ISSN 0971-0388

Aligarh Journal of Statistics
Vol. 43(2023) 149-189

Benefits of Preservation Technology, Synchronous Reworking, and Waste Management in Sustainable Production-Inventory System in Inflationary Environment and Fuzzy Learning

S.K. Shon and Dharmendra Yadav
[Received on December, 2022. Accepted on May, 2023]

ABSTRACT

In the degradation of environment, carbon emissions, deterioration, and waste play critical role. These three are very common phenomenon in the production firms. Government regulations compel production companies to establish some mechanisms to manage this. As a result, the authors of the current paper examine a production-inventory model while considering imperfect production, preservation technology, green technology, reworking, waste management, and the carbon tax. The product's demand is assumed to be price-dependent. A specific proportion of manufactured goods are flawed, and this proportion is a linear function of manufacturing rate. The impact of inflation is considered throughout the entire analysis. Objective of the current study is to maximize the profit of production firm taking selling price, production rate, and cycle time as decision variables. Solution methodology is provided to obtain the optimal result. Numerical analysis is used to examine the model, and for managerial insights, sensitivity analysis is employed. Study suggests that decision-maker with learning attitude can earn more profit for the production system.

1. Introduction

Environmental sustainability is the big challenge in front of governments all over the world. Recognizing its importance, 196 parties at COP 21 in Paris came to an agreement, known as the "Paris Agreement." This agreement came into force on November 4, 2016. In accordance with this agreement, all nations committed to keeping global warming to 2 degrees Celsius over pre-industrial levels. In this

---

✉ : Dharmendra Yadav
Email: dharmendrayadav3580@gmail.com

Extended authors information available after reference list of the article.

submission, all the countries reached the common goal of reducing greenhouse gas emissions as much as possible. In this submission, it was also concluded that production firms are also big contributors of greenhouse gases in the environment.[2] As per the US Environmental Protection Agency, in the US, 21% of the total greenhouse gas emissions were due to industry. Various countries all over the world have observed that by adopting advanced technology, carbon emissions can be controlled efficiently.

It is observed that waste generated in production firms has a hazardous impact on the environment as well as society. All this should draw the attention of governments all over the world to designing some mechanism to reduce waste in the environment as much as possible. In this direction, it is observed that this is not only beneficial for the environment but also important from an economic point of view.[3] It is observed that in Germany, around 270,000 people are employed by companies that are engaged in waste management. This results in an annual turnover of around 70 billion euros. [4]As per the World Bank report, industrial waste production will rise by 70 percent from 2018 to 2050. These reports show the advantage of systematic waste management, as no production system is perfect.

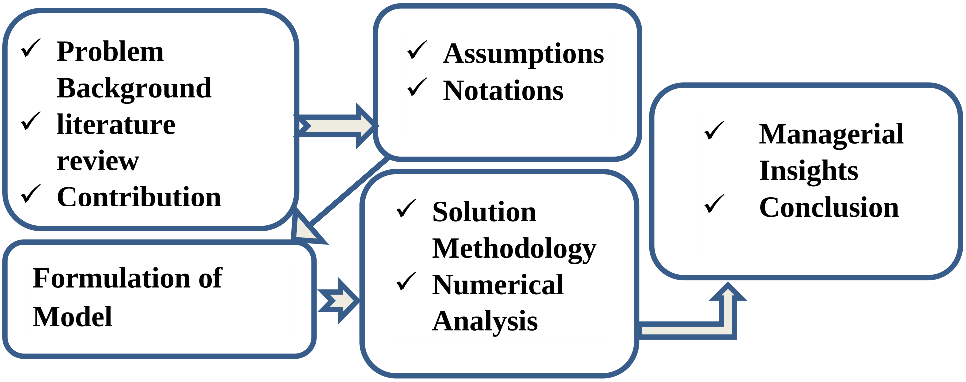

Fig.1 : The Flow of Current Study.

In this study, an attempt is made to integrate the issues of financial sustainability and environmental sustainability into the production-inventory model where the production process is not perfect. Green investment and preservation technology

---

[2] https://www.epa.gov/ visited on 20.10.2022.
[3] https://en.wikipedia.org/wiki/Waste_management_industry visited on 11.10.2022
[4] https://en.wikipedia.org/wiki/World_Bank visited on 12.10.2022

are applied in the model to achieve the goal of social sustainability. In the current work, the sections listed below are formed: An introduction is presented in Section 1, and a literature review is carried out in Section 2 to get the motivation for the current work. Basic assumptions and notations required for modeling purposes are presented in Section 3. Section 4 contains the mathematical formulation of the current problem. Solution methodology is included in Section 5, whereas the model is illustrated with the help of numerical analysis in Section 6. Managerial insights are presented in Section 7, and the final concluding remark is presented in Section 8. Flow of the study is presented in fig.1.

2. Literature Review

2.1 Models with Carbon Emissions

The main driving force of the study is to investigate the challenges associated with production systems and make production firms aware of how carbon emissions damage the environment. In this direction, the authors investigated how government regulation can reduce the carbon emissions from various operations associated with production systems. **Hua *et al.* (2011)** analysed how carbon footprints can be control in the inventory system with the help of carbon emission trading regulations. **Choi (2013)** presented an inventory model that considered quick response systems for fashionable products and employed a carbon footprint tax scheme for environmental sustainability. **Bazan *et al.* (2015)** applied the EU Emissions Trading System, i.e., the penalty tax mechanism, to curb greenhouse gas emissions from the manufacturing process, remanufacturing process, and transportation activities in the production-inventory system. **Tang *et al.* (2018)** designed an inventory model to control carbon emissions from transportation by employing three different regulations governed through the government. **Mishra *et al.* (2021)** adopted preservation technology to reduce the deterioration of goods produced during the summer and applied green investment and a carbon tax to curb carbon emissions. They discovered that using green technology yields more sustainable results than other technologies. Sustainable supply chain was presented by **Yadav *et al.* (2021)** with the objective of reducing waste and considered carbon emissions in various operational activities associated with the inventory system. They observed that preservation technology has a positive impact on the environment. **Singh *et al.* (2021)** presented a flexible manufacturing system that takes into account the costs of energy and carbon emissions in the system in an inflationary environment. **Yadav *et al.* (2022)** established a smart manufacturing-inventory model with the objective to reduce waste and control pollution with the help of different regulatory mechanisms. They observed a significant reduction in pollution while applying variable pollution costs. **Mishra *et al.* (2022)** presented a smart and energy-efficient manufacturing system and considered a variable production rate and a stochastic demand rate. They observed that there is a 4.01% reduction in carbon emissions due to reduction technology. **Yadav *et al.* (2022)** developed production model and observed that there is reduction of 34.37, 0.83, and 0.62% in pollution level because of pollution-controlling strategies: pollution cap, pollution cap and trade, and pollution tax respectively. **Kumar *et al.* (2023)** presented an EOQ model considering the issue of environment and society. They observed that rise in the investment of green technology by 33% results 4% decline in carbon footprints.

2.2 Models with Waste Reduction

The goal of any production company nowadays is to cut costs related to the production system and waste generated during production. This is helpful for environmental sustainability as well as financial sustainability. **Manzouri *et al*.** (2014) investigated lean tools to reduce waste and costs in the supply chain system in this direction. They observed that this can be achieved through collaboration on demand and continuous improvement practices. **Karmakar *et al.* (2018)** modelled a production inventory model and opted preservation technology to restrict deterioration rates. They discovered the functional relationship between production rate and pollution. They observed that proper disposal of waste plays a vital role in environmental sustainability. **Mohammadi *et al.* (2019)** presented a supply chain model to reduce the waste that materials such as plastic, paper, metal, and glass produce during transportation and production. **Saxena *et al.* (2020)** suggested that the efficiency of the integrated system can be enhanced with the reduction of waste item and waiting time. They suggested that the utilization of waste from the primary supply market has a positive influence on the secondary supply chain. **Debrah *et al.* (2021)** presented a literature review on solid waste management and observed that fragile environments in developing countries (like India, Pakistan, Bangladesh, etc.) are due to a lack of environmental education. **Bhatnagar *et al.* (2022)** developed a single-stage production system. They considered reworking and waste management in order to make the cleaner production system. **Beullens and Ghiami (2022)** designed some means to calculate the financial implications of waste for the retailer. They also suggested the benefits of logistics, technology, and market strategy for the reduction of waste.

2.3 Models with Reworking

Imperfect products are unavoidable in any production process. There are two different strategies adopted by the production firms with these imperfect products: disposal and reworking. But disposing of the imperfect products has a hazardous impact on the environment. In this direction, a single-stage imperfect production-inventory model was modelled by **Sonntag and Kiesmüller (2018).** Further, they investigated the two different mechanisms for imperfect items: disposal or rework. They suggested that reworking is preferred by production firms when the reworking duration is short. A production-inventory system for imperfect production process was modelled by **Khanna *et al.* (2020).** In this model, they considered inspection, reworking, price-dependent demand, and carbon emissions. **Cárdenas-Barrón *et al.* (2021)** presented an inventory model considering reworking, inspection, selling price-dependent demand, and different holding costs for perfect and imperfect items. **Berling and Sonntag (2022)** presented a production-inventory model considering random yield and random demand. They employed the reworking option to reduce the waste in the system. They suggested that reworking is more reasonable as the reworking cost is always lower than the production cost. **Yadav *et al.* (2023)** developed smart production-inventory model considering imperfect production process. They considered random defective rate. They observed that due to manufacturing process inventory cost rises by 6%. **Bachar *et al.* (2023)** designed a flexible production model considering the reworking of repairable products and green investment. They outsourced the products to prevent backlogging.

2.4 Model with Fuzzy Learning

The literature reveals an increase in the quantity of research papers on the fuzzy inventory model. Many authors analysed the effect of fuzzy on the decision-making process. They used constant fuzzy parameters. The concept of constant fuzziness is relaxed by very few researchers by considering the learning in fuzziness. In this line, **Kazemi *et al.* (2015)** investigated the fuzzy inventory model by taking learning into account. They discovered that learning in fuzzyness reduced total inventory cost. **Soni *et al.* (2017)** explored the continuous review inventory model, considering backorders and lost sales. They considered the effect of learning fuzzy parameters to reduce errors and costs. They observed that the learning effect on fuzzy reduced the ambiguity present in the decision-making process. **Khatua *et al.* (2019)** presented a model with the objective of maximizing the profit of the fuzzy dynamic system, considering learning by doing. **Soni and Suthar (2021)** investigated the effect of learning on impreciseness on the EOQ model taking fuzzy demand, and a finite time horizon.

They suggested that learning in fuzziness leads to more effective decisions whenever imprecision in parameters is high. **Mahapatra *et al.* (2022)** presented the EOQ model under three different environments: a crisp environment, a fuzzy environment, and a fuzzy learning environment. They noticed that as the preservation factor rises, the order quantity rises as well. **Yadav *et al.* (2022)** developed an integrated model considering learning-forgetting phenomenon in the setup cost. They observed that the effect of learning-forgetting, profit of the centralized system is increases. **Vandana *et al.* (2023)** presented a production-inventory model considering the effect of learning-forgetting, volume agility, and carbon emissions.

2.5 Models under the Effect of Inflation

Involvement of inflation in modeling is a practical approach as it has an impact on the economy in various ways. Due to high inflation, the purchasing power of unit money decreases. In this direction, **Misra (1975)** and **Buzacott (1975)** explored an inventory system simultaneously under the influence of inflation. **Shah and Vaghela (2018)** explored the EPQ model considering imperfect production, advertisement sensitive demand, and product reliability under the effect of inflation. A reserve logistics integrated model was developed by **Singh and Sharma (2019)** considering reuse, repair, and recycling under the effect of inflation. In the model, they considered a time-dependent backlog rate. Many authors, such as **Padiyar *et al.* (2021)**, **Barman *et al.* (2021)**, **Singh and Mani (2021)**, **Padiyar *et al.* (2022a), and Padiyar *et al.* (2022b), Singh and Choudhary (2023), Padiyar *et al.* (2023)** enrich the inventory literature by analysing the impact of inflation on the decision-making process of an inventory model.

2.6 Contribution of Current Research Work

The above-mentioned research papers illustrate that researchers have focused on areas like carbon emissions, waste management, reworking, fuzzy learning, and inflation while developing the production-inventory model independently. From the literature, it is evident that rarely have researchers explored the impact of all these issues simultaneously. All these issues are related. Therefore, it is important to explore the impact of all these on the decision-making process for the production-inventory problem. The following is the current work's major contribution:

i. Carbon emissions in the production process cannot be stopped. It can only be controlled with investments in advanced technology. Thus, in the current study, green investment is considered along with a carbon tax mechanism.
ii. Waste reduction is critical for environmental and social sustainability. Reworking imperfect items costs less than production costs. Therefore, reworking, along with waste management, is considered in the current study.
iii. Additionally, preservation technology is considered to slow down deterioration because it costs the system revenue.
iv. Learning in impreciseness is considered in the different cost parameters to accommodate human learning in the model.
v. In the model, market disruption is handled by taking inflation into account.

Currently, no study is reported in the literature on the imperfect production model, considering all these practical issues. Thus, it is the main contribution of the current study that formulates the imperfect production-inventory model, considering all these issues simultaneously.

3. Assumptions and Notations

In this section, required assumptions and notations are incorporated for the formulation of mathematical model.

3.1 Assumptions

1. A cleaner production-inventory system is considered in which production rate is finite and confined between two pre-determined levels i.e., $P_{min} \leq P \leq P_{max}$.
2. Because of various reasons such as human error, improper raw materials, mishandling, machine failure, etc., imperfect production is unavoidable in the production system. Further, it is observed that the proportion of defective production is a linearly increasing function of production rate.
3. An automated screening system is installed to segregate the perfect and imperfect items. Additionally, defective items are divided into reworkable and non-reworkable categories.
4. Production systems are cleaned with the help of reworking defective items and proper waste management of non-reworkable items. This helps to achieve the goal of environmental sustainability. Here, synchronous reworking at the same pace as production is considered.
5. Production rates, especially for newly launched products, are different during the planning horizon. Hence, three different levels of production are considered.

