ISSN 0971-0388

Aligarh Journal of Statistics
Vol. 42 (2022), 77-90

Three Component Optimal and Nearly Optimal Orthogonally Blocked Designs For Husain and Sharma's Model

Bushra Husain and Afrah Hafeez
[Received on August, 2020. Accepted on October, 2022]

ABSTRACT

Three component orthogonal block designs are obtained using pairs of latin squares called mates. All three ingredient optimal mixture designs discussed in the literature consist primarily of two component blends barring the centroid points which are added to remove the singularity of the designs. For a mixture to be practically viable, there could be situations requiring the presence of all the ingredients. In such cases, we use nearly optimal orthogonally blocked designs. Latin square based D- and A- optimal and nearly optimal orthogonally blocked mixture designs for Husain and Sharma's (2017) reduced cubic canonical model are obtained here for q = 3.

1. Introduction

Experiments having conditions imposed on mixture ingredients have another set of complications when some extraneous factors known as process variables are involved. In order to deal with process variables, the sets of runs are organized in blocks in such a manner that the estimation of the terms involving mixture are independent of the terms involving the process variables. The advantage of using orthogonal blocking is that the number of observations needed to obtain least square estimates for the model are smaller in comparison to the simplex centroid designs. Nigam (1970) gave the conditions for orthogonal blocking which were later modified by John (1984). For three component mixture experiments, mates of latin squares are used to construct optimal designs which are basically consisting of $q = 2$ components. Hence, such designs are not true mixtures practically. For example, architectural bronze consists of 57% copper, 3% lead and 40% zinc. It is regarded as a brass alloy since it contains zinc as the main alloying ingredient. Prescott (1998) introduced an interesting plan of obtaining

✉ : Bushra Husain
bushra_husain@rediffmail.com

Extended authors information available after reference list of the article.

nearly optimal designs to meet such requirements of true mixtures and constructed nearly D- optimal designs for $q = 3$ and 4 by reparameterising each component thereby preserving the orthogonality property of the designs.
In this paper, we have used John's (1984) design to attain three ingredient latin square based D- and A- optimal and nearly optimal orthogonally blocked mixture designs for Husain and Sharma's (2017) reduced cubic canonical model.

2. The Mixture Model and Conditions for Orthogonal Blocks

Let $x_1, x_2, \ldots, x_q$ denote the proportions of the mixture ingredients subject to the condition that they sum to unity and being component proportions, are non-negative. General model takes the form of a polynomial of selected orders in z_1, $z_2, \ldots, z_n$, the process variables, whose coefficients are polynomials of selected order in $x_1, x_2, \ldots, x_q$ in case if both mixture variables and process variables are present in the study. Orthogonal blocking ensures the estimation of parameters of mixture terms independently to those of the process variables. A linear model for the i-th observed value y_i of the response of interest is

$$y_i = \eta_i + e_i \qquad i = 1, 2, \ldots, N \tag{2.1}$$

where η_i is the i^{th} expected response, e_i is the experimental error for the i^{th} observation, the e_i's are assumed to be independent and identically distributed normal variables with mean 0 and variance σ^2, and there are N observations. If the N observations or blends are arranged in the blocks then blocking is orthogonal if the least square estimates of the coefficients involving mixture terms are uncorrelated with the terms involving the process variables. John (1984) gave the following blocking conditions for each block.

$$\sum_k x_{ik} = u_i \qquad \text{for each } i = 1, 2, \ldots, q$$

$$\sum_k x_{ik} x_{jk} = u_{ij} \qquad \text{for each } i, j = 1, 2, \ldots, q \ (i < j) \tag{2.2}$$

Scheffé's (1958) full cubic canonical model is as given in (2.3).

$$E(Y) = \sum_{i=1}^{q} \beta_i x_i + \sum_{1 \le i \le j \le q} \beta_{ij} x_i x_j + \sum_{1 \le i \le j \le q} \beta_{ij} x_i x_j (x_i - x_j) + \sum_{1 \le i \le j \le k \le q} \beta_{ijk} x_i x_j x_k \tag{2.3}$$

Husain and Sharma (2017) gave the model given in (2.4).

$$E(Y) = \sum_{i=1}^{q} \beta_i x_i + \sum_{1 \le i \le j \le q} \beta_{ij} x_i x_j \,|\, x_i - x_j \,| \tag{2.4}$$

In case of two blocks, say Block I and Block II with m_1 and m_2 blends such that $m_1 + m_2 = m$ mixture blends, model (2.4) with block effect γ is as given in (2.5).

