ISSN 0971-0388

Aligarh Journal of Statistics
Vol. 39 (2019*)*, 75-94

Bayes Prediction limits of Pareto distribution when observations are mid type II censored

Vastoshpati Shastri, Rakesh Ranjan* and Deependra S. Pal
[Received on May 22, 2018. Accepted on June 3, 2020]

ABSTRACT

This paper deals with the two sample Bayesian prediction problem for Pareto distribution when observations are mid-type II censored. Bayesian predictive distribution and corresponding prediction limits are obtained for a general k^{th} ordered future observation. Finally performances of the proposed prediction limits have been studied for the smallest ordered future observation on the basis of Monte Carlo simulation study of 1000 randomly generated samples.

1. Introduction

Pareto family started getting its applicability in the context of modelling and predicting tools in wide variety of socio-economic and naturally occurring phenomenon with observations in very long right tail. Later it has been pointed out that the family has potential for modelling and predicting in reliability and life time data as well especially in the situation where product or system development results in an improved performance as the development proceeds. Hsu et. al. (2011), Ahmadi and Doostparast (2019) and Smadi et. al. (2019) among others have studied the Pareto distribution extensively in the context of analysing life testing and reliability data.

Several forms and extensions of the Pareto distribution have been studied and a systematic literature is available in Arnold (2015). Among the various members of Pareto family, the simplest is known as classical Pareto distribution with probability density function (pdf).

*Correspondence author**:* Rakesh Ranjan, and Vastoshpati Shastri, DST-Centre of Interdisciplinary Mathematical Sciences Banaras Hindu University, Varanasi, India-221001, E-mail: rakesh.stats88@gmail.com, Deependra S. Pal, Department of Sciences, Govt. Arts & Science College, Ratlam.

$$f(x|\tau,\vartheta) = (\tau\vartheta)(\tau x)^{-(\vartheta+1)} \qquad ;(\tau x) > 1,\ \vartheta > 0 \qquad (1.1)$$

with shape or inequality parameter ϑ and precision parameter τ.

Its cumulative distribution function is given by

$$F(x|\tau,\vartheta) = 1 - (\tau x)^{-\vartheta} \qquad ;(\tau x) > 1,\ \vartheta > 0 \qquad (1.2)$$

Bayesian analysis of above distribution was done by Upadhyay and Shastri (1997) for doubly type II censored observations using sample based approach. Notable work on the classical Pareto distribution has been made by Arnold and Press (1983), which includes Bayesian inferences about the inequality and precision parameter along with prediction about the behaviour of future observation.

Prediction problems are widely used in life testing and reliability theory where we infer the value of unknown future variable using current available information. The prediction of future ordered observation(s) shows how long a sample of units might run until all fail in life testing. For making inferences about future sample characteristics, its predictive density is determined by combining the posterior distribution with the pdf of future characteristics given parameter(s). Integration with respect to each of parameter of this combination yields the predictive distribution for the future characteristics which summarizes the knowledge about future sample in the light of information provided by the data in hand (see, Aitchen and Dunsmore (1975)).

Bayes prediction in Pareto distribution was studied by Soliman (2000) when sample size is random variable. Prediction interval for Pareto distribution was obtained by Ali Mousa (2003) for doubly type II censored data. Raqab et. al. (2010) studied predictive inference for Pareto distribution for progressively type-II censored sample. El-Din et. al (2017) drew Bayesian inference and prediction for Pareto distribution based on ordered ranked set sampling. Shafay et. al. (2017) considered the problem of Bayesian prediction of order statistics when sample size is both fixed and random for Pareto distribution. For Pareto distribution several authors have focused on the Bayesian prediction problem of future observations based on various types of censoring, but there appears to be nothing in the literature of prediction of Pareto observables for mid type II censoring scheme which is reverse of doubly type II censoring scheme.