6. It is assumed that the cost of production depends on the rate of production. This cost consists of three different components: (i) raw material cost, which is fixed; (ii) labor cost, which decreases as the production rate increases; (iii) tool and die cost, which increases as the production rate increases.
7. A product's demand is greatly impacted by its selling price. Therefore, it is assumed that demand rate is a linearly diminishing function of selling price.
8. Market turbulence can be incorporated into the model with the help of inflation. Thus, in the current study, the effect of inflation is considered.
9. Carbon emissions occur during various stages of the production system, such as the preparation of the production setup, the production process, reworking, waste management of scrap, carrying the products in stock, and deterioration.
10. For every unit of carbon that the manufacturing system puts into the air, a carbon tax is put in place.
11. Manufacturing systems are more inclined toward environmentally sustainable systems. Thus, through investment in energy-efficient machinery, advanced technology, etc., the production system moves towards a green production system.

3.2 Notations

Decision Variables

P	: Production rate	(unit/unit time)
s	: Selling price	($ per unit)

Imprecise Parameters

$\widetilde{S_A}$	: Setup cost	($ per cycle)
$\widetilde{C_r}$	: Cost of reworking	($ per unit item)
$\widetilde{C_m}$	: Waste management cost	($ per unit item)
$\widetilde{C_h}$	: Carrying cost	($ per unit)
$\widetilde{C_d}$	: Deterioration cost	($ per unit item)
$\widetilde{S_c}$	: Screening cost	($ per unit item)

Crisp Parameters

$D(s)$	: Price dependent demand rate $(= d_0 - d_1 s)$	
θ	: Deterioration rate	(unit/per unit time)
w	: Preservation cost	($ per unit)

$\pi(w)$: Preservation technology function $\left(= \theta(1 - e^{-p_1 w})\right)$

$\varphi(P)$: Production cost comprises following components ($ per unit)

(a). R_m = raw material cost

(b). L_c/P = labor and energy cost

(c). $T_c P$ = tool/die cost

r : Inflation rate (%)

$\alpha(P)$: Proportion of defective units during production process $\left(= \gamma_1 + \gamma_2 \left(\frac{P - P_{min}}{P_{max} - P_{min}}\right)\right)$ where γ_1 and γ_2 are positive constant

β : Proportion of defective units which can be reworked

C_E : Total carbon emissions due to the various process

ϖ : Fraction of emissions reduced due to capital investment in green technology

F : $\left(= \varpi\left(1 - e^{-\vartheta G}\right)\right)$ Fraction of reduction of emissions due to capital investment G where $\vartheta \geq 0$

C_E' : Carbon emissions after the investment in green technology (Kg/cycle)

S_e : Carbon emissions due to setup (Kg per setup)

P_e : Carbon emissions due to production (Kg/unit)

I_e : Carbon emissions due to screening (Kg/unit)

R_e : Carbon emissions due to reworking (Kg/unit)

W_e : Carbon emissions due to waste management (Kg/unit)

C_e : Carbon emissions due to holding (Kg/unit)

D_e : Carbon emissions due to deterioration (Kg/unit)

P_e : Carbon emissions due to preservation technology (Kg/cycle)

ζ : Tax on emitted unit of carbon ($/Kg)

T : Cycle time (unit time)

4. Mathematical Formulation under Fuzzy Learning Environment

The objective of this section is to formulate a mathematical model for a production-inventory system where the production process is not perfect. The automated screening process segregated the produced products into two categories: serviceable and non-serviceable items. Further, non-serviceable items are segregated into two categories: reworkable and non-reworkable items. In the model, production, screening, and reworking take place simultaneously.

Presently, production cycle $[0, T]$ is divided into four sub-cycles: $[0, t_1]$, $[t_1, t_2], [t_2, t_3]$, and $[t_3, T]$. In first three intervals, production continues with different production rates and last cycle is non-production period (see fig.2). Following equations can be used to depict the inventory situations:

$$\frac{dQ_1(t)}{dt} = (1 - \alpha(P))P + \beta\alpha(P)P - D(s) - \big(\theta - \pi(w)\big)Q_1(t), 0 \le t \le t_1, \quad (1)$$

$$\frac{dQ_2(t)}{dt} = \big((1 - \alpha(P))aP + \beta\alpha(P)aP - D(s)\big) - \big(\theta - \pi(w)\big)Q_2(t), t_1 \le t \le t_2, \quad (2)$$

$$\frac{dQ_3(t)}{dt} = \big((1 - \alpha(P))bP + \beta\alpha(P)bP - D(s)\big) - \big(\theta - \pi(w)\big)Q_3(t), \ \ t_2 \le t \le t_3, \quad (3)$$

$$\frac{dQ_4(t)}{dt} = -\Big(D(s) + \big(\theta - \pi(w)\big)Q_4(t)\Big), t_3 \le t \le T. \quad (4)$$

On solving the above equations under the conditions $Q_1(0) = 0, Q_1(t_1) = Q_2(t_1), Q_2(t_2) = Q_3(t_2),\ Q_4(T) = 0,$ we get

$$Q_1(t) = \frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\big(1 - e^{-(\theta-\pi(w))t}\big), \quad t\epsilon[0, t_1], \quad (5)$$

$$Q_2(t) = \frac{\big((1-\alpha(P))aP+\beta\alpha(P)aP-D(s)\big)}{(\theta-\pi(w))}\big(1 - e^{-(\theta-\pi(w))(t-t_1)}\big) - \frac{\big((1-\alpha(P))P+\beta\alpha(P)P-D(s)\big)}{(\theta-\pi(w))}\big(e^{-(\theta-\pi(w))(t-t_1)} - e^{-(\theta-\pi(w))t}\big) \ t\epsilon[t_1, t_2], \quad (6)$$

$$Q_3(t) = \frac{\big((1-\alpha(P))bP+\beta\alpha(P)bP-D(s)\big)}{(\theta-\pi(w))}\big(1 - e^{-(\theta-\pi(w))(t-t_2)}\big) - \frac{\big((1-\alpha(P))aP+\beta\alpha(P)aP-D(s)\big)}{(\theta-\pi(w))}\big(e^{-(\theta-\pi(w))(t-t_2)} - e^{-(\theta-\pi(w))(t-t_1)}\big) + \frac{\big((1-\alpha(P))P+\beta\alpha(P)P-D(s)\big)}{(\theta-\pi(w))}\big(e^{-(\theta-\pi(w))(t-t_1)} - e^{-(\theta-\pi(w))t}\big), t\epsilon[t_2, t_3], \quad (7)$$

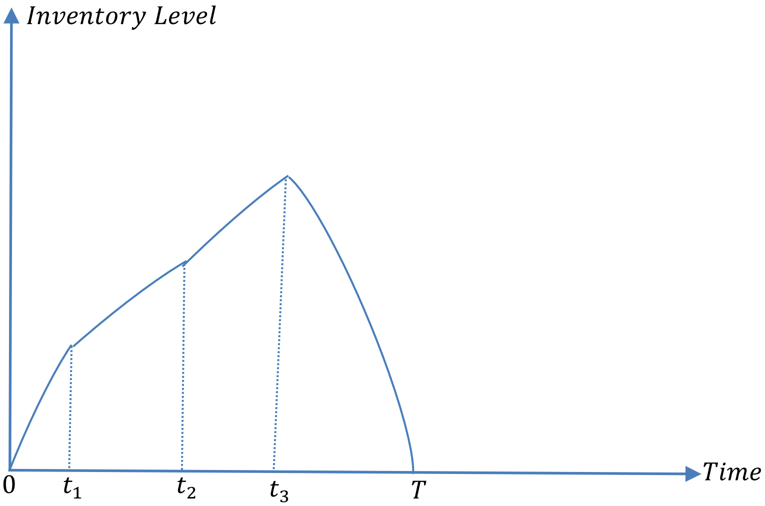

Fig. 2: Graphical Representation of Inventory Level.

$$Q_4(t) = \frac{D(s)}{(\theta-\pi(w))}\left(e^{(\theta-\pi(w))(T-t)} - 1\right), \ t\epsilon[t_3, T]. \quad (8)$$

Following result is obtained on applying the condition $Q_1(t_1) = I_1$ and using the approximation:

$$I_1 = \big((1-\alpha(P))P + \beta\alpha(P)P - D(s)\big)t_1. \quad (9)$$

Following result is obtained on applying the condition $Q_2(t_2) = I_2$ and using the approximation:

$$I_2 = \big((1-\alpha(P))aP + \beta\alpha(P)aP - D(s)\big)(t_2 - t_1) - \big((1-\alpha(P))P + \beta\alpha(P)P - D(s)\big)t_1. \quad (10)$$

Following result is obtained on applying the condition $Q_3(t_3) = I_3$ and using the approximation:

$$I_3 = \big((1-\alpha(P))bP + \beta\alpha(P)bP - D(s)\big)(t_3 - t_2) - \Big(\big(1-\alpha(P)\big)aP + \beta\alpha(P)aP - D(s)\Big)(t_2 - t_1) + \big((1-\alpha(P))P + \beta\alpha(P)P - D(s)\big)t_1. \quad (11)$$

In order to obtain the expression for profit, we evaluate the different cost components one by one as follows:

Inflation induced Setup Cost for Manufacturing System: Initially, manufacturers make an investment to prepare the basic infrastructure, which is termed the "setup cost." Hence, inflation-induced setup cost is

$$ST = S_A.$$

Inflation induced Production Cost: After preparing the basic setup for production, the producer procures raw material, arranges the labor, and acquires different tolls for production. Thus, some cost is associated with this, which is termed "production cost." Hence, inflation-induced production cost is

$$PC = \int_0^{t_1} \varphi(P)Pe^{-rt}dt + \int_{t_1}^{t_2} \varphi(P)aPe^{-rt}dt + \int_{t_2}^{t_3} \varphi(P)bPe^{-rt}dt$$
$$= \frac{\varphi(P)P}{r}(1 - e^{-rt_1}) + \frac{\varphi(P)aP}{r}(e^{-rt_1} - e^{-rt_2}) + \frac{\varphi(P)bP}{r}(e^{-rt_2} - e^{-rt_3}).$$

Inflation induced Screening Cost: To maintain goodwill in the market, it is important for the producer to carry out the screening process. Therefore, the producer carried out the screening process simultaneously with the production process. Thus, inflation-induced screening cost is

$$SC = \int_0^{t_1} S_c Pe^{-rt}dt + \int_{t_1}^{t_2} S_c aPe^{-rt}dt + \int_{t_2}^{t_3} S_c bPe^{-rt}dt$$
$$= \frac{S_c P}{r}(1 - e^{-rt_1}) + \frac{S_c aP}{r}(e^{-rt_1} - e^{-rt_2}) + \frac{S_c bP}{r}(e^{-rt_2} - e^{-rt_3}).$$

Inflation induced Reworking Cost: The aim of the producer is to reduce waste as much as possible in the production system. Reworking the reworkable items is one of the options. Thus, the inflation-induced reworking cost is

$$RC = \int_0^{t_1} C_r\beta\alpha(P)Pe^{-rt}dt + \int_{t_1}^{t_2} C_r\beta\alpha(P)aPe^{-rt}dt + \int_{t_2}^{t_3} C_r\beta\alpha(P)bPe^{-rt}dt$$

$$= \frac{C_r\beta\alpha(P)P}{r}(1 - e^{-rt_1}) + \frac{C_r\beta\alpha(P)aP}{r}(e^{-rt_1} - e^{-rt_2}) + \frac{C_r\beta\alpha(P)bP}{r}(e^{-rt_2} - e^{-rt_3}).$$

Inflation induced Waste Management Cost: For environmental sustainability, it is important to apply proper waste management technology to the non-reworkable items. Thus, inflation-induced waste management cost is

$$WM = \int_0^{t_1} C_m(1-\beta)\alpha(P)Pe^{-rt}dt + \int_{t_1}^{t_2} C_m(1-\beta)\alpha(P)aPe^{-rt}dt + \int_{t_2}^{t_3} C_m(1-\beta)\alpha(P)bPe^{-rt}dt$$

$$= \frac{C_m(1-\beta)\alpha(P)P}{r}(1 - e^{-rt_1}) + \frac{C_m(1-\beta)\alpha(P)aP}{r}(e^{-rt_1} - e^{-rt_2}) + \frac{C_m(1-\beta)\alpha(P)bP}{r}(e^{-rt_2} - e^{-rt_3})$$