Here, the process variable at two levels is represented by Z, where we put Z = -1 for the blend in block Block I and Z = +1 for the blends in Block II and e_u's are random errors assumed to follow N(0, σ^2).

$$Y_u = \sum_{i=1}^{q} \beta_i x_{iu} + \sum_{i\le 1\le j\le q} \beta_{ij} x_{iu} x_{ju} \,|\, x_{iu} - x_{ju} \,| + \gamma Z_u + e_u \tag{2.5}$$

Husain and Sharma (2017) obtained the blocking conditions given in (2.6).

$$\sum_{u=1}^{m_w} x_{iu} = k_i \,,\; \sum_{u=1}^{m_w} x_{iu} x_{ju} \,|\, x_{iu} - x_{ju} \,| = k_{ij} \quad \forall\, w = 1, 2 \text{ and } i = 1, 2, \ldots, q \tag{2.6}$$

where, k_i's and k_{ij}'s are constants.

3. Reparametrisation of the Coordinate System

D-, A- and E- optimal designs for q = 3 for Scheffé's (1958) model, Darroch and Waller's (1985) quadratic mixture model and Draper and Pukelsheim's (1997) K-models consist of two component blends with the exclusion of the overall centroid. Practically for a mixture to be feasible, there should be atleast a little quantity of all the ingredients components in the blend. For example in making coffee, we require three ingredients viz; milk, sugar and coffee powder to be present in the beverage. So, in order to obtain true mixtures, we have to attain nearly optimal orthogonal block designs so that the minimum proportions of all the ingredients may be included in all the mixture blends.

Prescott's (1998) Reparametrisation of P from (*a*, *b*, *c*) with *a* ≤ *b* ≤ *c* to (*f*, *s*).

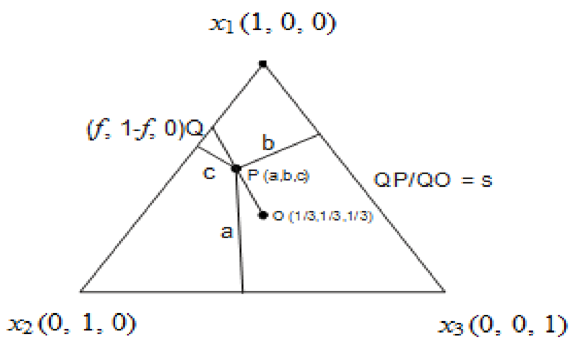

Fig.1

For q = 3, Prescott (1998) reparametrised the point P (a, b, c) with restriction $a < b < c$ by following a single shrinkage. O is the centroid whose coordinates are (1/3, 1/3, 1/3) and (f, 1-f, 0) are the coordinates of Q which are obtained by expanding the line OP to the edge of the triangle. If $\frac{PO}{QP} = \frac{1-s}{s}$ i.e., P is located at a proportion s along the line QO. Prescott (1998) obtained the transformation given in (3.1).

$$a = \frac{s}{3}$$

$$b = (1-s)(1-f) + \frac{s}{3}$$

$$c = (1-s)f + \frac{s}{3} \tag{3.1}$$

4. Two Orthogonal Blocks

John (1984) presented the following blocks of blends for q = 3 blending components with the addition of centroid point (1/3, 1/3, 1/3) to remove singularity of the design.

	x_1	x_2	x_3
Blend 1:	a	b	c
Blend 2:	b	c	a
Blend 3:	c	a	b
Blend 4:	1/3	1/3	1/3

	x_1	x_2	x_3
Blend 1:	a	c	b
Blend 2:	b	a	c
Blend 3:	c	b	a
Blend 4:	1/3	1/3	1/3

where a, b and c are numbers assuming values between 0 and 1 and add up to one. Both the blocks are based on latin squares. For three component mixtures, orthogonality conditions (2.6) represent twelve equations. The two blocks shown above satisfy the six equations for each block and hence these blocks are orthogonal. For q = 3 blending components, Husain and Sharma's (2017) model (2.4) for η reduces to (4.1).