A mid censored data could be advocated in distribution where early failures or late failures are more significant. In this situation inferences based on left censored or right censored observations could not give clear picture. Mid censoring arises when an experimenter while recording some data, find hurdles in observing some middle observations due to some unforeseen event and thereafter he records remaining data. Let $x_1, x_2, \dots, x_n$ be n items put on test, experimenter records first r observations $x_1 < \cdots < x_r$. Due to some unforeseen event he is not able to record some l middle observations, namely $x_{r+1} < \cdots < x_{r+l}$ and then he records the last (n-r-l) observations $x_{r+l+1} < \cdots < x_n$. Shastri et. al. (2020) obtained Bayes predictive inference for inverse Weibull distribution for mid type II censored observations. This censoring scheme, as a special case of multiply type II censoring, was discussed by Upadhyay et. al. (1996), Shastri and Pamari (2014) among others.

In next section, Bayes predictive density and prediction limits are derived for a general kth ordered future observation from a Pareto distribution under mid type II censoring scheme. Three different cases on parameters are considered, namely case (i) when parameter ϑ is known, case (ii) when parameter τ is known and case (iii) when both the parameters ϑ and τ are unknown. In section 3, prediction limits for the smallest future observation are obtained for all three cases. Results are tabulated and discussed in section 4 and concluded in the last section.

2. Prediction limits

Let us assume that $x_1, x_2, \dots, x_n$ be an informative sample of size n drawn from a Pareto distribution given in (1.1). Consider the mid type II censoring scheme as described in previous section and let $x_1 < \cdots < x_r$ and $x_{r+l+1} < \cdots < x_n$ be the observed life times where l middle observations are censored.

The likelihood function (LF) for this situation can be written as

$$L(\underline{x};\tau,\vartheta) = \frac{n!}{l!}\left[F(x_{r+l+1}) - F(x_r)\right]^l \prod_{i=1}^{r} f(x_i|\tau,\vartheta) \prod_{i=r+l+1}^{n} f(x_i|\tau,\vartheta) \qquad (2.1)$$

On substituting (1.1) and (1.2), and simplifying, it reduces to

$$L(\underline{x};\tau,\vartheta) = \frac{n!}{l!}\tau^{-\vartheta n}\vartheta^{(n-l)} \sum_{g=0}^{l} \Omega_g \, x_r{}^{-\vartheta(l-g)} \, x_{r+l+1}{}^{-\vartheta g} \prod_{i=1}^{r} x_i{}^{-(\vartheta+1)} \prod_{i=r+l+1}^{n} x_i{}^{-(\vartheta+1)} \qquad (2.2)$$

where

$$\Omega_g = (-1)^g \binom{l}{g}$$

Case I : When parameter ϑ is known

Consider a Pareto prior for the parameter τ, when other parameter ϑ is known

$$g_1(\tau|a_0,b_0)=b_0 a_0 (b_0\tau)^{-(a_0+1)}; \qquad b_0\tau > 1, a_0 > 0 \tag{2.3}$$

Combining LF (2.2) with prior (2.3) via Bayes theorem, the posterior distribution is defined and obtained as

$$p_1(\tau|\vartheta,\underline{x}) = \frac{L(\underline{x};\tau,\vartheta)g_1(\tau)}{\int L(\underline{x};\tau,\vartheta)g_1(\tau)d\tau}$$

$$p_1(\tau|\vartheta,\underline{x}) = (n\vartheta + a_0){b_0}^{-(n\vartheta+a_0)}\tau^{-(n\vartheta+a_0+1)} \tag{2.4}$$

Let $y_1, y_2, \dots, y_m$ be the second independent random sample of size m of future observations from the model (1.1), then the density of a general kth ordered future observation, where $1 \leq k \leq m$ will be obtained by

$$f(y_{(k)}|\tau,\vartheta) = \frac{m!}{(k-1)!\,(m-k)!}\left[F(y_{(k)})\right]^{k-1} f(y_{(k)})\left[1 - F(y_{(k)})\right]^{m-k}$$

Substituting from (1.1), (1.2) and solving

$$f(y_{(k)}|\tau,\vartheta) = \beta^{-1}(k,M)\vartheta \sum_{i=0}^{k-1} \Omega_i\, \tau^{-(M+i)\vartheta}\; {y_{(k)}}^{-(M+i)\vartheta-1} \tag{2.5}$$

where

$$\Omega_i = (-1)^i \binom{k-1}{i}$$

$$M = m - k + 1$$

and

$$\beta^{-1}(k,M) = \frac{m!}{(k-1)!\,(m-k)!}$$

Then the Bayes predictive density for future kth ordered observation will be

$$h(y_{(k)}|\underline{x}) = \int f(y_{(k)}|\tau,\vartheta)p_1(\tau|\vartheta,\underline{x})\, d\tau$$