Inflation induced Carrying Cost: Inventory is carried in stock by the producer to satisfy the demand of the customers. Thus, carrying cost is paid by the producer. Hence, inflation-induced carrying cost is

$$CC = \int_0^{t_1} C_h Q_1(t)e^{-rt}dt + \int_{t_1}^{t_2} C_h Q_2(t)e^{-rt}dt + \int_{t_2}^{t_3} C_h Q_3(t)e^{-rt}dt + \int_{t_3}^{T} C_h Q_4(t)e^{-rt}dt$$

$$= C_h\left[\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(\frac{1}{r}(1-e^{-rt_1}) + \frac{1}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_1} - 1\right)\right) + \frac{((1-\alpha(P))aP+\beta\alpha(P)aP-D(s))}{(\theta-\pi(w))}\right.$$

$$\left(\frac{1}{r}(e^{-rt_1} - e^{-rt_2}) + \frac{e^{(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2} - e^{-(r+\theta-\pi(w))t_1}\right)\right) -$$

$$\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(\frac{e^{(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_1} - e^{-(r+\theta-\pi(w))t_2}\right) + \right.$$

$$\left.\frac{1}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2} - e^{-(r+\theta-\pi(w))t_1}\right)\right) + \frac{((1-\alpha(P))bP+\beta\alpha(P)bP-D(s))}{(\theta-\pi(w))}$$

$$\left(\frac{1}{r}(e^{-rt_2} - e^{-rt_3}) + \frac{e^{(\theta-\pi(w))t_2}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_3} - e^{-(r+\theta-\pi(w))t_2}\right)\right) -$$

$$\frac{((1-\alpha(P))aP+\beta\alpha(P)aP-D(s))}{(\theta-\pi(w))}\left(\frac{e^{-(\theta-\pi(w))t_2}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2} - e^{-(r+\theta-\pi(w))t_3}\right) + \right.$$

$$\left.\frac{e^{-(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_3} - e^{-(r+\theta-\pi(w))t_2}\right)\right) + \frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}$$

$$\left(\frac{e^{-(r+\theta-\pi(w))t_1}\left(e^{-(r+\theta-\pi(w))t_2} - e^{-(r+\theta-\pi(w))t_3}\right)}{r+\theta-\pi(w)} + \frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_3} - \right.\right.$$

$$\left.\left. e^{-(\theta-\pi(w))t_2}\right)\right) + \frac{D(s)}{(\theta-\pi(w))}\left(\frac{1}{r+\theta-\pi(w)}\left(e^{(\theta-\pi(w))T-(r+\theta-\pi(w))t_3} - e^{-rT}\right) + \right.$$

$$\left.\left.\frac{1}{r}\left(e^{-rT} - e^{-rt_3}\right)\right)\right]$$

Inflation induced Deterioration Cost: The deterioration process is inevitable due to the physical presence of goods in the market. This is an extra financial liability for the manufacturing system. As a result, the cost of deterioration due to inflation is

$$DC = \int_0^{t_1}(\theta - \pi(w))C_d Q_1(t)e^{-rt}dt + \int_{t_1}^{t_2}(\theta - \pi(w))C_d Q_2(t)e^{-rt}dt + \int_{t_2}^{t_3}(\theta - \pi(w))C_d Q_3(t)e^{-rt}dt + \int_{t_3}^{T}(\theta - \pi(w))C_d Q_4(t)e^{-rt}dt$$

$$= C_d\Bigg[\left((1-\alpha(P))P + \beta\alpha(P)P - D(s)\right)\left(\frac{1}{r}(1-e^{-rt_1}) + \frac{1}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_1} - 1\right)\right) + \left(\left((1-\alpha(P))aP + \beta\alpha(P)aP - D(s)\right)\right)$$

$$\left(\frac{1}{r}(e^{-rt_1} - e^{-rt_2}) + \frac{e^{(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2} - e^{-(r+\theta-\pi(w))t_1}\right)\right)$$

$$-\left((1-\alpha(P))P + \beta\alpha(P)P - D(s)\right)$$

$$\left(\frac{e^{(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_1} - e^{-(r+\theta-\pi(w))t_2}\right) + \frac{1}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2} - e^{-(r+\theta-\pi(w))t_1}\right)\right) + \left(\left((1-\alpha(P))bP + \beta\alpha(P)bP - D(s)\right)\right)$$

$$\left(\frac{1}{r}(e^{-rt_2} - e^{-rt_3}) + \frac{e^{(\theta-\pi(w))t_2}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_3} - e^{-(r+\theta-\pi(w))t_2}\right)\right)$$

$$-\left(\left((1-\alpha(P))aP + \beta\alpha(P)aP - D(s)\right)\right)$$

$$\left(\frac{e^{-(\theta-\pi(w))t_2}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2} - e^{-(r+\theta-\pi(w))t_3}\right) + \frac{e^{-(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_3} - e^{-(r+\theta-\pi(w))t_2}\right)\right) + \left((1-\alpha(P))P + \beta\alpha(P)P - D(s)\right)$$

$$\left(\frac{e^{-(r+\theta-\pi(w))t_1}\left(e^{-(r+\theta-\pi(w))t_2 - e^{-(r+\theta-\pi(w))t_3}}\right)}{r+\theta-\pi(w)} + \frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_3} - e^{-(\theta-\pi(w))t_2}\right)\right) +$$

$$D(s)\left(\frac{1}{r+\theta-\pi(w)}\left(e^{(\theta-\pi(w))T - (r+\theta-\pi(w))t_3} - e^{-rT}\right) + \frac{1}{r}(e^{-rT} - e^{-rt_3})\right)\Bigg]$$

Inflation induced Preservation Technology Cost: To control the deterioration rate, producers make investments in preservation technology. Thus, inflation-induced investment in preservation technology is

$$PT = w$$

Inflation induced Green Technology Cost: In the beginning of the production process, the producer makes an investment in green technology. Thus, inflation-induced investment in green technology is
$GT = G$

Inflation induced Carbon Emissions Tax under the Effect of Green Technology: Carbon emissions take place due to setup, production, screening, reworking, waste management, carrying product, deterioration, and preservation technology. Thus,

Carbon emissions due to setup preparation=S_e

Carbon emissions due to production =$P_e(Pt_1 + aP(t_2 - t_1) + bP(t_3 - t_1))$

Carbon emissions due to screening = $I_e(Pt_1 + aP(t_2 - t_1) + bP(t_3 - t_1))$

Carbon emissions due to reworking = $R_e(\beta\alpha(P)Pt_1 + \beta\alpha(P)aP(t_2 - t_1) + \beta\alpha(P)bP(t_3 - t_1))$

Carbon emissions due to waste management = $W_e((1-\beta)\alpha(P)Pt_1 + (1-\beta)\alpha(P)aP(t_2 - t_1) + (1-\beta)\alpha(P)bP(t_3 - t_1))$

Carbon emissions due to carrying the product in the stock =
$C_e\left[\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(t_1 + \frac{\left(e^{-(\theta-\pi(w))t_1}-1\right)}{\theta-\pi(w)}\right) + \right.$
$\frac{\left((1-\alpha(P))aP+\beta\alpha(P)P-D(s)\right)}{(\theta-\pi(w))}\left((t_2 - t_1) + \frac{\left(e^{-(\theta-\pi(w))(t_2-t_1)}-1\right)}{\theta-\pi(w)}\right) -$
$\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(\frac{1}{\theta-\pi(w)}\left(1 - e^{-(\theta-\pi(w))(t_2-t_1)}\right) + \frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_2} - e^{-(\theta-\pi(w))t_1}\right)\right) +$
$\frac{\left((1-\alpha(P))bP+\beta\alpha(P)P-D(s)\right)}{(\theta-\pi(w))}\left((t_3 - t_2) + \frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_2)} - 1\right)\right) -$
$\frac{\left((1-\alpha(P))aP+\beta\alpha(P)P-D(s)\right)}{(\theta-\pi(w))}\left(\frac{\left(1-e^{-(\theta-\pi(w))(t_3-t_2)}\right)}{\theta-\pi(w)} + \frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_2)} - e^{-(\theta-\pi(w))(t_2-t_1)}\right)\right) + \frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}$

$\left(\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_1)}-e^{-(\theta-\pi(w))(t_2-t_1)}\right)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_3}-\right.\right.$
$\left.\left.e^{-(\theta-\pi(w))t_2}\right)\right)+\frac{D(s)}{(\theta-\pi(w))}\left(\frac{1}{\theta-\pi(w)}\left(e^{(\theta-\pi(w))(T-t_3)}-1\right)+(T-t_3)\right)\Bigg]$

Carbon emissions due to deterioration $= D_e\left[\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(t_1+\right.\right.$
$\left.\frac{\left(e^{-(\theta-\pi(w))t_1}-1\right)}{\theta-\pi(w)}\right)+\frac{\left((1-\alpha(P))aP+\beta\alpha(P)P-D(s)\right)}{(\theta-\pi(w))}\left((t_2-t_1)+\frac{\left(e^{-(\theta-\pi(w))(t_2-t_1)}-1\right)}{\theta-\pi(w)}\right)-$
$\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(\frac{1}{\theta-\pi(w)}\left(1-e^{-(\theta-\pi(w))(t_2-t_1)}\right)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_2}-\right.\right.$
$\left.\left.e^{-(\theta-\pi(w))t_1}\right)\right)+$
$\frac{\left((1-\alpha(P))bP+\beta\alpha(P)P-D(s)\right)}{(\theta-\pi(w))}\left((t_3-t_2)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_2)}-1\right)\right)-$
$\frac{\left((1-\alpha(P))aP+\beta\alpha(P)P-D(s)\right)}{(\theta-\pi(w))}\left(\frac{\left(1-e^{-(\theta-\pi(w))(t_3-t_2)}\right)}{\theta-\pi(w)}+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_2)}-\right.\right.$
$\left.\left.e^{-(\theta-\pi(w))(t_2-t_1)}\right)\right)+\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}$
$\left(\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_1)}-e^{-(\theta-\pi(w))(t_2-t_1)}\right)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_3}-\right.\right.$
$\left.\left.e^{-(\theta-\pi(w))t_2}\right)\right)+\frac{D(s)}{(\theta-\pi(w))}\left(\frac{1}{\theta-\pi(w)}\left(e^{(\theta-\pi(w))(T-t_3)}-1\right)+(T-t_3)\right)\Bigg]$

Carbon emissions due to preservation technology $=P_e$

Let, C_E be the total emissions due to all these processes. Thus,

$C_E=S_e+(P_e+I_e)(Pt_1+aP(t_2-t_1)+bP(t_3-t_1))+R_e(\beta\alpha(P)Pt_1+\beta\alpha(P)aP(t_2-t_1)+\beta\alpha(P)bP(t_3-t_1))+W_e((1-\beta)\alpha(P)Pt_1+(1-\beta)\alpha(P)aP(t_2-t_1)+(1-\beta)\alpha(P)bP(t_3-t_1))$

$+C_e\left[\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(t_1+\frac{\left(e^{-(\theta-\pi(w))t_1}-1\right)}{\theta-\pi(w)}\right)+\right.$
$\frac{\left((1-\alpha(P))aP+\beta\alpha(P)P-D(s)\right)}{(\theta-\pi(w))}\left((t_2-t_1)+\frac{\left(e^{-(\theta-\pi(w))(t_2-t_1)}-1\right)}{\theta-\pi(w)}\right)-$
$\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(\frac{1}{\theta-\pi(w)}\left(1-e^{-(\theta-\pi(w))(t_2-t_1)}\right)+\right.$

$$\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_2}-e^{-(\theta-\pi(w))t_1}\right)\Bigg)+\frac{((1-\alpha(P))bP+\beta\alpha(P)P-D(s))}{(\theta-\pi(w))}\Bigg((t_3-t_2)+$$

$$\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_2)}-1\right)\Bigg)-\frac{((1-\alpha(P))aP+\beta\alpha(P)P-D(s))}{(\theta-\pi(w))}$$

$$\Bigg(\frac{\left(1-e^{-(\theta-\pi(w))(t_3-t_2)}\right)}{\theta-\pi(w)}+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_2)}-e^{-(\theta-\pi(w))(t_2-t_1)}\right)\Bigg)+$$

$$\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}$$

$$\Bigg(\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_1)}-e^{-(\theta-\pi(w))(t_2-t_1)}\right)+\frac{1}{\theta-\pi(w)}\Big(e^{-(\theta-\pi(w))t_3}-$$

$$e^{-(\theta-\pi(w))t_2}\Big)\Bigg)+\frac{D(s)}{(\theta-\pi(w))}\Bigg(\frac{1}{\theta-\pi(w)}\left(e^{(\theta-\pi(w))(T-t_3)}-1\right)+(T-t_3)\Bigg)\Bigg]$$

$$+D_e\Bigg[\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\Bigg(t_1+\frac{\left(e^{-(\theta-\pi(w))t_1}-1\right)}{\theta-\pi(w)}\Bigg)+$$

$$\frac{((1-\alpha(P))aP+\beta\alpha(P)P-D(s))}{(\theta-\pi(w))}\Bigg((t_2-t_1)+\frac{\left(e^{-(\theta-\pi(w))(t_2-t_1)}-1\right)}{\theta-\pi(w)}\Bigg)-$$

$$\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\Bigg(\frac{1}{\theta-\pi(w)}\left(1-e^{-(\theta-\pi(w))(t_2-t_1)}\right)+$$

$$\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_2}-e^{-(\theta-\pi(w))t_1}\right)\Bigg)+\frac{((1-\alpha(P))bP+\beta\alpha(P)P-D(s))}{(\theta-\pi(w))}\Bigg((t_3-t_2)+$$

$$\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_2)}-1\right)\Bigg)-\frac{((1-\alpha(P))aP+\beta\alpha(P)P-D(s))}{(\theta-\pi(w))}$$

$$\Bigg(\frac{\left(1-e^{-(\theta-\pi(w))(t_3-t_2)}\right)}{\theta-\pi(w)}+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_2)}-e^{-(\theta-\pi(w))(t_2-t_1)}\right)\Bigg)+$$

$$\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}$$

$$\Bigg(\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_1)}-e^{-(\theta-\pi(w))(t_2-t_1)}\right)+\frac{1}{\theta-\pi(w)}\Big(e^{-(\theta-\pi(w))t_3}-$$

$$e^{-(\theta-\pi(w))t_2}\Big)\Bigg)+\frac{D(s)}{(\theta-\pi(w))}\Bigg(\frac{1}{\theta-\pi(w)}\left(e^{(\theta-\pi(w))(T-t_3)}-1\right)+(T-t_3)\Bigg)\Bigg]+P_e \tag{12}$$

Due to the investment in green technology, level of carbon emissions is reduced. Thus, net carbon emissions is

$C_E' = \left(1-\varpi\left(1-e^{-\vartheta G}\right)\right)C_E$ where expression C_E is given in equation (12).

