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A Genetic Algorithm Approach for Multi-Objective Transportation Problem with Hexagonal Fuzzy Number

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ABSTRACT

Several fuzzy approaches have been used for finding the compromise results in the context of "multi-objective transportation problem (MOTP)" with fuzzy parameters. In this work, we have examined a MOTP with "hexagonal fuzzy numbers (HFNs)" as its parameters, i.e., demand, supply and penalties of the problem are mold in HFNs with a new approach developed with the help of a genetic algorithm. Robust ranking is used for the defuzzified value of the hexagonal fuzzy parameters. We have found the BFS (basic feasible solution) of the problem by adopting the zero-point technique. Then the genetic algorithm is used for obtaining the compromising superlative solution by the set of feasible solutions obtained by the zero-point technique of the problem. An algorithm has been developed for the procedure. To figure out the adaptability of the proposed technique, a numerical example has been used.

1. Introduction

Transportation problems (TPs) is a kind of the optimization techniques, plays a crucial role in supply chain management to minimize total cost and for making the best service. Because of high competition in the market, it became too difficult to find the best system or way for distributing the product to the costumers at minimum cost or maximum profit by satisfying the demand of the costumers. Transportation problems TPs gave the best way to meet this challenge. The TPs are special kind of the linear programming problem. Because of its special structure, it is not comfortable to find the solution of this

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optimization problem, i.e., of transportation problem by using the Simplex method. There are a lot of methods or algorithms in the literature to find the optimal solution of the transportation problems (TPs). The first classical Transportation Problem was introduced by Hitchcock (1941). The major objective of a classical Transportation problem is minimizing the total transport cost, but generally, in the transportation problem, objective is not unique. Decision maker wants to send the product at minimum cost, minimum time and with minimum deterioration of the raw material, i.e., there are several objectives in the transportation problem and all the objectives are conflicting and incommensurable. So, in the TPs more than one objective is involved, thus the assignment of finding one or more than one optimal solution or the best one compromise solution is called Multi-Objective Transportation Problems (MOTPS). Lee and Moore (1973) gave a goal programming approach to evaluate the optimal solutions of TPs with multiple objectives.

But in real life, there are many conditions by which the parameters of optimization problems are not exact, i.e., the parameters of the problem are not represented in the exact manner. So due to several conditions in the transportation problem, supply, demand and penalties such as cost or time may be uncertain. Such transportation problems which are not exactly defined are known as fuzzy transportation problems. To deal with such uncertain or unreliable information in decision making, professor Zadeh (1965) gave the fuzzy set theory and fuzzy logic. After that, Bellman and Zadeh (1970) first used the fuzzy set approach in decision making problems. Firstly, Oheigeartaigh (1982) gave an algorithm to evaluate the solution of the TPs. Yager (1986) gave a substituted classification of the fuzzy approach-based extension principle. Chanas and discussed the perception of the optimal solution of the TPs under a fuzzy environment. Gen *et al.* (1997, 1999) improved the genetic algorithm (GA) for MOTPs and also recycled the spanning tree GA for a multi-objective transportation problem. EL-Wahed (2001) determined the optimal and comprehensive solution for crisp MOTPs with fuzzy parameters. Ammar and Youness (2005) discussed the stability of MOTPs with fuzzy criterions and also discussed the efficiency of the solutions of the problem. Ho and Ji (2005) used the genetic algorithm to find the solution of a generalized transportation problem. Sharma *et al.* (2012) gave the optimal solution of transportation problem by using a zero-point method. Zaki *et al.* (2012) used the GA for the efficient result of several multi objective problems like TPs, assignment problem and transshipment problem. Rajarajeswary *et al.* (2013) introduced the new operation of hexagonal fuzzy numbers (HFNs). Rajarajeswary and Sudha (2014) ordered the generalized HFNs by rank, divergence, mode and spread. Chandrasekaran *et al.* (2015) gave the result of fuzzy TPs with hexagonal fuzzy parameters by using α- cut and ranking technique. Bharathi and Vijayalakshmi (2016) used the GA for finding the optimal and compromise solution of a bi-objective TPs. Dhural and Parkagam (2016) introduced a new membership function for HFNs. Annie Christi and Sumitha (2017) used the fuzzy geometric programming for finding best optimal compromise solution of MOTPs with hexagonal fuzzy parameters and also by using genetic algorithm Karthy and Ganesan (2018) gave the optimal and compromise solution of MOTPs. Kaur *et al.* (2018) used a new approach to find the solution of MOTPs. Uddin *et al.* (2018) gave the new approach by using fuzzy logic-based goal programming and a GA for solving the multi-objective transportation problem. Gowthami and Prabhakaran (2019) also gave a new approach to find the solution of a MOTPs. In this paper we propose the GA approach for solving the MOTPs using HFNs as parameters. Dinagar and Narayanan (2016) studied some elementary operations on HFNs. Christi and Priyadharshini (2017) applied DSW (Dong, Shah, Wong) approach to define a membership function of the performance measures by using heptagonal as well as octagonal fuzzy number. Ghadle and Pathade (2017) gave the concept of generalized hexagonal and octagonal fuzzy number in the context of transportation problem by ranking method. Further, Rajarajeswari *et al.* (2013), Dhurai and Karpagam (2016), Thamaraiselvi and Santhi (2015), Sudha and Revathy (2016) applied the concept of generalized HFNs in the context of transportation problem. The developments of hexagonal fuzzy number with their arithmetic were introduced by Sudha and Revathy in 2014 used a linear membership and non-membership function. Christi and Kasthiri (2016) proposed the fuzzy ranking technique based on intuitionstic pentagonal fuzzy number in transportation problems. In this paper, we propose the GA approach for solving the MOTPs using HFNs as parameters.