On substituting the values

$$h\left(y_{(k)}\middle|\underline{x}\right) = \beta^{-1}(k,M)\,\vartheta(n\vartheta + a_0)\sum_{i=0}^{k-1}\Omega_i\;{y_{(k)}}^{-(M+i)\vartheta-1}\frac{{b_0}^{\vartheta(M+i)}}{\vartheta(M+i+n)+a_0}$$

(2.6)

In the context of Bayes prediction, we say here that (t_{1k}, t_{2k}) is a $100(1-\alpha)\%$ limit for k[th] ordered future observation, if

$$P_r\left[t_{1k} \le y_{(k)} \le t_{2k}\right] = 1-\alpha$$

Here t_{1k} and t_{2k} are said to be lower and upper Bayes prediction limits for k[th] ordered future observation $y_{(k)}$. One-sided Bayes prediction limits are obtained by solving

$$P_r\left[y_{(k)} \ge t_{1k}\right] = 1-\frac{\alpha}{2}$$
$$P_r\left[y_{(k)} > t_{2k}\right] = \frac{\alpha}{2}$$

Above can be rewritten as

$$\int_{t_{1k}}^{\infty} h\left(y_{(k)}|\underline{x}\right)dy = 1-\frac{\alpha}{2} \qquad (2.7)$$

and

$$\int_{t_{2k}}^{\infty} h\left(y_{(k)}|\underline{x}\right)dy = \frac{\alpha}{2} \qquad (2.8)$$

Using (2.6), (2.7) and (2.8), the one sided Bayes prediction limits are obtained by solving

$$\beta^{-1}(k,M)(n\vartheta + a_0)\sum_{i=0}^{k-1}\Omega_i\frac{{b_0}^{\vartheta(M+i)}}{\vartheta(M+n+i)+a_0}\left\{\frac{{t_{1k}}^{-(M+i)\vartheta}}{(M+i)}\right\} = 1-{}^{\alpha}\!/_{2} \qquad (2.9)$$

and

$$\beta^{-1}(k,M)(n\vartheta + a_0)\sum_{i=0}^{k-1}\Omega_i\frac{{b_0}^{\vartheta(M+i)}}{\vartheta(M+n+i)+a_0}\left\{\frac{{t_{2k}}^{-(M+i)\vartheta}}{(M+i)}\right\} = {}^{\alpha}\!/_{2} \qquad (2.10)$$

Case II : When parameter τ is known

Consider a gamma prior for the parameter ϑ, when other parameter τ is known as suggested by Arnold and Press (1989)

$$g_2(\vartheta|c_0,d_0)=\frac{{d_0}^{c_0}}{\Gamma c_0}\vartheta^{c_0-1}e^{-d_0\vartheta}; \qquad \vartheta > 0,\; c_0, d_0 > 0 \qquad (2.11)$$

Combining LF (2.5) with prior (2.11) via Bayes theorem, the posterior distribution is defined and obtained as

$$p_2(\vartheta|\tau,\underline{x}) = \frac{L(\underline{x},\tau,\vartheta)g_2(\vartheta)}{\int L(\underline{x},\tau,\vartheta)g_2(\vartheta)d\vartheta}$$

$$p_2(\vartheta|\tau,\underline{x}) = \frac{1}{\Gamma(n-l+c_0)\sum_{g=0}^{l}\Omega_g(S_x+nlog\tau)(^{-(n-l+c_0)}}\sum_{g=0}^{l}\Omega_g\,\vartheta^{n-l+c_0-1}$$

$$.\exp[-\vartheta(S_x+nlog\tau)] \qquad (2.12)$$

where

$$S_x = d_0 + (l-g)logx_r + glogx_{r+l+1} + \sum_{i=1}^{r} logx_i + \sum_{i=r+l+1}^{n} logx_i$$

Then the Bayes predictive density for the k^th^ ordered future observation will be

$$h(y_{(k)}|\underline{x}) = \int f(y_{(k)}|\tau,\vartheta)p_2(\vartheta|\tau,\underline{x})\,d\vartheta$$

where $f(y_{(k)}|\tau,\vartheta)$ is pdf of k^th^ ordered future observation as discussed in case I.