Producer have to pay the tax due to the emission of carbon at the end of cycle. Therefore, carbon tax under the effect of inflation is

$$CT = \zeta C_E' e^{-rT}$$

Revenue Generation under the effect of Inflation: Selling price per unit item is $'s'$. Let RG is the present worth of revenue generated by the producer after selling serviceable and reworked items. Thus

$$RG = \int_0^{t_1} s((1-\alpha(P))P + \beta\alpha(P)P)e^{-rt}dt + \int_{t_1}^{t_2} s(1-\alpha(P))aP + \beta\alpha(P)aP)e^{-rt}dt + \int_{t_2}^{t_3} s((1-\alpha(P))bP + \beta\alpha(P)bP)e^{-rt}dt$$

$$= \frac{s((1-\alpha(P))P+\beta\alpha(P)P)}{r}(1-e^{-rt_1}) + \frac{s((1-\alpha(P))aP+\beta\alpha(P)aP)}{r}(e^{-rt_1} - e^{-rt_2}) + \frac{s((1-\alpha(P))bP+\beta\alpha(P)bP)}{r}(e^{-rt_2} - e^{-rt_3}) \quad (13)$$

The difference between the system's total revenue and total inventory costs is its overall profit. The costs associated with setting up an inventory include setup, production, screening, reworking, waste management, deterioration, preservation technology costs, investments in green technologies, and carbon tax. Thus, the whole profit per unit of time is

$$TP = \frac{1}{T}(RG - TC)$$

$$= \frac{1}{T}\Big[\frac{s((1-\alpha(P))P+\beta\alpha(P)P)}{r}(1-e^{-rt_1}) + \frac{s((1-\alpha(P))aP+\beta\alpha(P)aP)}{r}(e^{-rt_1} - e^{-rt_2}) + \frac{s((1-\alpha(P))bP+\beta\alpha(P)bP)}{r}(e^{-rt_2} - e^{-rt_3}) - [S_A + \frac{\varphi(P)P}{r}(1-e^{-rt_1}) + \frac{\varphi(P)aP}{r}(e^{-rt_1} - e^{-rt_2}) + \frac{\varphi(P)bP}{r}(e^{-rt_2} - e^{-rt_3}) + \frac{S_cP}{r}(1-e^{-rt_1}) + \frac{S_caP}{r}(e^{-rt_1} - e^{-rt_2}) + \frac{S_cbP}{r}(e^{-rt_2} - e^{-rt_3}) + \frac{C_r\beta\alpha(P)P}{r}(1-e^{-rt_1}) + \frac{C_r\beta\alpha(P)aP}{r}(e^{-rt_1} - e^{-rt_2}) + \frac{C_r\beta\alpha(P)bP}{r}(e^{-rt_2} - e^{-rt_3}) \frac{C_m(1-\beta)\alpha(P)P}{r}(1-e^{-rt_1}) + \frac{C_m(1-\beta)\alpha(P)aP}{r}(e^{-rt_1} - e^{-rt_2}) + \frac{C_m(1-\beta)\alpha(P)bP}{r}(e^{-rt_2} - e^{-rt_3}) +$$

$$C_h\Bigg[\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(\frac{1}{r}(1-e^{-rt_1}) + \frac{1}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_1} - 1\right)\right) +$$

$$\frac{((1-\alpha(P))aP+\beta\alpha(P)aP-D(s))}{(\theta-\pi(w))}\left(\frac{1}{r}(e^{-rt_1} - e^{-rt_2}) + \frac{e^{(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2} - e^{-(r+\theta-\pi(w))t_1}\right)\right) -$$

$$\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(\frac{e^{(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_1}-e^{-(r+\theta-\pi(w))t_2}\right)+\right.$$

$$\left.\frac{1}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_1}\right)\right)+\frac{((1-\alpha(P))bP+\beta\alpha(P)bP-D(s))}{(\theta-\pi(w))}$$

$$\left(\frac{1}{r}(e^{-rt_2}-e^{-rt_3})+\frac{e^{(\theta-\pi(w))t_2}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_3}-e^{-(r+\theta-\pi(w))t_2}\right)\right)-$$

$$\frac{((1-\alpha(P))aP+\beta\alpha(P)aP-D(s))}{(\theta-\pi(w))}\left(\frac{e^{-(\theta-\pi(w))t_2}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_3}\right)+\right.$$

$$\left.\frac{e^{-(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_3}-e^{-(r+\theta-\pi(w))t_2}\right)\right)+\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}$$

$$\left(\frac{e^{-(r+\theta-\pi(w))t_1}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_3}\right)}{r+\theta-\pi(w)}+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_3}-\right.\right.$$

$$\left.\left.e^{-(\theta-\pi(w))t_2}\right)\right)+\frac{D(s)}{(\theta-\pi(w))}\left(\frac{1}{r+\theta-\pi(w)}\left(e^{(\theta-\pi(w))T-(r+\theta-\pi(w))t_3}-e^{-rT}\right)+\right.$$

$$\left.\left.\frac{1}{r}(e^{-rT}-e^{-rt_3})\right)\right]\quad+C_d\left[\left((1-\alpha(P))P+\beta\alpha(P)P-D(s)\right)\left(\frac{1}{r}(1-e^{-rt_1})+\right.\right.$$

$$\left.\frac{1}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_1}-1\right)\right)+$$

$$\left(\left((1-\alpha(P))aP+\beta\alpha(P)aP-D(s)\right)\right)\left(\frac{1}{r}(e^{-rt_1}-e^{-rt_2})+\right.$$

$$\left.\frac{e^{(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_1}\right)\right)-\left((1-\alpha(P))P+\beta\alpha(P)P-\right.$$

$$\left.D(s)\right)$$

$$\left(\frac{e^{(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_1}-e^{-(r+\theta-\pi(w))t_2}\right)+\frac{1}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2}-\right.\right.$$

$$\left.\left.e^{-(r+\theta-\pi(w))t_1}\right)\right)+\left(\left((1-\alpha(P))bP+\beta\alpha(P)bP-D(s)\right)\right)$$

$$\left(\frac{1}{r}(e^{-rt_2}-e^{-rt_3})+\frac{e^{(\theta-\pi(w))t_2}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_3}-e^{-(r+\theta-\pi(w))t_2}\right)\right)$$

$$-\left(\left((1-\alpha(P))aP+\beta\alpha(P)aP-D(s)\right)\right)$$

$$\left(\frac{e^{-(\theta-\pi(w))t_2}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_3}\right)+\frac{e^{-(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_3}-\right.\right.$$

$$\left.\left.e^{-(r+\theta-\pi(w))t_2}\right)\right)+\left((1-\alpha(P))P+\beta\alpha(P)P-D(s)\right)$$

$$\left(\frac{e^{-(r+\theta-\pi(w))t_1}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_3}\right)}{r+\theta-\pi(w)}+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_3}-e^{-(\theta-\pi(w))t_2}\right)\right)+$$

$$D(s)\left(\frac{1}{r+\theta-\pi(w)}\left(e^{(\theta-\pi(w))T-(r+\theta-\pi(w))t_3}-e^{-rT}\right)+\frac{1}{r}\left(e^{-rT}-e^{-rt_3}\right)\right)\Bigg]+w+G+$$

$$\zeta[S_e+(P_e+I_e)(Pt_1+aP(t_2-t_1)+bP(t_3-t_1))+R_e(\beta\alpha(P)Pt_1+$$
$$\beta\alpha(P)aP(t_2-t_1)+\beta\alpha(P)bP(t_3-t_1))+$$
$$W_e((1-\beta)\alpha(P)Pt_1+(1-\beta)\alpha(P)aP(t_2-t_1)+(1-\beta)\alpha(P)bP(t_3-t_1))+$$
$$C_e\left[\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(t_1+\frac{\left(e^{-(\theta-\pi(w))t_1}-1\right)}{\theta-\pi(w)}\right)+\right.$$

$$\frac{\left((1-\alpha(P))aP+\beta\alpha(P)P-D(s)\right)}{(\theta-\pi(w))}\left((t_2-t_1)+\frac{\left(e^{-(\theta-\pi(w))(t_2-t_1)}-1\right)}{\theta-\pi(w)}\right)-$$

$$\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(\frac{1}{\theta-\pi(w)}\left(1-e^{-(\theta-\pi(w))(t_2-t_1)}\right)+\right.$$

$$\left.\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_2}-e^{-(\theta-\pi(w))t_1}\right)\right)+\frac{\left((1-\alpha(P))bP+\beta\alpha(P)P-D(s)\right)}{(\theta-\pi(w))}\left((t_3-t_2)+\right.$$

$$\left.\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_2)}-1\right)\right)-$$

$$\frac{\left((1-\alpha(P))aP+\beta\alpha(P)P-D(s)\right)}{(\theta-\pi(w))}\left(\frac{\left(1-e^{-(\theta-\pi(w))(t_3-t_2)}\right)}{\theta-\pi(w)}+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_2)}-\right.\right.$$

$$\left.\left.e^{-(\theta-\pi(w))(t_2-t_1)}\right)\right)+\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}$$

$$\left(\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_1)}-e^{-(\theta-\pi(w))(t_2-t_1)}\right)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_3}-\right.\right.$$
$$\left.\left.e^{-(\theta-\pi(w))t_2}\right)\right)+\frac{D(s)}{(\theta-\pi(w))}\left(\frac{1}{\theta-\pi(w)}\left(e^{(\theta-\pi(w))(T-t_3)}-1\right)+(T-t_3)\right)\Bigg]$$

$$+D_e\left[\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(t_1+\frac{\left(e^{-(\theta-\pi(w))t_1}-1\right)}{\theta-\pi(w)}\right)+\right.$$

$$\frac{\left((1-\alpha(P))aP+\beta\alpha(P)P-D(s)\right)}{\left(\theta-\pi(w)\right)}\left((t_2-t_1)+\frac{\left(e^{-(\theta-\pi(w))(t_2-t_1)}-1\right)}{\theta-\pi(w)}\right)-$$

$$\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{\left(\theta-\pi(w)\right)}$$

$$\left(\frac{1}{\theta-\pi(w)}\left(1-e^{-(\theta-\pi(w))(t_2-t_1)}\right)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_2}-e^{-(\theta-\pi(w))t_1}\right)\right)+$$

$$\frac{\left((1-\alpha(P))bP+\beta\alpha(P)P-D(s)\right)}{\left(\theta-\pi(w)\right)}\left((t_3-t_2)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_2)}-1\right)\right)-$$

$$\frac{\left((1-\alpha(P))aP+\beta\alpha(P)P-D(s)\right)}{\left(\theta-\pi(w)\right)}\left(\frac{\left(1-e^{-(\theta-\pi(w))(t_3-t_2)}\right)}{\theta-\pi(w)}+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_2)}-\right.\right.$$

$$\left.\left.e^{-(\theta-\pi(w))(t_2-t_1)}\right)\right)+\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{\left(\theta-\pi(w)\right)}$$

$$\left(\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_1)}-e^{-(\theta-\pi(w))(t_2-t_1)}\right)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_3}-\right.\right.$$

$$\left.\left.e^{-(\theta-\pi(w))t_2}\right)\right)$$

$$+\frac{D(s)}{\left(\theta-\pi(w)\right)}\left(\frac{1}{\theta-\pi(w)}\left(e^{(\theta-\pi(w))(T-t_3)}-1\right)+(T-t_3)\right)\Bigg]+P_e]e^{-rT}]] \qquad (14)$$

Objective of the model is to maximize the profit of the production system. Thus,

$$Max\ TP(s,P)$$

$$s>0, P>0.$$

where $t_1 = x_1T$, $t_2 = x_2T$, $t_3 = x_3T$

4.1 Production inventory model under the effect of Fuzzy Learning

Practically, various costs such as setup cost, reworking cost, cost of waste management, carrying cost, cost due to deterioration, and screening cost associated with production systems are not precise in nature. These costs are imprecise by nature. Therefore, in the current study, these cost parameters are taken as imprecise in nature. Further, it is observed that the measure of imprecision depends on the number of repetitions of a task, i.e., it is affected by the learning effect. As a result, in the current study, it is assumed that learning in impreciseness follows the relationship proposed by Wright (1936). Thus, different cost components under the effect of learning can be represented by a triangular fuzzy number as follows:

$$\widetilde{S_A} = \left(S_A - \Delta_1 i^{-l}, S_A, S_A + \Delta_2 i^{-l}\right), \qquad \widetilde{S_C} = \left(S_C - \Delta_3 i^{-l}, S_C, S_C + \Delta_4 i^{-l}\right),$$

$$\widetilde{C_r} = \left(C_r - \Delta_5 i^{-l}, C_r, C_r + \Delta_6 i^{-l}\right), \qquad \widetilde{C_m} = \left(C_m - \Delta_7 i^{-l}, C_m, C_m + \Delta_6 i^{-l}\right),$$

$$\widetilde{C_h} = \left(C_h - \Delta_9 i^{-l}, C_h, C_h + \Delta_{10} i^{-l}\right), \qquad \widetilde{C_d} = \left(C_d - \Delta_{11} i^{-l}, C_d, C_d + \Delta_{12} i^{-l}\right).$$

where i is number of repetitions of task and l is the learning component.