This paper is disunited into seven segments. First segment includes an introduction; second segment includes basic concepts related to this paper. In the third and fourth segments, we have developed the algorithm for the zero-point method and discussed the basic concepts of genetic algorithm respectively. In the fifth segment of the paper, an approach of a genetic algorithm has been developed. In the sixth segment of the research paper a numerical example has been taken to describe the efficiency of our proposed algorithm. In this segment, we compared our method to another method. The last segment of the paper is devoted to the result and conclusion.

2. Basic Concepts

a) Fuzzy Number-

A fuzzy set $\tilde{A}$ on the real line is known is a fuzzy number if it satisfies the following three conditions defined as:

(i) It is "normal", i.e., height of the set is 1.

(ii) It is "convex".

(iii) Support i.e., $\{x\hat{I}X:\mu_{\tilde{A}}(x)f0\}$ is closed and bounded.

Where, $\mu_{\tilde{A}}(x)$ is called the membership grade of x in $\tilde{A}$.

b) Triangular Fuzzy Number-

A fuzzy number denoted by $\tilde{A}=(a_1,a_2,a_3)$ is represented as triangular fuzzy number, whose membership grade is a function $\mu_{\tilde{A}}: X \to [0,1]$ as follows:

$$\mu_{\tilde{A}}(x)=\begin{cases} 0, & x \le a_1 \,\&\, x \ge a_3 \\ \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & a_2 \le x \le a_3 \end{cases}$$

Where, a_1, a_2 *and* a_3 are real numbers and $\mu_{\tilde{A}}(x)$ is membership grade of x in $\tilde{A}$

c) Trapezoidal Fuzzy Number-

A fuzzy number denoted by $\tilde{A}=(a_1,a_2,a_3,a_4)$ is represented as trapezoidal fuzzy number, whose membership grade is a function $\mu_{\tilde{A}}: X \to [0,1]$ as follows:

$$\mu_{\tilde{A}}(x)=\begin{cases} 0, & x \le a_1 \,\&\, x \ge a_4 \\ \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3}, & a_3 \le x \le a_4 \end{cases}$$

Where a_1, a_2, a_3 *and* a_4 are real numbers and $\mu_{\tilde{A}}(x)$ is membership grade of x in $\tilde{A}$.