On substituting the values from (2.5) and (2.12), and on simplification we get

$$h(y_{(k)}|\underline{x}) = \frac{\beta^{-1}(k,M)(n-l+c_0)}{\sum_{g=0}^{l}\Omega_g\;(S_x+nlog\tau)^{-(n-l+c_0)}}\sum_{g=0}^{l}\sum_{i=0}^{k-1}\Omega_g\,\Omega_i\quad\frac{1}{y_{(k)}}$$

$$\{(S_x+nlog\tau)+(M+i)log\tau \quad +(M+i)logy_{(k)}\}^{-(n-l+c_0+1)} \qquad (2.13)$$

Using (2.7) and (2.8), the one sided Bayes prediction limits are obtained by solving

$$\frac{\beta^{-1}(k,M)}{\sum_{g=0}^{l}\Omega_g\;(S_x+nlog\tau)^{-(n-l+c_0)}}\sum_{g=0}^{l}\sum_{i=0}^{k-1}\frac{\Omega_g\Omega_i}{(M+i)}\;\{(S_x+nlog\tau)+(M+i)log\tau$$

$$+(M+i)logt_{1k}\}^{-(n-l+c_0)} = 1-{}^{\alpha}/_{2}$$

(2.14)

and

$$\frac{\beta^{-1}(k,M)}{\sum_{g=0}^{l}\Omega_g\ (S_x+nlog\tau)^{-(n-l+c_0)}}\sum_{g=0}^{l}\sum_{i=0}^{k-1}\frac{\Omega_g\Omega_i}{(M+i)}\ \{(S_x+nlog\tau)+(M+i)log\tau + (M+i)logt_{2k}\}^{-(n-l+c_0)} = {}^{\alpha}/_{2}$$

(2.15)

Case III : When both parameters τ and ϑ are unknown

As suggested by Arnold and Press (1989), we consider a gamma prior for the parameter ϑ and Pareto prior for parameter τ given ϑ

$$g_3(\vartheta)=\frac{{d_0}^{c_0}}{\Gamma c_0}\vartheta^{c_0-1}e^{-d_0\vartheta};\qquad \vartheta>o,\ c_0,d_0>0 \tag{2.16}$$

$$g_4(\tau|\vartheta)=\vartheta b_0 a_0(b_0\tau)^{-(\vartheta a_0+1)};\qquad b_0\tau>1,\ a_0>0 \tag{2.17}$$

Combining LF (4) with priors (2.16), (2.17) via Bayes theorem, the posterior distribution is defined and obtained as

$$p_3(\vartheta|\tau,\underline{x})=\frac{L(\underline{x},\tau,\vartheta)g_3(\vartheta)g_4(\tau|\vartheta)}{\int\int L(\underline{x},\tau,\vartheta)g_3(\vartheta)g_4(\tau|\vartheta)d\tau d\vartheta}$$

$$p_3(\vartheta,\tau|\underline{x})=\frac{(n+a_0)}{\Gamma(n-l+c_0)\sum_{g=0}^{l}\Omega_g(S_x-nlogb_0\ )^{-(n-l+c_0)}}\sum_{g=0}^{l}\Omega_g\,\vartheta^{n-l+c_0}$$

$$.\,\tau^{-\vartheta n-\vartheta a_0-1}\exp[-\vartheta(S_x+a_0logb_0\ )] \tag{2.18}$$

If $f(y_{(k)}|\tau,\vartheta)$ is the pdf of kth ordered future observation as discussed in case I, then the Bayes predictive density for kth ordered future observation will be

$$h(y_{(k)}|\underline{x})=\int\int f(y_{(k)}|\tau,\vartheta)p_3(\vartheta,\tau|\underline{x})\,d\vartheta d\tau$$

