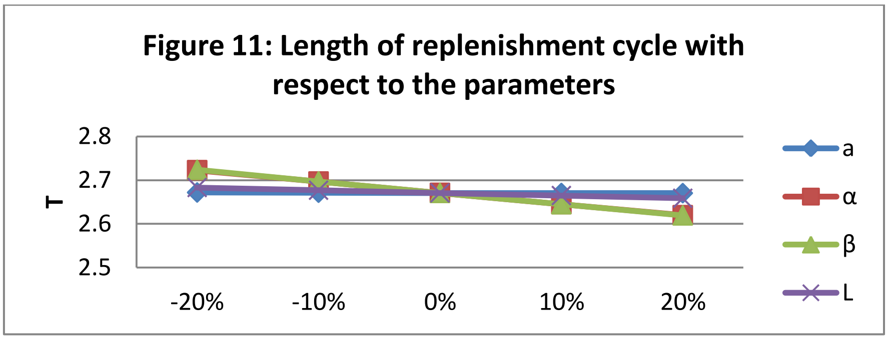


Figure 9: Order Quantity with respect to the parameters
Q
450
400
350
-20%
-10%
0%
10%
20%
b
α
β





6. Managerial Insights

Some implications are given based on a review of numerical examples along with the sensitivity analysis included in this inventory model. Any person in business or a manager can benefit from using it in their business model.

By following the policy of full advance payment by the businessman, the total average profit increases, which is much more than partial payment or cash payment; hence, it is suggested that the policy maker follow it.

This is recommended to the policymaker so that following the policy of full advance payment reduces the chances of the buyer backing out of the deal so that the businessman can also make other plans for the future.

If the seller gives a discount on the price of goods on the condition of full payment in advance, then the purchase cost decreases and the total average profit definitely increases; hence, the policymaker is advised to implement it.

7. Conclusion

This paper has no shortage, and demand depends on the selling price. The rate of product spoilage increases as the product gets closer to its shelf-life date. It is a profit-based model describing the optimal ordering cycle and quantity. Using the numerical values taken in the above example, the concavity of the profit function is displayed graphically. We also performed a sensitivity analysis to discuss the effects of parameter changes on total profit of the retailer, the order quantity and replenishment. The results of our study show that the proposed model can effectively manage inventory for deteriorating items. The model is also robust to changes in the model parameters.

The major limitation of this model is that there are no shortages in the assumption

This model can be developed in various other forms, such as under the consideration of shortages, partial backlogging, variable holding cost, time dependent demand rate etc. Another possible extension to this model could be to consider fuzzy environment and stochastic demand.

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