d) Hexagonal Type Fuzzy Numbers (HFNs)-

A fuzzy number denoted by $\tilde{A}=(a_1^6, a_2^6, a_3^6, a_4^6, a_5^6, a_6^6)$ is called hexagonal fuzzy number**,** whose membership grade is a function defined as:

$$\mu_{\tilde{A}}(x)=\begin{cases} 0, & x \geq a_6^6 \ \& \ x \leq a_1^6 \\ \frac{1}{2}(\frac{x-a_1^6}{a_2^6-a_1^6}), & a_1^6 \leq x \leq a_2^6 \\ \frac{1}{2}+\frac{1}{2}(\frac{x-a_2^6}{a_3^6-a_2^6}), & a_2^6 \leq x \leq a_3^6 \\ 1, & a_3^6 \leq x \leq a_4^6 \\ 1-\frac{1}{2}(\frac{x-a_3^6}{a_4^6-a_3^6}), & a_4^6 \leq x \leq a_5^6 \\ \frac{1}{2}(\frac{a_6^6-x}{a_6^6-a_5^6}), & a_5^6 \leq x \leq a_6^6 \end{cases}$$

Where, $a_1^6, a_2^6, a_3^6, a_4^6, a_5^6 \ and \ a_6^6$ are real numbers and $\mu_{\tilde{A}}(x)$ is membership grade of x in $\tilde{A}$.

e) Arithmetic Operations on HFNs-

Let $\tilde{A}=(a_1^6, a_2^6, a_3^6, a_4^6, a_5^6, a_6^6)$ and $\tilde{B}=(b_1^6, b_2^6, b_3^6, b_4^6, b_5^6, b_6^6)$ be two HFNs then the operation on HFNs numbers are stated as follows:

- **Addition:** $\tilde{A}+\tilde{B}=(a_1^6, a_2^6, a_3^6, a_4^6, a_5^6, a_6^6)+(b_1^6, b_2^6, b_3^6, b_4^6, b_5^6, b_6^6)$
  $=(a_1^6+b_1^6, a_2^6+b_2^6, a_3^6+b_3^6, a_4^6+b_4^6, a_5^6+b_5^6, a_6^6+b_6^6)$

- **Subtraction:** $\tilde{A}-\tilde{B}=(a_1^6, a_2^6, a_3^6, a_4^6, a_5^6, a_6^6)-(b_1^6, b_2^6, b_3^6, b_4^6, b_5^6, b_6^6)$
  $=(a_1^6-b_1^6, a_2^6-b_2^6, a_3^6-b_3^6, a_4^6-b_4^6, a_5^6-b_5^6, a_6^6-b_6^6)$

- **Multiplication:** $\tilde{A}*\tilde{B}=(a_1^6, a_2^6, a_3^6, a_4^6, a_5^6, a_6^6)*(b_1^6, b_2^6, b_3^6, b_4^6, b_5^6, b_6^6)$

  $=(a_1^6*b_1^6, a_2^6*b_2^6, a_3^6*b_3^6, a_4^6*b_4^6, a_5^6*b_5^6, a_6^6*b_6^6)$
- **Scalar multiplication:**
  $k(a_1^6, a_2^6, a_3^6, a_4^6, a_5^6, a_6^6)=(ka_1^6, ka_2^6, ka_3^6, ka_4^6, ka_5^6, ka_6^6)$, for any value of k except 0.

f) Linear and Non-Linear HFN with Symmetry and Asymmetry-

A hexagonal fuzzy set $\tilde{A}$ of the form $\tilde{A} = (a_1^6, a_2^6, a_3^6, a_4^6, a_5^6, a_6^6; \alpha, \beta, \gamma, \delta)$ then it is categorized as follows.

i) **Linear Hexagonal Fuzzy Number with Symmetry if** $m = n, \alpha = \beta = \gamma = \delta = 1$ and defined as (see Fig. 1)

$$\tilde{A}_L = \begin{cases} 0, & x \geq a_6 \;\&\; x \leq a_1 \\ m\left(\dfrac{x-a_1}{a_2-a_1}\right)^1, & a_1 \leq x \leq a_2 \\ 1-(1-m)\left(\dfrac{x-a_2}{a_3-a_2}\right)^1, & a_2 \leq x \leq a_3 \\ 1, & a_3 \leq x \leq a_4 \\ 1-(1-m)\left(\dfrac{x-a_4}{a_5-a_4}\right)^1, & a_4 \leq x \leq a_5 \\ m\left(\dfrac{a_6-x}{a_6-a_5}\right)^1, & a_5 \leq x \leq a_6 \end{cases}$$