On substituting the values from (2.5) and (2.18), and on simplification we get

$$h(y_{(k)}|\underline{x}) = \frac{\beta^{-1}(k,M)(n+a_0)(n-l+c_0)}{\sum_{g=0}^{l}\Omega_g\ (S_x-nlogb_0)^{-(n-l+c_0)}}\sum_{g=0}^{l}\sum_{i=0}^{k-1}\Omega_g\,\Omega_i\quad\frac{1}{y_{(k)}(M+n+a_0+i)}$$

$$\{(S_x+(M+i)logy_{(k)}-(M+n+i)logb_0\}^{-(n-l+c_0+1)} \tag{2.19}$$

Using (2.7) and (2.8), the one sided Bayes prediction bound limits are obtained by solving

$$\frac{\beta^{-1}(k,M)(n+a_0)}{\sum_{g=0}^{l}\Omega_g\ (S_x-nlogb_0)^{-(n-l+c_0)}}\sum_{g=0}^{l}\sum_{i=0}^{k-1}\frac{\Omega_g\Omega_i}{(M+n+a_0+i)(M+i)}$$

$$.\{(S_x+(M+i)logt_{1k}-(M+n+i)logb_0\}^{-(n-l+c_0)}=1-{}^{\alpha}/_{2} \qquad (2.20)$$

and

$$\frac{\beta^{-1}(k,M)(n+a_0)}{\sum_{g=0}^{l}\Omega_g\ (S_x-nlogb_0)^{-(n-l+c_0)}}\sum_{g=0}^{l}\sum_{i=0}^{k-1}\frac{\Omega_g\Omega_i}{(M+n+a_0+i)(M+i)}$$

$$\{S_x+(M+i)logt_{2k}-(M+n+i)logb_0\}^{-(n-l+c_0)}={}^{\alpha}/_{2} \qquad (2.21)$$

3. Prediction limits for the future order smallest observation

Prediction limits for the smallest observation can be obtained by substituting k=1 in all three cases on parameters discussed in previous section.

Case I : When parameter ϑ is known

Prediction limits obtained from (2.9) and (2.10) reduces to

$$t_{11}=b_0\left[\left(1-{}^{\alpha}/_{2}\right)\frac{\{\vartheta(m+n)+a_0\}}{(n\vartheta+a_0)}\right]^{-1/m\vartheta} \qquad (3.1)$$

$$t_{21}=b_0\left[\left({}^{\alpha}/_{2}\right)\frac{\{\vartheta(m+n)+a_0\}}{(n\vartheta+a_0)}\right]^{-1/m\vartheta} \qquad (3.2)$$

which are in nice closed form.

Case II: When parameter τ is known

Prediction limits obtained from (2.14) and (2.15) reduces to

$$\frac{\sum_{g=0}^{l}\ \Omega_g\{S_x+(m+n)log\tau+mlogt_{11}\}^{-(n-l+c_0)}}{\sum_{g=0}^{l}\Omega_g\ (S_x+nlog\tau)^{-(n-l+c_0)}}=1-{}^{\alpha}/_{2} \qquad (3.3)$$

and

$$\frac{\sum_{g=0}^{l}\ \Omega_g\{S_x+(m+n)log\tau+mlogt_{21}\}^{-(n-l+c_0)}}{\sum_{g=0}^{l}\Omega_g\ (S_x+nlog\tau)^{-(n-l+c_0)}}={}^{\alpha}/_{2} \qquad (3.4)$$

It can be seen that the above equations cannot be solved explicitly for t_{21} and t_{21}, one needs iterative method to solve them.

Case III: When both parameters τ *and* ϑ are unknown

Prediction limits obtained from (2.20) and (2.21) reduces to

$$\frac{(n+a_0)\ \sum_{g=0}^{l}\ \Omega_g\ \{(S_x+mlogt_{11}-(m+n)logb_0\}^{-(n-l+c_0)}}{(m+n+a_0)\sum_{g=0}^{l}\Omega_g\ (S_x-nlogb_0)^{-(n-l+c_0)}}=1-{}^{\alpha}/_{2} \tag{3.5}$$

and

$$\frac{(n+a_0)\ \sum_{g=0}^{l}\ \Omega_g\ \{(S_x+mlogt_{21}-(m+n)logb_0\}^{-(n-l+c_0)}}{(m+n+a_0)\sum_{g=0}^{l}\Omega_g\ (S_x-nlogb_0)^{-(n-l+c_0)}}={}^{\alpha}/_{2} \tag{3.6}$$

Since above equation are not in explicit form therefore bisection method is used for finding the prediction limits t_{11} and t_{21} .