Under this situation, total profit of the system is

$$\widetilde{TP} = (TP_1, TP_2, TP_3)\ , TP_1 < TP_2 < TP_3$$

Here, centroid method is adopted to defuzzify the total profit of the system. Thus, defuzzified total profit is as follows:

$$TP^{cd} = \frac{TP_1 + TP_2 + TP_3}{3}$$

where

$$TP_1 = \frac{1}{T}\Bigg[\frac{s((1-\alpha(P))P+\beta\alpha(P)P)}{r}(1 - e^{-rt_1}) + \frac{s((1-\alpha(P))aP+\beta\alpha(P)aP)}{r}(e^{-rt_1} - e^{-rt_2}) + \frac{s((1-\alpha(P))bP+\beta\alpha(P)bP)}{r}(e^{-rt_2} - e^{-rt_3}) - \Big[S_A + \Delta_2 i^{-l} + \frac{\varphi(P)P}{r}(1 - e^{-rt_1}) + \frac{\varphi(P)aP}{r}(e^{-rt_1} - e^{-rt_2}) + \frac{\varphi(P)bP}{r}(e^{-rt_2} - e^{-rt_3}) + \frac{\left(S_c+\Delta_4 i^{-l}\right)P}{r}(1 - e^{-rt_1}) + \frac{\left(S_c+\Delta_4 i^{-l}\right)aP}{r}(e^{-rt_1} - e^{-rt_2}) +$$

$$\frac{\left(S_c+\Delta_4 i^{-l}\right)bP}{r}(e^{-rt_2} - e^{-rt_3}) + \frac{\left(C_r+\Delta_6 i^{-l}\right)\beta\alpha(P)P}{r}(1 - e^{-rt_1}) + \frac{\left(C_r+\Delta_6 i^{-l}\right)\beta\alpha(P)aP}{r}(e^{-rt_1} - e^{-rt_2}) +$$

$$\frac{\left(C_r+\Delta_6 i^{-l}\right)\beta\alpha(P)bP}{r}(e^{-rt_2} - e^{-rt_3}) + \frac{\left(C_m+\Delta_8 i^{-l}\right)(1-\beta)\alpha(P)P}{r}(1 - e^{-rt_1}) + \frac{\left(C_m+\Delta_8 i^{-l}\right)(1-\beta)\alpha(P)aP}{r}(e^{-rt_1} - e^{-rt_2}) + \frac{\left(C_m+\Delta_8 i^{-l}\right)(1-\beta)\alpha(P)bP}{r}(e^{-rt_2} - e^{-rt_3}) +$$

$$\left(C_h + \Delta_{10} i^{-l}\right)\Bigg[\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(\frac{1}{r}(1 - e^{-rt_1}) + \frac{1}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_1} - 1\right)\right) +$$

$$\frac{\left((1-\alpha(P))aP+\beta\alpha(P)aP-D(s)\right)}{\left(\theta-\pi(w)\right)}\left(\frac{1}{r}(e^{-rt_1} - e^{-rt_2}) + \frac{e^{(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-\left(r+\theta-\pi(w)\right)t_2} - e^{-\left(r+\theta-\pi(w)\right)t_1}\right)\right) -$$

$$\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(\frac{e^{(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_1}-e^{-(r+\theta-\pi(w))t_2}\right)+\right.$$

$$\left.\frac{1}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_1}\right)\right)+\frac{((1-\alpha(P))bP+\beta\alpha(P)bP-D(s))}{(\theta-\pi(w))}$$

$$\left(\frac{1}{r}(e^{-rt_2}-e^{-rt_3})+\frac{e^{(\theta-\pi(w))t_2}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_3}-e^{-(r+\theta-\pi(w))t_2}\right)\right)-$$

$$\frac{((1-\alpha(P))aP+\beta\alpha(P)aP-D(s))}{(\theta-\pi(w))}\left(\frac{e^{-(\theta-\pi(w))t_2}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_3}\right)+\right.$$

$$\left.\frac{e^{-(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_3}-e^{-(r+\theta-\pi(w))t_2}\right)\right)+\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}$$

$$\left(\frac{e^{-(r+\theta-\pi(w))t_1}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_3}\right)}{r+\theta-\pi(w)}+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_3}-\right.\right.$$

$$\left.\left.e^{-(\theta-\pi(w))t_2}\right)\right)+\frac{D(s)}{(\theta-\pi(w))}\left(\frac{1}{r+\theta-\pi(w)}\left(e^{(\theta-\pi(w))T-(r+\theta-\pi(w))t_3}-e^{-rT}\right)+\right.$$

$$\left.\left.\frac{1}{r}(e^{-rT}-e^{-rt_3})\right)\right]+\left(C_d+\Delta_{12}i^{-l}\right)$$

$$\left[\left((1-\alpha(P))P+\beta\alpha(P)P-D(s)\right)\left(\frac{1}{r}(1-e^{-rt_1})+\right.\right.$$

$$\left.\frac{1}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_1}-1\right)\right)+$$

$$\left(\left((1-\alpha(P))aP+\beta\alpha(P)aP-D(s)\right)\right)\left(\frac{1}{r}(e^{-rt_1}-e^{-rt_2})+\right.$$

$$\left.\frac{e^{(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_1}\right)\right)-\left((1-\alpha(P))P+\beta\alpha(P)P-\right.$$

$$\left.D(s)\right)$$

$$\left(\frac{e^{(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_1}-e^{-(r+\theta-\pi(w))t_2}\right)+\frac{1}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2}-\right.\right.$$

$$\left.\left.e^{-(r+\theta-\pi(w))t_1}\right)\right)+$$

$$\left(\left((1-\alpha(P))bP+\beta\alpha(P)bP-D(s)\right)\right)\left(\frac{1}{r}(e^{-rt_2}-e^{-rt_3})+\right.$$

$$\frac{e^{(\theta-\pi(w))t_2}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_3}-e^{-(r+\theta-\pi(w))t_2}\right)\Bigg)-\Big(\big((1-\alpha(P))aP+$$

$$\beta\alpha(P)aP-D(s)\big)\Big)\left(\frac{e^{-(\theta-\pi(w))t_2}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_3}\right)+\right.$$

$$\left.\frac{e^{-(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_3}-e^{-(r+\theta-\pi(w))t_2}\right)\right)+\big((1-\alpha(P))P+\beta\alpha(P)P-$$

$$D(s)\big)$$

$$\left(\frac{e^{-(r+\theta-\pi(w))t_1}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_3}\right)}{r+\theta-\pi(w)}+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_3}-\right.\right.$$

$$\left.\left.e^{-(\theta-\pi(w))t_2}\right)\right)+$$

$$D(s)\left(\frac{1}{r+\theta-\pi(w)}\left(e^{(\theta-\pi(w))T-(r+\theta-\pi(w))t_3}-e^{-rT}\right)+\frac{1}{r}\left(e^{-rT}-e^{-rt_3}\right)\right)\Bigg]+w+G+$$

$$\zeta[S_e+(P_e+I_e)(Pt_1+aP(t_2-t_1)+bP(t_3-t_1))+R_e(\beta\alpha(P)Pt_1+$$
$$\beta\alpha(P)aP(t_2-t_1)+\beta\alpha(P)bP(t_3-t_1))+$$
$$W_e((1-\beta)\alpha(P)Pt_1+(1-\beta)\alpha(P)aP(t_2-t_1)+(1-\beta)\alpha(P)bP(t_3-t_1))+$$

$$C_e\left[\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(t_1+\frac{\left(e^{-(\theta-\pi(w))t_1}-1\right)}{\theta-\pi(w)}\right)+\right.$$

$$\frac{\big((1-\alpha(P))aP+\beta\alpha(P)P-D(s)\big)}{(\theta-\pi(w))}\left((t_2-t_1)+\frac{\left(e^{-(\theta-\pi(w))(t_2-t_1)}-1\right)}{\theta-\pi(w)}\right)-$$

$$\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(\frac{1}{\theta-\pi(w)}\left(1-e^{-(\theta-\pi(w))(t_2-t_1)}\right)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_2}-\right.\right.$$

$$\left.\left.e^{-(\theta-\pi(w))t_1}\right)\right)+$$

$$\frac{\big((1-\alpha(P))bP+\beta\alpha(P)P-D(s)\big)}{(\theta-\pi(w))}\left((t_3-t_2)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_2)}-1\right)\right)-$$

$$\frac{\big((1-\alpha(P))aP+\beta\alpha(P)P-D(s)\big)}{(\theta-\pi(w))}\left(\frac{\left(1-e^{-(\theta-\pi(w))(t_3-t_2)}\right)}{\theta-\pi(w)}+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_2)}-\right.\right.$$

$$\left.\left.e^{-(\theta-\pi(w))(t_2-t_1)}\right)\right)+\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}$$

$$\left(\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_1)}-e^{-(\theta-\pi(w))(t_2-t_1)}\right)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_3}-e^{-(\theta-\pi(w))t_2}\right)\right)+\frac{D(s)}{(\theta-\pi(w))}\left(\frac{1}{\theta-\pi(w)}\left(e^{(\theta-\pi(w))(T-t_3)}-1\right)+(T-t_3)\right)\Bigg]$$

$$+D_e\left[\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(t_1+\frac{\left(e^{-(\theta-\pi(w))t_1}-1\right)}{\theta-\pi(w)}\right)+\frac{((1-\alpha(P))aP+\beta\alpha(P)P-D(s))}{(\theta-\pi(w))}\left((t_2-t_1)+\frac{\left(e^{-(\theta-\pi(w))(t_2-t_1)}-1\right)}{\theta-\pi(w)}\right)-\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\right.$$

$$\left(\frac{1}{\theta-\pi(w)}\left(1-e^{-(\theta-\pi(w))(t_2-t_1)}\right)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_2}-e^{-(\theta-\pi(w))t_1}\right)\right)+\frac{((1-\alpha(P))bP+\beta\alpha(P)P-D(s))}{(\theta-\pi(w))}\left((t_3-t_2)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_2)}-1\right)\right)-$$

$$\frac{((1-\alpha(P))aP+\beta\alpha(P)P-D(s))}{(\theta-\pi(w))}\left(\frac{\left(1-e^{-(\theta-\pi(w))(t_3-t_2)}\right)}{\theta-\pi(w)}+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_2)}-e^{-(\theta-\pi(w))(t_2-t_1)}\right)\right)+\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}$$

$$\left(\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_1)}-e^{-(\theta-\pi(w))(t_2-t_1)}\right)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_3}-e^{-(\theta-\pi(w))t_2}\right)\right)$$

$$+\frac{D(s)}{(\theta-\pi(w))}\left(\frac{1}{\theta-\pi(w)}\left(e^{(\theta-\pi(w))(T-t_3)}-1\right)+(T-t_3)\right)\Bigg]+P_e]e^{-rT}]]$$

$$TP_2=\frac{1}{T}\Bigg[\frac{s((1-\alpha(P))P+\beta\alpha(P)P)}{r}(1-e^{-rt_1})+\frac{s((1-\alpha(P))aP+\beta\alpha(P)aP)}{r}(e^{-rt_1}-e^{-rt_2})+\frac{s((1-\alpha(P))bP+\beta\alpha(P)bP)}{r}(e^{-rt_2}-e^{-rt_3})-[S_A+\frac{\varphi(P)P}{r}(1-e^{-rt_1})+\frac{\varphi(P)aP}{r}(e^{-rt_1}-e^{-rt_2})+\frac{\varphi(P)bP}{r}(e^{-rt_2}-e^{-rt_3})+\frac{S_cP}{r}(1-e^{-rt_1})+\frac{S_caP}{r}(e^{-rt_1}-e^{-rt_2})+\frac{S_cbP}{r}(e^{-rt_2}-e^{-rt_3})+$$

$$\frac{C_r\beta\alpha(P)P}{r}(1-e^{-rt_1})+\frac{C_r\beta\alpha(P)aP}{r}(e^{-rt_1}-e^{-rt_2})+\frac{C_r\beta\alpha(P)bP}{r}(e^{-rt_2}-e^{-rt_3})\frac{C_m(1-\beta)\alpha(P)P}{r}(1-e^{-rt_1})+\frac{C_m(1-\beta)\alpha(P)aP}{r}(e^{-rt_1}-e^{-rt_2})+\frac{C_m(1-\beta)\alpha(P)bP}{r}(e^{-rt_2}-e^{-rt_3})+$$

$$C_h\left[\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(\frac{1}{r}(1-e^{-rt_1})+\frac{1}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_1}-1\right)\right)+\right.$$

$$\frac{\left((1-\alpha(P))aP+\beta\alpha(P)aP-D(s)\right)}{(\theta-\pi(w))}\left(\frac{1}{r}(e^{-rt_1}-e^{-rt_2})+\frac{e^{(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2}-\right.\right.$$

$$\left.\left.e^{-(r+\theta-\pi(w))t_1}\right)\right)-$$

$$\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(\frac{e^{(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_1}-e^{-(r+\theta-\pi(w))t_2}\right)+\right.$$

$$\left.\frac{1}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_1}\right)\right)+\frac{\left((1-\alpha(P))bP+\beta\alpha(P)bP-D(s)\right)}{(\theta-\pi(w))}$$

$$\left(\frac{1}{r}(e^{-rt_2}-e^{-rt_3})+\frac{e^{(\theta-\pi(w))t_2}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_3}-e^{-(r+\theta-\pi(w))t_2}\right)\right)-$$

$$\frac{\left((1-\alpha(P))aP+\beta\alpha(P)aP-D(s)\right)}{(\theta-\pi(w))}\left(\frac{e^{-(\theta-\pi(w))t_2}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_3}\right)+\right.$$

$$\left.\frac{e^{-(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_3}-e^{-(r+\theta-\pi(w))t_2}\right)\right)+\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}$$

$$\left(\frac{e^{-(r+\theta-\pi(w))t_1}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_3}\right)}{r+\theta-\pi(w)}+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_3}-\right.\right.$$

$$\left.\left.e^{-(\theta-\pi(w))t_2}\right)\right)+\frac{D(s)}{(\theta-\pi(w))}\left(\frac{1}{r+\theta-\pi(w)}\left(e^{(\theta-\pi(w))T-(r+\theta-\pi(w))t_3}-e^{-rT}\right)+\right.$$

$$\left.\left.\frac{1}{r}(e^{-rT}-e^{-rt_3})\right)\right]+$$

$$C_d\left[\left((1-\alpha(P))P+\beta\alpha(P)P-D(s)\right)\left(\frac{1}{r}(1-e^{-rt_1})+\right.\right.$$

$$\left.\frac{1}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_1}-1\right)\right)+$$

$$\left(\left((1-\alpha(P))aP+\beta\alpha(P)aP-D(s)\right)\right)\left(\frac{1}{r}(e^{-rt_1}-e^{-rt_2})+\right.$$

$$\left.\frac{e^{(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_1}\right)\right)-\left((1-\alpha(P))P+\beta\alpha(P)P-D(s)\right)$$

$$\left(\frac{e^{(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_1}-e^{-(r+\theta-\pi(w))t_2}\right)+\frac{1}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_1}\right)\right)+\left(\left((1-\alpha(P))bP+\beta\alpha(P)bP-D(s)\right)\right)$$