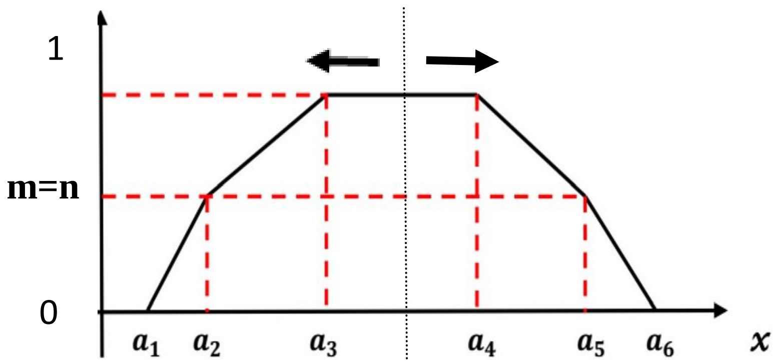

Fig. 1

ii) **Linear Hexagonal Fuzzy Number with Asymmetry if** $m \neq n, \alpha = \beta = \gamma = \delta = 1$ and defined as(see Fig. 2)

$$\tilde{A}_L = \begin{cases} 0, & x \geq a_6 \,\&\, x \leq a_1 \\ m(\frac{x-a_1}{a_2-a_1})^1, & a_1 \leq x \leq a_2 \\ 1-(1-m)(\frac{x-a_2}{a_3-a_2})^1, & a_2 \leq x \leq a_3 \\ 1, & a_3 \leq x \leq a_4 \\ 1-(1-n)(\frac{x-a_4}{a_5-a_4})^1, & a_4 \leq x \leq a_5 \\ n(\frac{a_6-x}{a_6-a_5})^1, & a_5 \leq x \leq a_6 \end{cases}$$

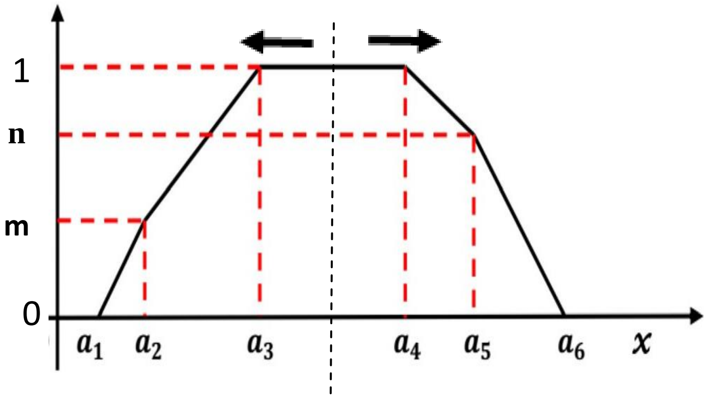

Fig. 2

iii) **Non-Linear Hexagonal Fuzzy Number with Symmetry if** $m = n, \alpha = \beta = \gamma = \delta \neq 1$ and defined as(see Fig. 3)

$$\tilde{A}_L = \begin{cases} 0, & x \geq a_6 \,\&\, x \leq a_1 \\ m(\frac{x-a_1}{a_2-a_1})^\alpha, & a_1 \leq x \leq a_2 \\ 1-(1-m)(\frac{x-a_2}{a_3-a_2})^\beta, & a_2 \leq x \leq a_3 \\ 1, & a_3 \leq x \leq a_4 \\ 1-(1-m)(\frac{x-a_4}{a_5-a_4})^\gamma, & a_4 \leq x \leq a_5 \\ m(\frac{a_6-x}{a_6-a_5})^\delta, & a_5 \leq x \leq a_6 \end{cases}$$

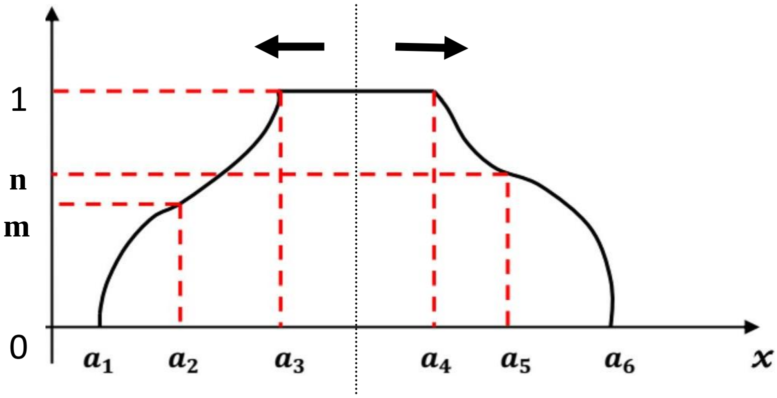

Fig. 3

iv) **Non-Linear Hexagonal Fuzzy Number with Symmetry if** $m \neq n, \alpha = \beta = \gamma = \delta \neq 1$ and defined as(see Fig. 4)