4. Discussion

The present section computes proposed Bayes prediction intervals for the smallest ordered future observation on the basis of Monte Carlo simulation technique of 1000 randomly generated samples from Pareto distribution by taking parameters τ=1 and ϑ =0.5. Two different sample sizes namely 6 and 20 were considered for both informative and future samples i.e. n=m=6 and n=m=20. During computation we have considered two levels of α namely 5% and 1%, while on reporting we found similar behaviour of proposed prediction limits. Therefore due to paucity of space we have reported 95% coverage probability of prediction intervals for all the three cases.

Case I: When parameter ϑ is known

Bayes prediction limits are computed for two different values of parameter ϑ (=0.5, 2.0) and Bayes prediction intervals are tabulated in the tables 1 and 2. Number of values, namely 0.5, 1.0, 2.0, 3.0 are assigned to the hyperparameter a_0 and 0.5(0.5)3.0 to the hyperparameter b_0. In this case prediction intervals are independent to censoring fraction so no variation in censoring fraction is reported. From both the tables, it may be noted that prediction intervals increase with the increase in hyperparameters a_0 and b_0. It is obvious that as value of parameter ϑ increases prediction intervals also increases. Prediction interval decreases with the increase in sample size.

Case II: When parameter τ is known

In this case parameter τ is known and kept fixed at 0.5, 1.0, 2.0, 4.0. Three different values 2.0, 3.0, 4.0 are assigned to hyperparameter c_0 and four different values 0.5, 1.0, 2.0, 3.0 are assigned to the hyperparameter d_0. Different censoring fractions are also considered for the study. For n=m=6, 33% and 66% observations are censored and hence resulted in l=2 and 4. Where as in case of n=m=20, values of l has been taken as 4, 8, 12, 16 so as 20%, 40%, 60%, 80% observations are censored. For the case results are reported in tables 3 and 4. It is evident from the results that the prediction intervals increase with the increase in hyperparameter d_o but reverse is the case with the effect of hyperparameter c_0. As c_0 increases, prediction intervals decrease. It may be deduced that with the increase in parameter τ, prediction intervals decrease. Similar is the case with the parameter ϑ, prediction intervals decrease with the increase in the value of ϑ. As far as censoring fraction is concerned, as we increase number of censored observations, results are found to be better i.e. if l increases prediction intervals decrease. Same trend may be noted when size of samples is 20, with shorter prediction intervals everywhere. Moreover large sample size and large numbers of censored observations, provide the shortest width of prediction intervals.

Case III : When parameter τ and ϑ both are unknown

Tables 5 to 8 report prediction intervals for the case when τ and ϑ both are unknown. As it may be deduced from tables 1 and 2, prediction interval is shorter for a_0=0.5, so by keeping value of hyperparameter a_0 fixed at 0.5 and considering b_0=1.0, effect of other hyperparameters and parameters on prediction interval is studied and reported in tables 5 and 6.

For the purpose three different values namely 2.0, 3.0, 4.0 are assigned to the hyperparameter c_0 and four different values 0.5, 1.0, 2.0, 3.0 to hyperparameter d_0.. Effect of censored observations is also studied by taking 33% and 66% of observations censored at n=m=6 and 20%, 40%, 60% and 80% of observations censored when n=m=20 which yields l=2, 4 and l=4, 8, 12 respectively.