$$\left(\frac{1}{r}\left(e^{-rt_2}-e^{-rt_3}\right)+\frac{e^{(\theta-\pi(w))t_2}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_3}-e^{-(r+\theta-\pi(w))t_2}\right)\right)-$$

$$\left(\left((1-\alpha(P))aP+\beta\alpha(P)aP-D(s)\right)\right)$$

$$\left(\frac{e^{-(\theta-\pi(w))t_2}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_3}\right)+\frac{e^{-(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_3}-e^{-(r+\theta-\pi(w))t_2}\right)\right)+\left((1-\alpha(P))P+\beta\alpha(P)P-D(s)\right)$$

$$\left(\frac{e^{-(r+\theta-\pi(w))t_1}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_3}\right)}{r+\theta-\pi(w)}+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_3}-e^{-(\theta-\pi(w))t_2}\right)\right)+$$

$$D(s)\left(\frac{1}{r+\theta-\pi(w)}\left(e^{(\theta-\pi(w))T-(r+\theta-\pi(w))t_3}-e^{-rT}\right)+\frac{1}{r}\left(e^{-rT}-e^{-rt_3}\right)\right)\Bigg]+w+G+$$

$$\zeta[S_e+(P_e+I_e)(Pt_1+aP(t_2-t_1)+bP(t_3-t_1))+R_e(\beta\alpha(P)Pt_1+\beta\alpha(P)aP(t_2-t_1)+\beta\alpha(P)bP(t_3-t_1))+$$

$$W_e((1-\beta)\alpha(P)Pt_1+(1-\beta)\alpha(P)aP(t_2-t_1)+(1-\beta)\alpha(P)bP(t_3-t_1))+$$

$$C_e\left[\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(t_1+\frac{\left(e^{-(\theta-\pi(w))t_1}-1\right)}{\theta-\pi(w)}\right)+\right.$$

$$\frac{\left((1-\alpha(P))aP+\beta\alpha(P)P-D(s)\right)}{(\theta-\pi(w))}\left((t_2-t_1)+\frac{\left(e^{-(\theta-\pi(w))(t_2-t_1)}-1\right)}{\theta-\pi(w)}\right)-\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}$$

$$\left(\frac{1}{\theta-\pi(w)}\left(1-e^{-(\theta-\pi(w))(t_2-t_1)}\right)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_2}-e^{-(\theta-\pi(w))t_1}\right)\right)+$$

$$\frac{\left((1-\alpha(P))bP+\beta\alpha(P)P-D(s)\right)}{(\theta-\pi(w))}\left((t_3-t_2)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_2)}-1\right)\right)-$$

$$\frac{\left((1-\alpha(P))aP+\beta\alpha(P)P-D(s)\right)}{\left(\theta-\pi(w)\right)}\left(\frac{\left(1-e^{-\left(\theta-\pi(w)\right)(t_3-t_2)}\right)}{\theta-\pi(w)}+\frac{1}{\theta-\pi(w)}\left(e^{-\left(\theta-\pi(w)\right)(t_3-t_2)}-\right.\right.$$

$$\left.\left.e^{-\left(\theta-\pi(w)\right)(t_2-t_1)}\right)\right)+\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{\left(\theta-\pi(w)\right)}$$

$$\left(\frac{1}{\theta-\pi(w)}\left(e^{-\left(\theta-\pi(w)\right)(t_3-t_1)}-e^{-\left(\theta-\pi(w)\right)(t_2-t_1)}\right)+\frac{1}{\theta-\pi(w)}\left(e^{-\left(\theta-\pi(w)\right)t_3}-\right.\right.$$

$$\left.\left.e^{-\left(\theta-\pi(w)\right)t_2}\right)\right)+\frac{D(s)}{\left(\theta-\pi(w)\right)}\left(\frac{1}{\theta-\pi(w)}\left(e^{\left(\theta-\pi(w)\right)(T-t_3)}-1\right)+(T-t_3)\right)\Bigg]$$

$$+D_e\left[\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{\left(\theta-\pi(w)\right)}\left(t_1+\frac{\left(e^{-\left(\theta-\pi(w)\right)t_1}-1\right)}{\theta-\pi(w)}\right)+\right.$$

$$\frac{\left((1-\alpha(P))aP+\beta\alpha(P)P-D(s)\right)}{\left(\theta-\pi(w)\right)}\left((t_2-t_1)+\frac{\left(e^{-\left(\theta-\pi(w)\right)(t_2-t_1)}-1\right)}{\theta-\pi(w)}\right)-$$

$$\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{\left(\theta-\pi(w)\right)}$$

$$\left(\frac{1}{\theta-\pi(w)}\left(1-e^{-\left(\theta-\pi(w)\right)(t_2-t_1)}\right)+\frac{1}{\theta-\pi(w)}\left(e^{-\left(\theta-\pi(w)\right)t_2}-e^{-\left(\theta-\pi(w)\right)t_1}\right)\right)+$$

$$\frac{\left((1-\alpha(P))bP+\beta\alpha(P)P-D(s)\right)}{\left(\theta-\pi(w)\right)}\left((t_3-t_2)+\frac{1}{\theta-\pi(w)}\left(e^{-\left(\theta-\pi(w)\right)(t_3-t_2)}-1\right)\right)-$$

$$\frac{\left((1-\alpha(P))aP+\beta\alpha(P)P-D(s)\right)}{\left(\theta-\pi(w)\right)}\left(\frac{\left(1-e^{-\left(\theta-\pi(w)\right)(t_3-t_2)}\right)}{\theta-\pi(w)}+\frac{1}{\theta-\pi(w)}\left(e^{-\left(\theta-\pi(w)\right)(t_3-t_2)}-\right.\right.$$

$$\left.\left.e^{-\left(\theta-\pi(w)\right)(t_2-t_1)}\right)\right)+\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{\left(\theta-\pi(w)\right)}$$

$$\left(\frac{1}{\theta-\pi(w)}\left(e^{-\left(\theta-\pi(w)\right)(t_3-t_1)}-e^{-\left(\theta-\pi(w)\right)(t_2-t_1)}\right)+\frac{1}{\theta-\pi(w)}\left(e^{-\left(\theta-\pi(w)\right)t_3}-\right.\right.$$

$$\left.\left.e^{-\left(\theta-\pi(w)\right)t_2}\right)\right)$$

$$+\frac{D(s)}{\left(\theta-\pi(w)\right)}\left(\frac{1}{\theta-\pi(w)}\left(e^{\left(\theta-\pi(w)\right)(T-t_3)}-1\right)+(T-t_3)\right)\Bigg]+P_e]e^{-rT}]]$$

$$TP_3=\frac{1}{T}\Big[\frac{s((1-\alpha(P))P+\beta\alpha(P)P)}{r}(1-e^{-rt_1})+\frac{s((1-\alpha(P))aP+\beta\alpha(P)aP)}{r}(e^{-rt_1}-$$

$$e^{-rt_2})+\frac{s((1-\alpha(P))bP+\beta\alpha(P)bP)}{r}(e^{-rt_2}-e^{-rt_3})-[S_A-\Delta_1 i^{-l}+\frac{\varphi(P)P}{r}(1-$$

$$e^{-rt_1}) + \frac{\varphi(P)aP}{r}(e^{-rt_1} - e^{-rt_2}) + \frac{\varphi(P)bP}{r}(e^{-rt_2} - e^{-rt_3}) + \frac{(S_c - \Delta_3 i^{-l})P}{r}(1 - e^{-rt_1}) + \frac{(S_c - \Delta_3 i^{-l})aP}{r}(e^{-rt_1} - e^{-rt_2}) +$$

$$\frac{(S_c - \Delta_3 i^{-l})bP}{r}(e^{-rt_2} - e^{-rt_3}) + \frac{(C_r - \Delta_5 i^{-l})\beta\alpha(P)P}{r}(1 - e^{-rt_1}) + \frac{(C_r - \Delta_5 i^{-l})\beta\alpha(P)aP}{r}(e^{-rt_1} - e^{-rt_2}) +$$

$$\frac{(C_r - \Delta_5 i^{-l})\beta\alpha(P)bP}{r}(e^{-rt_2} - e^{-rt_3}) + \frac{(C_m - \Delta_7 i^{-l})(1-\beta)\alpha(P)P}{r}(1 - e^{-rt_1}) + \frac{(C_m - \Delta_7 i^{-l})(1-\beta)\alpha(P)aP}{r}(e^{-rt_1} - e^{-rt_2}) + \frac{(C_m - \Delta_7 i^{-l})(1-\beta)\alpha(P)bP}{r}(e^{-rt_2} - e^{-rt_3}) +$$

$$(C_h - \Delta_9 i^{-l})\left[\frac{(1-\alpha(P))P + \beta\alpha(P)P - D(s)}{(\theta - \pi(w))}\left(\frac{1}{r}(1 - e^{-rt_1}) + \frac{1}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_1} - 1\right)\right) + \right.$$

$$\frac{((1-\alpha(P))aP + \beta\alpha(P)aP - D(s))}{(\theta - \pi(w))}\left(\frac{1}{r}(e^{-rt_1} - e^{-rt_2}) + \frac{e^{(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2} - e^{-(r+\theta-\pi(w))t_1}\right)\right) -$$

$$\frac{(1-\alpha(P))P + \beta\alpha(P)P - D(s)}{(\theta - \pi(w))}\left(\frac{e^{(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_1} - e^{-(r+\theta-\pi(w))t_2}\right) + \frac{1}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2} - e^{-(r+\theta-\pi(w))t_1}\right)\right) + \frac{((1-\alpha(P))bP + \beta\alpha(P)bP - D(s))}{(\theta - \pi(w))}$$

$$\left(\frac{1}{r}(e^{-rt_2} - e^{-rt_3}) + \frac{e^{(\theta-\pi(w))t_2}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_3} - e^{-(r+\theta-\pi(w))t_2}\right)\right) -$$

$$\frac{((1-\alpha(P))aP + \beta\alpha(P)aP - D(s))}{(\theta - \pi(w))}\left(\frac{e^{-(\theta-\pi(w))t_2}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2} - e^{-(r+\theta-\pi(w))t_3}\right) + \frac{e^{-(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_3} - e^{-(r+\theta-\pi(w))t_2}\right)\right) + \frac{(1-\alpha(P))P + \beta\alpha(P)P - D(s)}{(\theta - \pi(w))}$$

$$\left(\frac{e^{-(r+\theta-\pi(w))t_1}\left(e^{-(r+\theta-\pi(w))t_2} - e^{-(r+\theta-\pi(w))t_3}\right)}{r+\theta-\pi(w)} + \frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_3} - e^{-(\theta-\pi(w))t_2}\right)\right) + \frac{D(s)}{(\theta - \pi(w))}\left(\frac{1}{r+\theta-\pi(w)}\left(e^{(\theta-\pi(w))T - (r+\theta-\pi(w))t_3} - e^{-rT}\right) + \right.$$

$$\left.\left.\frac{1}{r}(e^{-rT} - e^{-rt_3})\right)\right] + (C_d - \Delta_{11} i^{-l})$$

$$\Bigg[\big((1-\alpha(P))P+\beta\alpha(P)P-D(s)\big)\Bigg(\frac{1}{r}(1-e^{-rt_1})+$$

$$\frac{1}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_1}-1\right)\Bigg)+$$

$$\Big(\big((1-\alpha(P))aP+\beta\alpha(P)aP-D(s)\big)\Big)\Bigg(\frac{1}{r}(e^{-rt_1}-e^{-rt_2})+$$

$$\frac{e^{(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_1}\right)\Bigg)-\big((1-\alpha(P))P+\beta\alpha(P)P-D(s)\big)$$

$$\Bigg(\frac{e^{(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_1}-e^{-(r+\theta-\pi(w))t_2}\right)+\frac{1}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2}-\right.$$

$$\left.e^{-(r+\theta-\pi(w))t_1}\right)\Bigg)+\Big(\big((1-\alpha(P))bP+\beta\alpha(P)bP-D(s)\big)\Big)$$

$$\Bigg(\frac{1}{r}(e^{-rt_2}-e^{-rt_3})+\frac{e^{(\theta-\pi(w))t_2}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_3}-e^{-(r+\theta-\pi(w))t_2}\right)\Bigg)-$$

$$\Big(\big((1-\alpha(P))aP+\beta\alpha(P)aP-D(s)\big)\Big)$$

$$\Bigg(\frac{e^{-(\theta-\pi(w))t_2}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_3}\right)+\frac{e^{-(\theta-\pi(w))t_1}}{r+\theta-\pi(w)}\left(e^{-(r+\theta-\pi(w))t_3}-\right.$$

$$\left.e^{-(r+\theta-\pi(w))t_2}\right)\Bigg)+\big((1-\alpha(P))P+\beta\alpha(P)P-D(s)\big)$$

$$\Bigg(\frac{e^{-(r+\theta-\pi(w))t_1}\left(e^{-(r+\theta-\pi(w))t_2}-e^{-(r+\theta-\pi(w))t_3}\right)}{r+\theta-\pi(w)}+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_3}-\right.$$

$$\left.e^{-(\theta-\pi(w))t_2}\right)\Bigg)+$$

$$D(s)\left(\frac{1}{r+\theta-\pi(w)}\left(e^{(\theta-\pi(w))T-(r+\theta-\pi(w))t_3}-e^{-rT}\right)+\frac{1}{r}\left(e^{-rT}-e^{-rt_3}\right)\right)\Bigg]+w+G+$$

$$\zeta[S_e+(P_e+I_e)(Pt_1+aP(t_2-t_1)+bP(t_3-t_1))+R_e(\beta\alpha(P)Pt_1+$$
$$\beta\alpha(P)aP(t_2-t_1)+\beta\alpha(P)bP(t_3-t_1))+$$
$$W_e((1-\beta)\alpha(P)Pt_1+(1-\beta)\alpha(P)aP(t_2-t_1)+(1-\beta)\alpha(P)bP(t_3-t_1))+$$