$$\tilde{A}_L = \begin{cases} 0, & x \geq a_6 \,\&\, x \leq a_1 \\ m(\frac{x-a_1}{a_2-a_1})^{\alpha}, & a_1 \leq x \leq a_2 \\ 1-(1-m)(\frac{x-a_2}{a_3-a_2})^{\beta}, & a_2 \leq x \leq a_3 \\ 1, & a_3 \leq x \leq a_4 \\ 1-(1-n)(\frac{x-a_4}{a_5-a_4})^{\gamma}, & a_4 \leq x \leq a_5 \\ n(\frac{a_6-x}{a_6-a_5})^{\delta}, & a_5 \leq x \leq a_6 \end{cases}$$

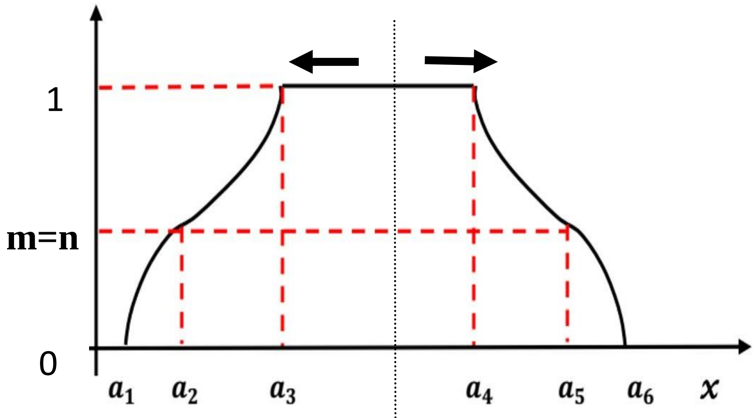

Fig. 4

Where, $0 \leq m, n \leq 1$.

g) Robust's Ranking of HFN-

Let $\tilde{A}=(a_1^6,a_2^6,a_3^6,a_4^6,a_5^6,a_6^6)$ be given hexagonal fuzzy number. Hexagonal fuzzy number can be defuzzied by Robust's ranking method [6] which satisfies the properties as additive, linearity. If $\tilde{A}=(a_1^6,a_2^6,a_3^6,a_4^6,a_5^6,a_6^6)$ is a fuzzy number then the Robust's ranking index is defined by

$R(\tilde{A})=\int_0^1 0.5(\tilde{A}_\alpha^L,\tilde{A}_\alpha^U)d\alpha$, where, $(\tilde{A}_\alpha^L,\tilde{A}_\alpha^U)$ is the α-cut of the hexagonal fuzzy number.

i.e.,

$$(\tilde{A}_\alpha^L,\tilde{A}_\alpha^U)=(a_2^6-a_1^6)\alpha+a_1^6,a_4^6-(a_4^6-a_3^6)\alpha,(a_4^6-a_3^6)\alpha+a_3^6,a_6^6-(a_6^6-a_5^6)\alpha$$

3. Algorithm for the Zero-Point Approach

In this segment, we have written the zero-point approach in the form of algorithm shown below as:

Step 1: Consider a transportation problem, then check whether given problem is balanced or not. If not, then make it balanced by dummy row or column with zero cost to the given cost matrix.

Step 2: Determine the smallest element in each horizontal line and after that reduce each element from the corresponding horizontal line by this element. Do the same procedure for each vertical line of the cost matrix.

Step 3: Then check there will be at least one zero in each horizontal line and each vertical line. Now choose that each horizontal line which has only one zero. If there is a tie then choose which has the minimum unit cost. If there are more than one zero in each horizontal line, then choose any of one row then that cell which has the minimal unit cost. Then allocate the cell by minimal demand or supply and cross that horizontal line whose supply or demand is exhausted. Do the same process in the vertical line.