$$\eta = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 \mid x_1 - x_2 \mid + \beta_{13} x_1 x_3 \mid x_1 - x_3 \mid + \beta_{23} x_2 x_3 \mid x_2 - x_3 \mid \tag{4.1}$$

Using this model, the design matrix for the two blocks is as given in (4.2).

$$\mathbf{X} = \begin{bmatrix} a & b & c & ab(b-a) & ac(c-a) & bc(c-b) \\ b & c & a & bc(c-b) & ab(b-a) & ca(c-a) \\ c & a & b & ca(c-a) & cb(c-b) & ab(b-a) \\ 1/3 & 1/3 & 1/3 & 0 & 0 & 0 \\ a & c & b & ac(c-a) & ab(b-a) & cb(c-b) \\ b & a & c & ba(b-a) & bc(c-b) & ac(c-a) \\ c & b & a & cb(c-b) & ac(c-a) & ab(b-a) \\ 1/3 & 1/3 & 1/3 & 0 & 0 & 0 \end{bmatrix} \quad (4.2)$$

The matrix X given in (4.2) for the model given in (4.1) is obtained on assuming $a < b < c$ since this condition is necessary for satisfying the orthogonality conditions (2.6).

$$k_1 = k_2 = k_3 = a + b + c + \frac{1}{3}$$
$$k_{12} = k_{13} = k_{23} = ab^2 - a^2b + bc^2 - b^2c + c^2a - a^2c \quad (4.3)$$

For three component mixtures, Husain and Sharma's (2017) orthogonality conditions given in (2.6) yield (4.3). In order to attain D- and A- optimality, we construct matrix **X'X** which is as given in (4.4).

$$\mathbf{X'X} = \begin{bmatrix} A & B & B & C & C & D \\ B & A & B & C & D & C \\ B & B & A & D & C & C \\ C & C & D & E & F & F \\ C & D & C & F & E & F \\ D & C & C & F & F & E \end{bmatrix} \quad (4.4)$$

where,

$$A = \frac{2}{9} + 2a^2 + 2b^2 + 2c^2$$

$$B = \frac{2}{9} + 2ab + 2ac + 2bc$$

$$C = a^2b(b-a) + b^2a(b-a) + a^2c(c-a) + b^2c(c-b) + c^2a(c-a) + c^2b(c-b)$$

$$D = 2abc(b-a) + 2abc(c-a) + 2abc(c-b)$$

$$E = ab^2(b-a)^2 + a^2b(b-a)^2 + ac^2(c-a)^2 + bc^2(c-b)^2 + a^2c(c-a^2) + b^2c(c-b)^2$$

$$F = 2a^2bc(b-a)(c-a) + 2b^2ac(b-a)(c-b) + 2abc^2(c-a)(c-b) \qquad (4.5)$$

5. Optimal Designs for *q* = 3

To find D- and A- optimality for model (2.4), values of *a*, *b* and *c* that maximize determinant of the matrix given in (4.4) and minimize T = trace $(\mathbf{X'X})^{-1}$, respectively are required. The general expression of $|\mathbf{X'X}|$ is given in (5.1) and the expression of T is very lengthy and not discussed here. We have obtained the same results on all the boundary points because $|\mathbf{X'X}|$ and T are symmetric functions of *a*, *b* and *c*. We have considered the case *a* = 0 as the requirement $a < b < c$ is to be met in our set-up. On putting *b* = 1- *c*, we express $|\mathbf{X'X}|$ and T as functions of *c* alone as given in (5.2) and (5.3), respectively, while their graphs are depicted in Figure 2.

$$|\mathbf{X'X}| = 12(a-b)^4(a-c)^2(b-c)^4(ab-b^2+ac+bc)^2(a^2-ab-2ac-bc+c^2)^4 \qquad (5.1)$$

$$|\mathbf{X'X}| = 12(1-2c)^{10}(c-1)^6c^6 \qquad (5.2)$$

$$\mathbf{T} = \frac{20+(c-1)c(72+(1-2c)^2(c-1)c(17+36(c-1)c))}{6(1-2c)^4(c-1)^2c^2} \qquad (5.3)$$

Graphs of $|\mathbf{X'X}|$ and T against *c* for Husain and Sharma's reduced cubic canonical model in three components.