$$C_e\left[\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(t_1+\frac{\left(e^{-(\theta-\pi(w))t_1}-1\right)}{\theta-\pi(w)}\right)+\right.$$

$$\frac{\big((1-\alpha(P))aP+\beta\alpha(P)P-D(s)\big)}{(\theta-\pi(w))}\left((t_2-t_1)+\frac{\left(e^{-(\theta-\pi(w))(t_2-t_1)}-1\right)}{\theta-\pi(w)}\right)-$$

$$\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(\frac{1}{\theta-\pi(w)}\left(1-e^{-(\theta-\pi(w))(t_2-t_1)}\right)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_2}-e^{-(\theta-\pi(w))t_1}\right)\right)+$$

$$\frac{((1-\alpha(P))bP+\beta\alpha(P)P-D(s))}{(\theta-\pi(w))}\left((t_3-t_2)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_2)}-1\right)\right)-$$

$$\frac{((1-\alpha(P))aP+\beta\alpha(P)P-D(s))}{(\theta-\pi(w))}\left(\frac{\left(1-e^{-(\theta-\pi(w))(t_3-t_2)}\right)}{\theta-\pi(w)}+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_2)}-e^{-(\theta-\pi(w))(t_2-t_1)}\right)\right)+\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}$$

$$\left(\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_1)}-e^{-(\theta-\pi(w))(t_2-t_1)}\right)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_3}-e^{-(\theta-\pi(w))t_2}\right)\right)+\frac{D(s)}{(\theta-\pi(w))}\left(\frac{1}{\theta-\pi(w)}\left(e^{(\theta-\pi(w))(T-t_3)}-1\right)+(T-t_3)\right)\Bigg]$$

$$+D_e\Bigg[\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}\left(t_1+\frac{\left(e^{-(\theta-\pi(w))t_1}-1\right)}{\theta-\pi(w)}\right)+$$

$$\frac{((1-\alpha(P))aP+\beta\alpha(P)P-D(s))}{(\theta-\pi(w))}\left((t_2-t_1)+\frac{\left(e^{-(\theta-\pi(w))(t_2-t_1)}-1\right)}{\theta-\pi(w)}\right)-$$

$$\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}$$

$$\left(\frac{1}{\theta-\pi(w)}\left(1-e^{-(\theta-\pi(w))(t_2-t_1)}\right)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_2}-e^{-(\theta-\pi(w))t_1}\right)\right)+$$

$$\frac{((1-\alpha(P))bP+\beta\alpha(P)P-D(s))}{(\theta-\pi(w))}\left((t_3-t_2)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_2)}-1\right)\right)-$$

$$\frac{((1-\alpha(P))aP+\beta\alpha(P)P-D(s))}{(\theta-\pi(w))}\left(\frac{\left(1-e^{-(\theta-\pi(w))(t_3-t_2)}\right)}{\theta-\pi(w)}+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_2)}-e^{-(\theta-\pi(w))(t_2-t_1)}\right)\right)+\frac{(1-\alpha(P))P+\beta\alpha(P)P-D(s)}{(\theta-\pi(w))}$$

$$\left(\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))(t_3-t_1)}-e^{-(\theta-\pi(w))(t_2-t_1)}\right)+\frac{1}{\theta-\pi(w)}\left(e^{-(\theta-\pi(w))t_3}-e^{-(\theta-\pi(w))t_2}\right)\right)$$

$$+\frac{D(s)}{(\theta-\pi(w))}\left(\frac{1}{\theta-\pi(w)}\left(e^{(\theta-\pi(w))(T-t_3)}-1\right)+(T-t_3)\right)\Bigg]+P_e]e^{-rT}]]$$

Objective of the model is to maximize the profit of the production system. Thus,

$$Max\ TP^{cd}(s,P)$$

$$s>0, P>0. T>0$$

5. Solution Methodology

Following steps have been followed to obtain the optimal solution:

Step-1: Find the following derivatives:

$$\frac{\partial TP}{\partial s}, and\ \frac{\partial TP}{\partial P}$$

Step-2: Find the necessary conditions for optimality:

$$\frac{\partial TP}{\partial s}=0, and\ \frac{\partial TP}{\partial P}=0$$

Step-3: Obtain the solution of above system of equations say (s, P).

Step-4: Determine the nature of the Hessian matrix at obtain point. If $H_{11}<0\ and\ H_{22}>0$ at the point (s, P). then the objective function is concave in nature and the point (s^*, P^*) gives the optimal solution.

6. Numerical Analysis

Example-1(Crisp Case): Following input parameters are used the analyse the production-inventory model in the case of crisp parameters:

$S_A=100,\ C_r=30,\ C_m=2,\ C_h=5,\ C_d=14,\ S_c=0.5,\ d_0=800,\ d_1=0.8,\ \theta=0.02, w=230,$
$p_1=0.01,\ w=230,\ R_m=50,\ L_c=150,\ T_c=0.01,\ r=0.06,\ \gamma_1=0.2,\ \gamma_2=0.4, P_{min}=800, P_{max}=1200, \beta=0.70, \varpi=0.6,\ \vartheta=0.01, G=306,\ S_e=2,\ P_e=0.5,\ I_e=0.3,\ R_e=0.5,\ W_e=1,\ C_e=0.6,\ D_e=1.5,\ P_e=2,\ \zeta=0.20, T=1, x_1=0.7, x_2=0.8, x_3=0.9.$

On applying the said methodology to obtain the optimal solution, we get the optimal solution as follows:

Table 1: Optimum Solution in Crisp Case.

s^*	P^*	TP^*
132	879	74953

Example-2(Fuzzy Learning Case): Following additional input parameters are used to analyse the production-inventory model in the case of fuzzy learning:

$\Delta_1=15,\ \Delta_2=20,\ \Delta_3=0.05,\ \Delta_4=0.06,\ \Delta_5=4,\ \Delta_6=5,\ \Delta_7=0.20,\ \Delta_6=0.21,\ \Delta_9=0.45, \Delta_{10}=0.59, \Delta_{11}=1.2, \Delta_{12}=1.3, i=2, l=0.31$

On applying the said methodology to obtain the optimal solution, we get the optimal solution as follows:

Table 2 : Optimum Solution in Fuzzy Learning Case.

s^*	P^*	TP^{cd*}
134	889	76602

From Table-1 and Table-2, it is observed that due to learning in fuzziness, total profit of the production-inventory increases. This, result shows the importance of learning in fuzziness.

6.1 Sensitivity Analysis

This section contains the sensitivity analysis with respect to key parameters to analyse their impact on the model behaviour.

a. Effect of Learning in Fuzziness

Fig.1 reflects the effect of learning parameter (l) and number of repetitions (i) on the profit of the production-inventory system. It is observed that for fixed learning parameter, as the number of repetitions of task increases total profit of the system increases. Initially, slope of profit curve is high and as the values of i increases curves become straight. Further, for the fixed value of i, as the learning rate increases profit of the production-inventory system increases. From the figure, it is observed that for high learning rate with high number of repetition of task, profit of the system is very high.

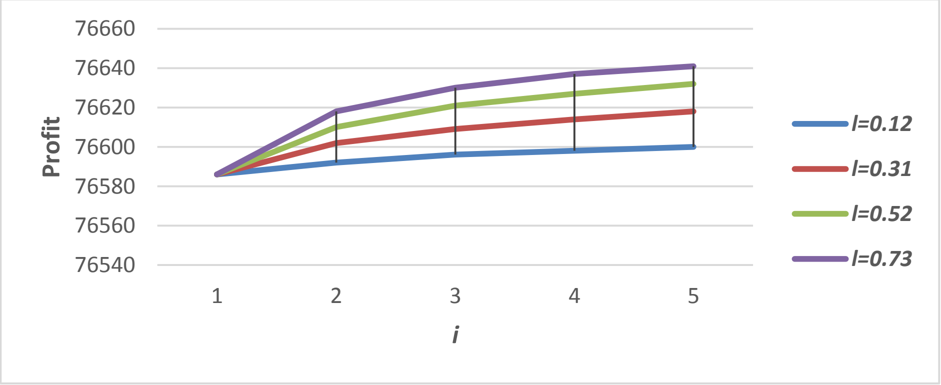

Fig.1 : Effect of learning in Fuzziness.

b. Effect of Green Investment

The profit of the production-inventory system and the amount of carbon emitted throughout the entire operational process of production are shown in Figure 2 as a result of investments in green technology. It is observed that the capital investment in green technology in increases, total emitted units of carbon decreases whereas profit of the production system increases. Thus, investment in green technology is not beneficial only financially but also environmentally.

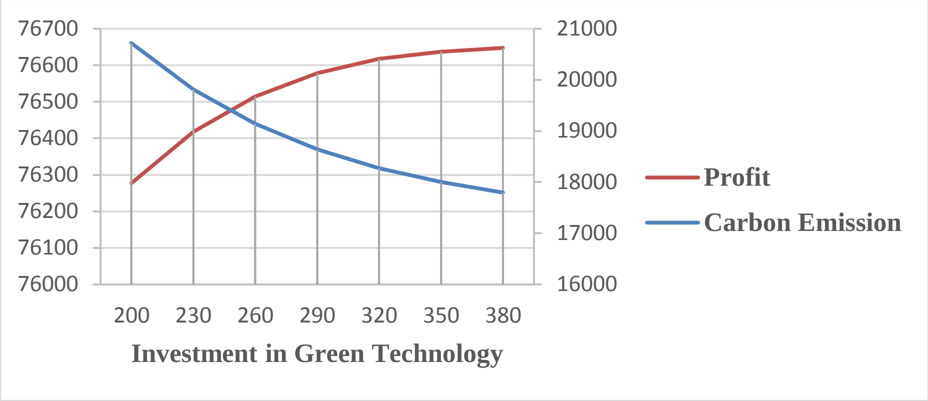

Fig. 2 : Effect of Green Investment.

c. Effect of Inflation

The impact of inflation on the production-inventory model's profit is depicted in Figure 3. It demonstrates how the system's profit declines when inflation rates rise. Thus, instability in market causes negative effect on the profit. As the inflation increases from 0.04 to 0.08, profit of the system in increased by 1.47%.

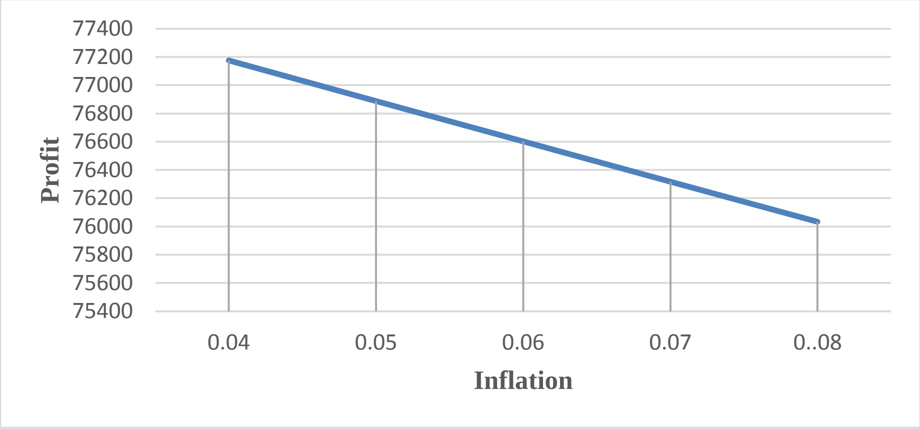

Fig. 3 : Effect of Green Investment.

d. Effect of Deterioration Rate

Figure 4 illustrates how the system's profit is negatively impacted by the pace of deterioration. Because of the change in the rate of deterioration, it has been noted that profit has decreased by 2.55%. This is because as the rate of deterioration increases, deterioration cost and carbon emission due to the deterioration increases.

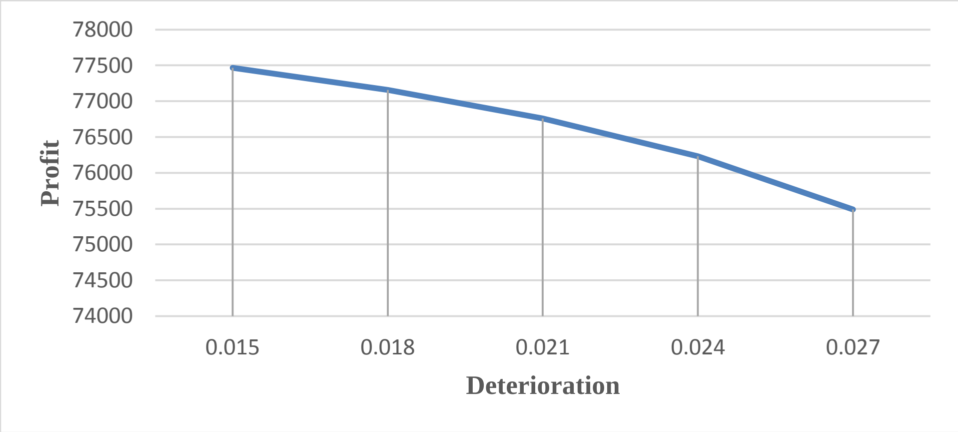

Fig. 4 : Effect of Green Investment.

e. Effect of Other Key Parameters on Profit

Here, values of parameters are increased and decreased by 25% and 50% respectively. Results are summarized in Table-3.

Table 3: Sensitivity Analysis w.r.t. Key Parameters.