Step 4: Now check all demand and supply are fulfilled. If yes, then a basic feasible solution is obtained. If not, then draw the vertical and horizontal lines so that each zero is shaded with these lines Now select the minimal uncovered element in the matrix and subtract this number from each entry of the matrix which is uncovered, add to which elements are at the intersection of the lines and leave the elements which are covered by one time. Then we get a modified cost matrix.

Step 5: Again, apply steps from step 2 and 4 till all supply and demands are fulfilled. Then we get a basic feasible solution of the given problem.

4. Genetic Algorithm

There are three processes in Genetic Algorithm that can be described as follows:

a) Selection: First, find the Basic Feasible Solution of the given multi-objective transportation problem. Then select some obtained solutions as a parent.

b) Crossover: There are 3 types crossover - (i) Single point (ii) Two-point (iii) Uniform.

i) Single point: Choose one random crossover point in parent. Then allocations from point are copied from first the parents whenever allocations located after that point are copied from another parent.

ii) Two-point: Choose two-point crossover randomly in parent, then allocations which are located from the first point and after the second point are copied the from first parent whenever allocations between these two points are copied from another parent.

iii) Uniform: Allocations of the parent are copied from the first or second parent randomly.

c) Mutation: In this process one or more allocations (genes) changes from initial state. By this process the entire solution may be changed from the initial solution. Hence, by mutation we can get the more optimal solution. In this paper, we applied the single point-based crossover to our problem.

5. Proposed Algorithm for the Problem with Genetic Algorithm

Step 1: Initially construct the table for Multi objective transportation problems with linear or non-linear hexagonal fuzzy parameters with symmetry or asymmetry. We consider a bi-objective Multi objective transportation problem**.**

Step 2: Convert the both penalty matrices of the problem into crisp form by using Robust's ranking method. Then check whether the given problem is balanced or not. If not, then we will convert it into a balanced problem.

Step 3: Now**,** according to Genetic Algorithm, select the method to find the BFS of the problem. We select Zero-point Method for BFS of MOTP with hexagonal fuzzy parameters. Solve the given problem by using the zero-point approach by considering problem as a single objective TP taking one objective at a time and ignoring all other objectives. Then we get a set of the basic feasible solutions. Then choose some solution as a parent if we have more than two objectives in the problem.

Step 4: Then we get two parents. Now crossover the parent by the one-point crossover then we get two new result**s**.

Step 5: Now, after crossover**,** we apply mutation to each new result. Then we obtain the compromise optimal solution by applying Genetic Algorithm for the problem.

6. Numerical Computation

Step 1: Let us consider a bi-objective Transportation Problem with linear hexagonal fuzzy parameters with asymmetry with the supplies:

$a_1 = (4,6,7,10,12,14)$, $a_2 = (5,7,10,13,16,20)$,
$a_3 = (6,8,10,12,14,15)$, $a_4 = (7,9,12,14,15,18)$;

and demands:
$b_1 = (6,7,9,12,14,18)$, $b_2 = (3,5,8,9,11,15)$, $b_3 = (9,11,12,15,17,18)$,
$b_4 = (4,7,10,13,15,16)$;
and penalties as follows:

$$C_1 = \begin{bmatrix} (2,3,5,7,8,10)(3,5,7,9,10,12)(3,7,11,14,15,16)(7,9,10,12,15,20) \\ (4,6,7,10,12,15)(3,6,8,12,14,15)(8,10,11,13,14,16)(6,8,9,11,13,18) \\ (7,8,12,13,15,19)(9,10,11,13,17,19)(2,4,5,8,12,16)(5,8,9,12,14,20) \\ (3,4,7,9,13,17)(6,9,11,14,18,22)(2,5,6,8,11,15)(7.11.13.16.17,21) \end{bmatrix}$$

$$C_2 = \begin{bmatrix} (5,9,12,13,15,19)(4,7,10,12,14,16)(3,5,7,10,12,14)(2,3,4,6,7,10) \\ (6,7,9,10,13.14)(8,10,11,13,14,16)(9,10,12,15,18,22)(2,4,5,8,11,17) \\ (8,10,12,15,17,20)(3,5,8,10,11,15)(6,7,9,13,15,16)(5,6,8,11,12,16) \\ (6,9,11,14,15,19)(5,8,9,13,17,22)(8,11,12,16,18,23)(2,5,7,10,14,17) \end{bmatrix}$$