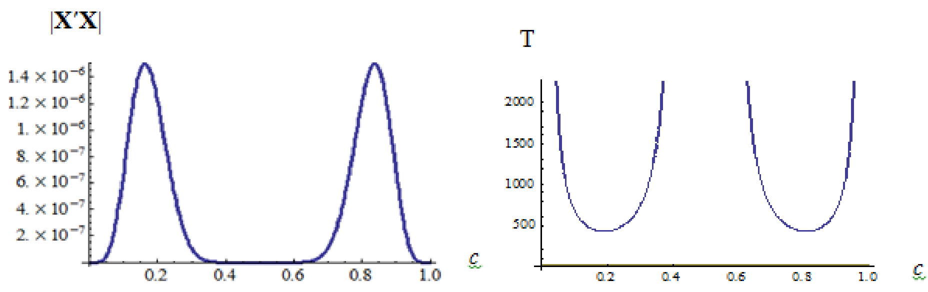

Fig. 2

It is observed numerically as well as pictorially from Figure 2 that the curve of $|\mathbf{X'X}|$ is an m-shaped curve. The value of the determinant of $\mathbf{X'X}$ equals zero at *c* = 0, 1/2, 1 and peak (= 0.00000149713) is achieved at *b* = 0.162907 and *c* = 0.837113. Furthermore, A-optimality (= 429.69) is attained at *b* = 0.191161 and *c* = 0.808839.

c

6. Nearly Optimal Design

In this section, we will obtain nearly optimal orthogonally blocked designs for Husain and Sharma's model (2.4). The idea of obtaining nearly optimal designs was first given by Prescott (1998) who used John's (1984) design and obtained minimum proportions of all the ingredients by using the transformation given in (3.1). The D- efficiency is given in (6.1)

D-Efficiency = $[|\mathbf{X}'\mathbf{X}|^{1/p} / |\mathbf{X}'\mathbf{X}|_0^{1/p}] \times 100$ percent (6.1)

where, $|\mathbf{X}'\mathbf{X}|_0$ is a function of *f* alone on substituting different values of *s* and p is the number of unknown parameters. The expressions of $|\mathbf{X}'\mathbf{X}|$ as functions of *f* and *s* and in terms of *f* alone are as given in (6.2) and (6.3), respectively.

$|\mathbf{X}'\mathbf{X}|=$

$$2.5801171\times10^{-6}(-1+f)^4 f^2(-1+2f)^4(-1+s)^{10}(9-27f+18f^2-18s+48fs-36f^2s7s^2-21fs^2+18f^2s^2)^2(-9f+18f^2-6s+21f-36f^2s+4s^2-12fs^2+18f^2s^2)^4 \quad (6.2)$$

$$|\mathbf{X}'\mathbf{X}|_0 = 12(1-2f)^{10}(-1+f)^6 f^6 \quad (6.3)$$

$|\mathbf{X}'\mathbf{X}|$ is maximised for $f = 0.162907, 0.837113$ at $s = 0$. By using (6.1), we obtain the D-efficiency of the nearly optimal designs for Husain and Sharma's (2017) model. Table 1 presents them for s = 0, 0.5, 0.1, 0.15 and 0.2, respectively. We have obtained maximum value of $|\mathbf{X}'\mathbf{X}|$ (= 1.49713×10^{-6}) at $f = 0.162907$, 0.837113 for $s = 0$. Figure 3 presents the graph of $|\mathbf{X}'\mathbf{X}|$ against *f* for $s = 0.05$.

Graph of |X′X| against *f* for *s* = 0.05 for Husain and Sharma's reduced cubic canonical model.