Parameter	% Change	TP^{cd}	Parameter	% Change	TP^{cd}
S_A	-50%	76652	γ_1	-50%	81552
	-25%	76627		-25%	79077
	25%	76577		25%	74126
	50%	76552		50%	71651
C_r	-50%	79555	γ_2	-50%	78805
	-25%	78078		-25%	77703
	25%	75125		25%	75500
	50%	73648		50%	74398

S_c	-50%	76845	β	-50%	69860
	-25%	76723		-25%	73231
	25%	76480		25%	79972
	50%	76358		50%	83343
C_m	-50%	76686	ζ	-50%	78338
	-25%	76644		-25%	77470
	25%	76559		25%	75734
	50%	76517		50%	74866
C_h	-50%	77097	ω	-50%	69274
	-25%	76849		-25%	73948
	25%	76354		25%	78117
	50%	76106		50%	78991
d_0	-50%	75451	d_1	-50%	76447
	-25%	76026		-25%	76524
	25%	77177		25%	76679
	50%	77752		50%	76756

i. From Table 3, it is observed that parameters S_A, C_r, S_c, C_m, C_h, γ_1, γ_2, and ζ have an adverse effect on the production-inventory system's profitability. whereas d_0, ω, and β are associated with the system's earnings in a good way.

ii. The production-inventory system's overall profit varies by 7.42% as a result of variations in reworking costs.

iii. Due to the change in the parameter γ_1, total profit of the system is declined by 12.14% whereas 5.59% decline in the profit is observed due to the variation of γ_2. This outcome demonstrates how important the system relies on the components of the defective production rate.

iv. As the carbon tax increases, the tax component increases, and the total profit of the system declines by 4.43%.

v. Preservation technology contributes to the system's financial success. The number of degraded units declines as preservation technology investment rises, which also lowers the cost of deterioration. The system benefits from all of this.

vi. It can be seen in Table 2 that the system's overall profit improves by 19.30% as the proportion of defective units that can be repaired rises.

vii. Table 2 shows that the total profit is only marginally responsive to setup, screening, and waste handling costs.

viii. From the table 2, it is observed that profit is highly sensitive with respect to C_r, γ_1, γ_2, and ζ in negative direction whereas highly sensitive in positive direction with respect to d_0, β, and ω.

7. Managerial Insights

Present section shows the managerial insights of the proposed model. These are as follows:

1. Results suggested that a regular training program must be organized in the system to enhance the decision-making capacity. Through this program, they can determine the various parameters as precisely as possible.
2. The decision-maker must focus on the investment in green technology as it is helpful for the environment. This also helps to achieve the goal of financial sustainability. This contributes to achieving the goal of a cleaner production system.
3. The decision-maker choose effective preservation technology since it helps to lower deterioration units and their negative environmental effects.
4. According to the current study, the decision-maker must make decisions considering inflation, or else some incorrect financial situation is noted in the documents.
5. A current study suggests that a flawless screening process must be carried out as the profit of the system is highly sensitive with respect to the proportion of defective units that can be reworked.
6. A current study suggests that the policy should be designed to lower the carbon emissions due to the operational activities of the production system.

8. Concluding Remarks with Future Extensions

Practically, carbon emissions cannot be stopped during the production and remanufacturing processes in the manufacturing system. Production, reworking, holding the stock in the system, deterioration, preservation technology, etc., are the main operations activities that cause carbon emissions in the environment. The current study aims to develop a model that will provide direction to the decision-makers regarding how they will make decisions regarding the production policy under the carbon regulatory authority. Green investment and preservation technology are the main features of the current study that help to minimize the negative impact of carbon emissions on the environment. Further, analysis is carried out under the effect of inflation to get the real picture of the financial position. Results indicate that reworking and waste management are helpful to obtain the objective of a cleaner production system. Results indicate that green investment, preservation technology, an increase in the proportion of defective units that can be reworked, and base demand boost the profit of the system, whereas reworking costs and taxes levied by the regulatory authority reduce the profit of the system.

There are numerous areas where current research could be expanded. The current model is analyzed with a carbon tax policy. In the future, models can be analyzed with different carbon mechanisms, such as carbon caps and carbon cap and trade policies. Further, shortages can be considered in the current model. Fractional calculus can be applied to analyze the effect of memory on the optimal decision.

References

Bachar, R. K., Bhuniya, S., Ghosh, S. K., AlArjani, A., Attia, E., Uddin, M. S., and Sarkar, B. (2023): Product outsourcing policy for a sustainable flexible manufacturing system with reworking and green investment, *Mathematical Biosciences and Engineering*, ***20*****(1)**, 1376-1401.

Barman, H., Pervin, M., Roy, S. K., and Weber, G. W. (2021): Back-ordered inventory model with inflation in a cloudy-fuzzy environment, *Journal of Industrial & Management Optimization*, ***17*****(4)**, 1913.

Bazan, E., Jaber, M. Y., and El Saadany, A. M. (2015): Carbon emissions and energy effects on manufacturing–remanufacturing inventory models, *Computers & Industrial Engineering*, ***88***, 307-316.

Berling, P., and Sonntag, D. R. (2022): Inventory control in production–inventory systems with random yield and rework: The unit- tracking approach, *Production and Operations Management*.

Beullens, P., & Ghiami, Y. (2022): Waste reduction in the supply chain of a deteriorating food item–Impact of supply structure on retailer performance, *European Journal of Operational Research*, ***300*****(3)**, 1017-1034.

Bhatnagar, P., Kumar, S., and Yadav, D. (2022): A single-stage cleaner production system with waste management, reworking, preservation technology, and partial backlogging under inflation, *RAIRO-Operations Research*, ***56*****(6)**, 4327-4346.

Buzacott, J. A. (1975): Economic order quantities with inflation, *Journal of the Operational Research Society*, ***26*****(3)**, 553-558.

Cárdenas-Barrón, L. E., Plaza-Makowsky, M. J. L., Sevilla-Roca, M. A., Núñez-Baumert, J. M., and Mandal, B. (2021): An inventory model for imperfect quality products with rework, distinct holding costs, and nonlinear demand dependent on price, *Mathematics*, ***9*****(12)**, 1362.

Choi, T. M. (2013): Local sourcing and fashion quick response system: The impacts of carbon footprint tax, *Transportation Research Part E: Logistics and Transportation Review*, ***55***, 43-54.

Debrah, J. K., Vidal, D. G., and Dinis, M. A. P. (2021): Raising awareness on solid waste management through formal education for sustainability: A developing countries evidence review, *Recycling*, ***6*****(1)**, 6.

Hua, G., Cheng, T. C. E., and Wang, S. (2011): Managing carbon footprints in inventory management, *International Journal of Production Economics*, ***132*****(2)**, 178-185.

Karmakar, S., De, S. K., and Goswami, A. (2017): A pollution sensitive dense fuzzy economic production quantity model with cycle time dependent production rate, *Journal of cleaner production*, ***154***, 139-150.

Kazemi, N., Shekarian, E., Cárdenas-Barrón, L. E., and Olugu, E. U. (2015): Incorporating human learning into a fuzzy EOQ inventory model with backorders, *Computers & Industrial Engineering*, ***87***, 540-542.

Khanna, A., Gautam, P., Hasan, A., and Jaggi, C. K. (2020): Inventory and pricing decisions for an imperfect production system with quality inspection, rework and carbon-emissions, *Yugoslav Journal of Operations Research*, ***30*****(3)**, 339-360.

Khatua, D., Maity, K., and Kar, S. (2019): A Fuzzy Optimal Control Inventory Model of Product–Process Innovation and Fuzzy Learning Effect in Finite Time Horizon, *International Journal of Fuzzy Systems*, ***21*****(5)**, 1560-1570.

Kumar, S., Sami, S., Agarwal, S., and Yadav, D. (2023): Sustainable fuzzy inventory model for deteriorating item with partial backordering along with social and environmental responsibility under the effect of learning, *Alexandria Engineering Journal*, ***69***, 221-241.

Mahapatra, A. S., Mahapatra, M. S., Sarkar, B., and Majumder, S. K. (2022): Benefit of preservation technology with promotion and time-dependent deterioration under fuzzy learning, *Expert Systems with Applications*, ***201***, 117169.

Manzouri, M., Ab-Rahman, M. N., Che Mohd Zain, C. R., and Jamsari, E. A. (2014): Increasing production and eliminating waste through lean tools and techniques for halal food companies, *Sustainability*, ***6*****(12)**, 9179-9204.

Mishra, M., Ghosh, S. K., and Sarkar, B. (2022): Maintaining energy efficiencies and reducing carbon emissions under a sustainable supply chain management, *AIMS Environmental Science*, ***9*****(5)**, 603-635.

Mishra, U., Wu, J. Z., and Sarkar, B. (2021): Optimum sustainable inventory management with backorder and deterioration under controllable carbon emissions, *Journal of Cleaner Production*, ***279***, 123699.

Misra, R. B. (1975): A study of inflationary effects on inventory systems, *Logistics Spectrum*, ***9***, 51-55.

Mohammadi, H., Ghazanfari, M., Pishvaee, M. S., and Teimoury, E. (2019): Fresh-product supply chain coordination and waste reduction using a revenue-and-preservation-technology-investment-sharing contract: A real-life case study, *Journal of cleaner production*, ***213***, 262-282.

Mohammadi, M., Jämsä-Jounela, S. L., and Harjunkoski, I. (2019): Optimal planning of municipal solid waste management systems in an integrated supply chain network, *Computers & Chemical Engineering*, ***123***, 155-169.

Padiyar, S. V. S., Bhagat, N., and Rajput, N. (2021): Integrated supply chain model for imperfect production and fuzzy parameters with probabilistic demand pattern and variable production rate under the environment of inflation, *Journal of Emerging Technologies and Innovative Research*, ***8*****(11)**, 630-641.

Padiyar, S. V. S., Kuraie, V. C., Bhagat, N., Singh, S. R., and Chaudhary, R. (2022a): An integrated inventory model for imperfect production process having preservation facilities under fuzzy and inflationary environment, *International Journal of Mathematical Modelling and Numerical Optimisation*, ***12*****(3)**, 252-286.

Padiyar, S. V. S., Singh, S. R., Singh, D., Sarkar, M., Dey, B. K., and Sarkar, B. (2023): Three-Echelon Supply Chain Management with Deteriorated Products under the Effect of Inflation, *Mathematics*, ***11*****(1)**, 104.

Padiyar, S. V. S., Vandana, V., Bhagat, N., Singh, S. R., and Sarkar, B. (2022b): Joint replenishment strategy for deteriorating multi-item through multi-echelon supply chain model with imperfect production under imprecise and inflationary environment, *RAIRO-Operations Research*, ***56*****(4)**, 3071-3096.

Saxena, N., Sarkar, B., and Singh, S. R. (2020): Selection of remanufacturing/production cycles with an alternative market: A perspective on waste management, *Journal of Cleaner Production*, ***245***, 118935.

Shah, N. H., and Vaghela, C. R. (2018): Imperfect production inventory model for time and effort dependent demand under inflation and maximum reliability, *International Journal of Systems Science: Operations & Logistics*, ***5*****(1)**, 60-68.

Singh, S. R., and Rani, M. (2021, April): An EPQ model with life-time items with multivariate demandwith markdown policy under shortages and inflation, In *Journal of Physics: Conference Series* (**Vol. 1854, No. 1**, p. 012045). IOP Publishing.

Singh, S. R., and Sharma, S. (2019): A partially backlogged supply chain model for deteriorating items under reverse logistics, imperfect production/remanufacturing and inflation, *International Journal of Logistics Systems and Management*, ***33*****(2)**, 221-255.

Singh, S. R., Yadav, D., Sarkar, B., and Sarkar, M. (2021): Impact of energy and carbon emission of a supply chain management with two-level trade-credit policy, *Energies*, ***14*****(6)**, 1569.

Singh, S., and Chaudhary, R. (2023): Effect of inflation on EOQ model with multivariate demand and partial backlogging and carbon tax policy, *Journal of Future Sustainability*, ***3*****(1)**, 35-58.

Singh, S. R., Sarkar, M., and Sarkar, B. (2023): Effect of Learning and Forgetting on Inventory Model under Carbon Emission and Agile Manufacturing, *Mathematics*, ***11*****(2)**, 368.

Soni, H. N., and Suthar, S. N. (2021): EOQ model of deteriorating items for fuzzy demand and learning in fuzziness with finite horizon, *Journal of Control and Decision*, ***8*****(2),** 89-97.

Soni, H. N., Sarkar, B., and Joshi, M. (2017): Demand uncertainty and learning in fuzziness in a continuous review inventory model, *Journal of Intelligent & Fuzzy Systems*, ***33*****(4)**, 2595-2608.

Sonntag, D., and Kiesmüller, G. P. (2018): Disposal versus rework–Inventory control in a production system with random yield, *European Journal of Operational Research*, ***267*****(1)**, 138-149.

Tang, S., Wang, W., Cho, S., and Yan, H. (2018): Reducing emissions in transportation and inventory management:(R, Q) Policy with considerations of carbon reduction, *European Journal of Operational Research*, ***269*****(1)**, 327-340.

Yadav, D., Chand, U., Goel, R., and Sarkar, B. (2023): Smart Production System with Random Imperfect Process, Partial Backordering, and Deterioration in an Inflationary Environment, *Mathematics*, ***11*****(2)**, 440.

Yadav, D., Kumari, R., Kumar, N., and Sarkar, B. (2021): Reduction of waste and carbon emission through the selection of items with cross-price elasticity of demand to form a sustainable supply chain with preservation technology, *Journal of Cleaner Production*, ***297***, 126298.

Yadav, D., Singh, R., Kumar, A., and Sarkar, B. (2022): Reduction of Pollution through Sustainable and Flexible Production by Controlling By-Products, *Journal of Environmental Informatics*, ***40*****(2)**, 106-124.

Yadav, D., Singh, S. R., Kumar, S., and Cárdenas-Barrón, L. E. (2022): Manufacturer-retailer integrated inventory model with controllable lead time and service level constraint under the effect of learning-forgetting in setup cost, *Scientia Iranica*, ***29*****(2)**, 800-815.

Authors and Affiliations

S.K. Shon[1] and Dharmendra Yadav[2]

S.K. Shon
Email: sksvishnu@gmail.com

[1,2]Department of Mathematics, Vardhaman College, Bijnor (UP) India