For small sample sizes i.e. n=m=6, prediction intervals increase everywhere with the increase in the value of hyperparameter d_0. As far as effect of hyperparameter c_0 is concerned, for smaller choices of ϑ, prediction intervals decrease with the increase in c_0. As we increase ϑ, prediction intervals decrease everywhere with smaller values of τ i.e. τ<=2. For τ>2, with the increase in c_0, prediction intervals first decrease then increase. In this range, it can be minimized with the proper choices of hyperparameters c_0 and d_0. As far as the effect of parameter τ is concerned, prediction intervals decrease as τ increases. In case as we increase number of censored observations, prediction intervals increase for $\tau \leq 2$ and then decreases for $\tau > 2$. It has been studied and reported that as we increase sample sizes i.e. (informative and future) n=m=20, prediction intervals decrease everywhere. It may be further shorten by increasing the value of parameter ϑ.

Similarly, it is obvious from the results of previous case that prediction intervals found to be shorter with minimum value of hyperparameter d_0 and maximum value of hyperparameter c_0. So keeping c_0 fixed at 4.0 and d_0 at 0.5 effect of other hyperparameters have studied and reported in tables 7 and 8. Four different values are assigned to the hyperparameter b_0 namely 0.5, 1.0, 2.0, 3.0 and three different values 2.0, 3.0 and 4.0 are assigned to the hyperparameter a_0. Results are studied and reported for ϑ =0.5, 2.0 and τ=0.5, 1.0, 2.0, 4.0. Effect of censored observations has also been studied by increasing the number of censored observations.

It can be deducted from the table 7 that for n=m=6 and for ϑ =0.5 prediction intervals increase with the increase in the value of hyperparameter b_0. While trend is reverse in the majority of the cases with the hyperparameter a_0. With less number of observations censored i.e. for l=2 and b_0<=1, prediction intervals decrease with increase in a_0. For b_0>1, prediction intervals can be minimized with the proper selection of hyperparameters a_0, b_0 and parameters τ and ϑ. With the increase in the value of τ, prediction intervals decrease for majority of cases except for l=4, τ=4 and b_0=2. As we increase number of censored observations prediction intervals first decrease for τ<=1 and c>1 then it increase. If size of informative and future samples increase, prediction intervals decrease everywhere i.e. for n=m=20, it is found and reported shorter. It can further be minimized at larger number of observation censored.

5. Conclusion

Under the mid censoring, prediction interval for future observation is obtained in nice closed form in the case when inequality parameter is known. For other two cases i.e. when precision parameter is known and when both the parameters are unknown, even though Bayes prediction limits cannot be obtained in explicit form, it can be computed using iterative methods. From the computation and discussion, it is clear that proposed limits perform better almost everywhere, especially for small values of hyperparameters except c_0. However for other values of hyperparameters, one can still consider proposed limits with proper choice of parameters. For mid censoring with large number of observations are censored in middle, prediction interval is found to be shortest. When size of both informative and future sample is large proposed prediction intervals are shorter everywhere. A similar study can be performed for any other future observation/characteristic of interest which will be useful to the situation where such type of life test is needed.

Acknowledgement

The authors would like to extend their sincere thanks to referees for their valuable and remarkable suggestions to improve the paper.

Table 1 : Prediction intervals when parameter ϑ is known for m=n=6 and τ =1.

b_0	ϑ =0.5				ϑ =2.0			
	a_0=0.5	a_0=1.0	a_0=2.0	a_0=3.0	a_0=0.5	a_0=1.0	a_0=2.0	a_0=3.0
0.5	0.16990	0.17074	0.17252	0.17295	0.96310	0.96950	0.98090	0.99120
1.0	0.33979	0.34148	0.34404	0.34504	1.92690	1.93910	1.96180	1.98240
1.5	0.50969	0.51222	0.51606	0.51755	2.89030	2.90870	2.94270	2.97361
2.0	0.67959	0.68296	0.68808	0.69179	3.85380	3.87830	3.92370	3.96481
2.5	0.84948	0.85370	0.86259	0.86474	4.81730	4.84794	4.90465	4.95601
3.0	1.01938	1.02444	1.03212	1.03768	5.78077	5.81753	5.88558	5.94721

Table 2 : Prediction intervals when parameter ϑ is known for m=n=20 and τ =1.