Step 2: By Robust's ranking problem convert into crisp form:
Supplies:

a_1 =17.5 a_2 =23.5 a_3 =21.75 a_4 =25.25, and $\sum_{i=1}^{4} a_i$ =88;

Demands:

b_1 =21.75 b_2 =17 b_3 =27.25 b_4 =22 and $\sum_{j=1}^{4} b_j$ =88;

$$\sum_{i=1}^{4} a_i = \sum_{j=1}^{4} b_j$$

This implies given problem is balanced.
Now penalty matrices:

$C_1 =$

12.25	12.25	12.25	12.25
17.75	19.5	24	21.25

24.75	25.75	15	20.25
17.25	26.75	14.75	28.5

Step 3: $C_2 =$

24.5	20.75	17	10.5
19.5	24	27.25	15
27.25	17.5	22	16.25
24.75	24	29	18

BFS obtained by zero-point method:

$C_1 =$

	17		0.5
21.75			1.75
		2	19.75
		25.25	

Then $Z_1 = 1440$, $Z_2 = 1905.56$.

$C_2 =$

		17.5	
		1.5	22
	17	4.75	
21.75		3.5	

Then $Z_1 = 1653.69$, $Z_2 = 1710.19$.

Step 4: After crossover obtained result:

	17	0.5	
19			4.5
		4.25	17.5
2.75		22.5	

Then $Z_1 =$ 1444.69, $Z_2 =$ 1897.69.

		17.5	
	17	1.5	5
		4.75	17
21.75		3.5	

$Z_1 =$ 1524.31, $Z_2 =$ 1841.94

After Mutation obtained result:

	17		0.5
19			4.5
		4.75	17
2.75		22.5	

Then $Z_1 =$ 1442.06, $Z_2 =$1888.81.

Hence After mutation compromise optimal solution is:

	17		0.5
19			4.5
		4.75	17
2.75		22.5	

Hence solution is $Z_1 =$ 1442.06, $Z_2 =$ 1888.81.

Table 1. Comparison between existing and proposed method

By Kaur L. *et al.* Method	$Z_1 = 1801.5$ and $Z_2 = 2038.69$.
By Proposed Algorithm	**$Z_1 =$ 1442.06 and $Z_2 =$ 1888.81.**

7. Conclusion

Linear and non-linear hexagonal fuzzy numbers with symmetry and asymmetry can be tested in various real-life applications of special domain like; in optimization, Multi criteria decision makings, in economics etc. In the specific cases where the decision variables may be linear or non-linear or they may be symmetric or asymmetric for assignment problem/ transportations problems, then the triangular and trapezoidal fuzzy numbers are insufficient to deal such situations. If the number of decision variables increases, then there will arise some complications in characterizing the nature of solutions by using the notion of triangular type or trapezoidal type fuzzy numbers. Hence, HFNs are being involved in solving such type of problems. The main objective of taking hexagonal fuzzy number is to obtain more comprehensive solution. The entire work illustrates the following points: -

i) Due to the capability of HFNs to denote the flawed knowledge with extra probability rather than the triangular type or trapezoidal type fuzzy number. HFNs gives rise to formulate various real-life problems and shows many uncertain and vague information in more appropriate and complete way. Hence, we used the hexagonal fuzzy number for solving the transportation problem.

ii) We get the basic feasible solution of the MOTP with hexagonal fuzzy parameters by Zero-point method easily. It is easy and less computational.

iii) Genetic algorithm used to get the compromise optimal solution by reducing the steps. Hence the approach of genetic algorithm has been developed gives the best optimal solution in less time with less steps.

iv) A Comparative study is also been studied and obtained results are better than Kaur L. *et al.* method. Computation in table [1] with the help of the numerical example shows the ability of our proposed algorithm. By using previous Kaur L. approach, we have the optimized values $Z_1 =$ 1801.5 and $Z_2 =$ 2038.69, which are not optimal than the values obtained by proposed approach ($Z_1 = 1442.06$ and $Z_2 =$ 1888.81.).

v) Genetic algorithm approach is applied due to its optimization nature. It gives us an appropriate solution rather than any other metaheuristic approaches.

vi) Additionally, for the statistical analysis, to reduce the computational complexity, we have used the LINGO software. It is less time-consuming and easy in many real-life complex structural computations.

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