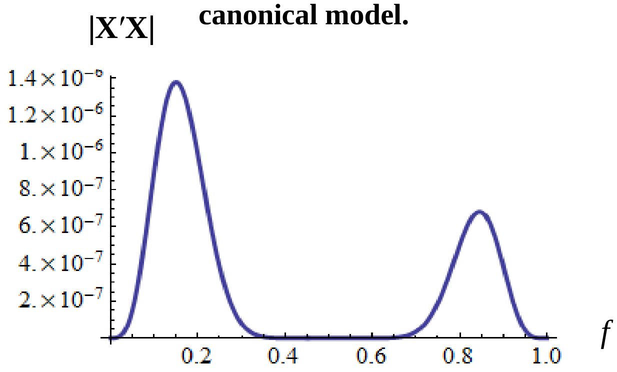

Fig. 3

From Figure 3, we observe that the maximum of |**X′X**| occur at f = 0.150032 but this value of f is not considered because it does not give the feasible values of a, b and c as orthogonality conditions (4.3) are satisfied for $a < b < c$. Hence we consider the maximum value of |**X′X**| as 6.82078×10^{-7} at f = 0.845398. Similarly for s = 0.10, 0.15 and 0.20, feasible values of f are 0.85345, 0.859783, 0.866657, respectively.

Table 1: Efficiency of the nearly D-optimal design against the shrinkage parameter s for Husain and Sharma's model.

s	Opt f	|**X′X**|	D= $\lvert X'X \rvert^{1/7}$	$D_0 = \lvert X'X \rvert_0^{1/7}$	D-efficiency
0	0.837113	1.49713×10^{-6}	0.147195	0.147195	100
0.05	0.845398	6.82078×10^{-7}	0.131559	0.146957	89.50
0.1	0.85345	2.76516×10^{-7}	0.115639	0.146254	79.06
0.15	0.859783	9.8941×10^{-8}	0.099848	0.145356	68.69
0.2	0.866657	3.08075×10^{-8}	0.0845183	0.144201	58.68

Chan and Guan (2001) gave the formula given in (6.4) for obtaining A-efficiency.

$$\text{A-efficiency} = [T_0 / \text{Min}(T)] \times 100 \quad (6.4)$$

where T_0 denotes the minimum value of T obtained on replacing optimal f in original T. We get T as a function of f alone by putting different values of s. The point of minima of T is obtained at s = 0 and f = 0.191161, 0.808839. Figure 4 presents the graph of T against f for s = 0.05.

Graph of T against *f* for *s* = 0.05 for Husain and Sharma's reduced cubic canonical model.

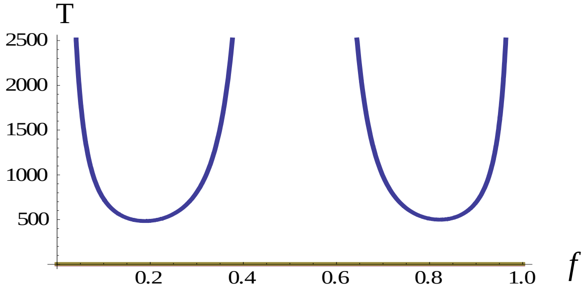

Fig. 4

From Figure 4, we observe that the minimum of T occur at f = 0.188994 but this value of f is not considered because it does not give the feasible values of a, b and c. In this case, we consider the minimum value of T as 499.784 at f = 0.821977. Similarly for s = 0.10, 0.15 and 0.20, feasible values of f are 0.831337, 0.838488, 0.845269, respectively.

Table 2: Efficiency of the Nearly A-optimal design against the shrinkage parameter s for Husain and Sharma's model.

s	Opt f	Min(T)	T_0	A-efficiency
0	0.808839	429.69	429.69	100
0.05	0.821977	499.784	433.107	86.65
0.10	0.831337	639.459	439.934	68.29
0.15	0.838488	897.616	447.863	49.89
0.20	0.845269	1378.37	457.788	33.21

7. Design Using Two Pairs of Latin Squares

The idea of using two pairs of latin squares was given by Prescott (1998). He suggested that the designs become more flexible by adding extra latin squares and the orthogonality remains unaffected by adding those extra squares. Consider the following design given by Prescott (1998) containing the extra latin square.

Table 3: Three component orthogonal block design with two squares.

Run	x_1	x_2	x_3	Run	x_1	x_2	x_3
1	a	b	c	8	a	c	b
2	b	c	a	9	b	a	c
3	c	a	b	10	c	b	a
4	a'	c'	b'	11	a'	b'	c'
5	b'	a'	c'	12	b'	c'	a'
6	c'	b'	a'	13	c'	a'	b'
7	1/3	1/3	1/3	14	1/3	1/3	1/3

7.1. Nearly D- and A- Optimal Design Formed by Shrinking Both the Pairs of Latin Squares

In this section, we consider the case when $a' = a$, $b' = b$, $c' = c$. Design 7.1 is obtained by using the reparametrisation of the coordinate system as done previously in section 6, i.e., by constricting both the pairs of latin squares towards the centroid. The form of the general determinant for both the pairs of latin squares in terms of *a*, *b* and *c* is given in (7.1.1).

$$|\mathbf{X'X}| = 384(a-b)^4(a-c)^2(b-c)^4(ab-b^2+ac+bc)^2(a^2-ab-2ac-bc+c^2)^4 \quad (7.1.1)$$

$|\mathbf{X'X}|$ is 32 times the determinant obtained for the orthogonal design based on a single square. The maximum of $|\mathbf{X'X}|$ obtained for the design given in Table 3 is same on all the boundary points and here we take $a = 0$ and $b = 1 - c$. The D-optimality (0.0000479081) is attained at $b = 0.162907$ and $c = 0.831503$. The expression of the general form of determinant obtained on constricting both the pairs of latin squares is given in (7.1.2). Here, also we observe that the maximum of $|\mathbf{X'X}|$ occur at $f = 0.150041$ but this value of f is not considered because it does not give the feasible values of *a*, *b* and *c*. Hence we consider the maximum value of $|\mathbf{X'X}|$ as 2.18266×10^{-5} at $f = 0.845585$ for $s = 0.05$. Similarly for $s = 0.10$, 0.15 and 0.20, feasible values of f are 0.85345, 0.860323, 0.866975, respectively.

$$|\mathbf{X'X}| = 0.00072256f^2(-1+2f)^4(-1+s)^{10}(9-27f+18f^2-18s+48fs-36f^2s+7s^2$$

$$-21fs^2+18f^2s^2)^2(-9f+18f^2-6s+21f-36f^2s+4s^2-12fs^2+18f^2s^2)^4 \quad (7.1.2)$$

Table 4: Efficiency of the nearly D-optimal design against the shrinkage parameter *s* applied to Design 7.1.

s	Opt *f*	$|\mathbf{X'X}|$	$D=|\mathbf{X'X}|^{1/7}$	$D_0=|\mathbf{X'X}|_0^{1/7}$	D-efficiency
0	0.837113	0.0000479081	0.241499	0.241499	100
0.05	0.845585	0.0000218266	0.215844	0.24109	89.52
0.1	0.853450	0.0000088485	0.189725	0.239954	79.06
0.15	0.860323	0.00000316627	0.163819	0.228331	68.73
0.2	0.866975	0.00000098589	0.138668	0.236175	58.71

We observe from Table 4 that by shrinking the design points towards the centroid by a parameter *s*, we are able to obtain true mixtures with a little loss in D-efficiency. For Design 7.1, we obtain Min (T) = 297.522 for $s = 0$ at $a = 0$, $b = f = 0.195989$ and $c = 1 - f = 0.804011$.

Table 5: Efficiency of the nearly A-optimal design against the shrinkage parameter *s* applied to Design 7.1.

s	Opt *f*	Min T	T_0	A-efficiency
0	0.804011	297.522	297.522	100
0.05	0.819101	323.257	300.5	92.9
0.1	0.830481	386.319	306.943	79.45
0.15	0.838709	510.8	314.135	61.49
0.2	0.86373	975.609	298.719	30.61

7.2. Nearly D- and A- Optimal Design Formed by Shrinking One Pair of Latin Square

Prescott (1998) presented another method to obtain more flexible designs by shrinking only one pair of latin square towards the centroid and leaving the other pair on the edges of the simplex. For model (2.4), the general expression in terms of *a*, *b*, *c* and *f* is very lengthy and therefore, not given here. We obtain the maximum of $|\mathbf{X}'\mathbf{X}|$ at $a = 0$, $b = f = 0.181897$ and $c = 1 - f = 0.818103$. Here also, the maximum value of $|\mathbf{X}'\mathbf{X}|$ occurs at $f = 0.175812$ for $s = 0.05$. But this value of f violates the condition $a < b < c$. Hence the feasible value of f is 0.820917. Similarly for $s = 0.10$, 0.15 and 0.20, feasible values of f are 0.823722, 0.826448 and 0.828989, respectively.

Table 6 represents the D- efficiency applied to Design 7.2, i.e, the case when we shrink only one pair of latin square towards the centroid.

Table 6: Efficiency of the nearly D-optimal design against the shrinkage parameter *s* applied to Design 7.2.

s	Opt *f*	$\lvert\mathbf{X}'\mathbf{X}\rvert$	$D = \lvert\mathbf{X}'\mathbf{X}\rvert^{1/7}$	$D_0 = \lvert\mathbf{X}'\mathbf{X}\rvert_0^{1/7}$	D-efficiency
0	0.818103	0.00042259	0.329601	0.329601	100
0.05	0.820917	0.00031709	0.316351	0.329554	95.90
0.1	0.823722	0.000243683	0.304672	0.329413	92.48
0.15	0.826448	0.000191492	0.29436	0.329183	89.42
0.2	0.828989	0.000153235	0.285135	0.328886	86.69

Table 7: Efficiency of the nearly A-optimal design against the shrinkage parameter *s* applied to Design 7.2.

s	Opt *f*	Min T	T_0	A-efficiency
0	0.787791	115.11	115.11	100
0.05	0.788828	123.476	115.115	93.20
0.1	0.789032	131.893	115.116	87.27
0.15	0.788695	139.875	115.114	82.29
0.2	0.788081	147.162	115.111	78.22

For Design 7.2, Min(T) = 115.11 for $s = 0$ at $a = 0$, $b = f = 0.212209$, $c = 1 - f = 0.787791$ In this case also, the same situation arises and we obtain minimum value of T at $f = 0.208327$ for $s = 0.05$ but this value violates the condition $a < b < c$. Hence the feasible value of f for $s = 0.05$ is 0.7888280. Similarly for $s = 0.10$, 0.15 and 0.20, feasible values of f are 0.789032, 0.788695 and 0.788081, respectively. We observe from tables 6 and 7 that with a minimal loss in D- and A- efficiencies, true mixtures are obtained.

8. Discussions

In this paper, we have obtained the D- and A- optimal and nearly optimal orthogonally blocked designs based on latin squares for Husain and Sharma's (2017) model. The D-optimality (0.0000149713) is attained at $b = 0.162907$ and $c = 0.837113$ and A- optimality (429.69) is attained at $b = 0.191161$ and $c = 0.808839$, respectively. We obtain the same optimality on all the boundary points $a = 0$, $b = 0$ and $c = 0$ but we consider the case $a = 0$ due to the restriction $a < b < c$ for the orthogonality conditions to be satisfied. We observe that three component optimal designs based on latin squares for the reduced cubic canonical model presented by Husain and Sharma (2017) comprise of binary mixtures with the exception of the ternary mixture, viz; (1/3, 1/3, 1/3).

In sections 6 and 7, we have obtained nearly optimal three component mixtures for Husain and Sharma's (2017) model. Nearly D- optimal designs for $s = 0$ are obtained at f = 0.162907, 0.837113 and nearly A- optimal designs for s = 0 are obtained at f = 0.191161 and 0.808839, respectively. Further by shrinking both the pairs of the latin squares towards the centroid viz; Design 7.1, we observe from Table 4 and Table 5 that the D- and A- efficiency at s = 0.05 is 89.2% and 92.9%, respectively. We have also obtained the D- and A- efficiencies by shrinking only one pair of latin square towards the centroid. For Design 7.2, we observe from tables 6 and 7 that the D- and A- efficiencies at s = 0.05 is 95% and 93%, respectively. Note that D- and A- efficiencies are higher for Design 7.2 as compared to Design 7.1.

Acknowledgments

The second author gratefully acknowledges the UGC- Maulana Azad National Fellowship for supporting the research project vide grant no- F1-17.1-MANF 2104-15-MUS –UTT-38109.

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Authors and Affiliations

Bushra Husain[1] and Afrah Hafeez[2]

Afrah Hafeez
E-mail: afrahhafeez987@gmail.com

[1,2]Department of Statistics & O.R., Women's College, Aligarh Muslim University-Aligarh-202002.