b_0	ϑ =0.5				ϑ =2.0			
	a_0=0.5	a_0=1.0	a_0=2.0	a_0=3.0	a_0=0.5	a_0=1.0	a_0=2.0	a_0=3.0
0.5	0.04719	0.04722	0.04727	0.04731	0.20707	0.20720	0.20744	0.20768
1.0	0.09438	0.09444	0.09453	0.09462	0.41414	0.41439	0.41488	0.41536
1.5	0.14157	0.14165	0.14180	0.14192	0.62121	0.62159	0.62232	0.62304

2.0	0.18877	0.18887	0.18906	0.18923	0.82828	0.82879	0.82977	0.83071
2.5	0.23596	0.23609	0.23633	0.23654	1.03535	1.03598	1.03721	1.03839
3.0	0.28315	0.28331	0.28360	0.28385	1.24242	1.24318	1.24465	1.24607

Table 3 : Prediction intervals when parameter τ is known for m=n=6.

l	τ	c_0		ϑ =0.5				ϑ =2.0		
			d_0=0.5	d_0=1.0	d_0=2.0	d_0=3.0	d_0=0.5	d_0=1.0	d_0=2.0	d_0=3.0
2	0.5	2.0	5.66303	6.43009	6.63947	7.82495	0.83529	0.97120	1.29380	1.61053
		3.0	4.38005	4.61511	5.19989	5.81399	0.70589	0.80454	1.06533	1.34397
		4.0	3.37272	3.73158	4.11115	4.62659	0.60952	0.69358	0.90573	1.12939
	1.0	2.0	2.72667	3.14047	3.58390	3.90281	0.41342	0.47968	0.62341	0.80753
		3.0	2.23518	2.25965	2.63084	2.75828	0.34756	0.40542	0.53336	0.67616
		4.0	1.73492	1.77359	1.79430	2.24217	0.30235	0.34752	0.46385	0.57243
	2.0	2.0	1.44070	1.54508	1.73939	1.96591	0.20188	0.24519	0.32482	0.40599
		3.0	1.07733	1.14132	1.34698	1.49042	0.17563	0.20828	0.26657	0.33625
		4.0	0.85683	0.88623	1.00009	1.09158	0.15223	0.17382	0.22869	0.28375
	4.0	2.0	0.69012	0.78171	0.84583	1.02633	0.10537	0.12333	0.16094	0.20600
		3.0	0.56427	0.57586	0.66718	0.73279	0.08634	0.10288	0.13322	0.16676
		4.0	0.42310	0.46473	0.50415	0.56691	0.07801	0.08690	0.11358	0.14010
4	0.5	2.0	4.53206	4.62070	6.19499	8.16095	0.86312	0.97076	1.32416	1.72214
		3.0	4.40764	4.27823	5.05713	6.34378	0.67476	0.80555	1.04288	1.61831
		4.0	2.85346	3.67336	3.29189	4.72130	0.57097	0.69599	0.98533	1.31029
	1.0	2.0	2.59581	2.50160	3.07945	4.43347	0.45132	0.52388	0.70029	0.87752
		3.0	1.74106	2.04434	2.20787	2.90352	0.43074	0.42200	0.56363	0.71445
		4.0	1.54268	1.43221	2.19217	2.32049	0.25209	0.35357	0.47338	0.60547
	2.0	2.0	1.61396	1.46542	2.40068	2.32416	0.21460	0.26276	0.34841	0.44213
		3.0	1.15978	1.30764	1.54580	1.58656	0.17358	0.21099	0.27902	0.36232
		4.0	0.69657	0.86281	1.02332	1.15333	0.14301	0.17458	0.23641	0.30094
	4.0	2.0	0.70576	0.62206	0.95193	1.07388	0.10926	0.13203	0.17391	0.27896
		3.0	0.40886	0.54915	0.59253	0.79721	0.08866	0.10500	0.13835	0.21974
		4.0	0.38741	0.44179	0.46491	0.51306	0.07174	0.08712	0.11611	0.18664

Table 4 : Prediction intervals when parameter τ is known for m=n=20.

l	τ	c_0	ϑ =0.5			ϑ =2.0		
			d_0=0.5	d_0=1.0	d_0=2.0	d_0=0.5	d_0=1.0	d_0=2.0
4	1.0	2.0	0.451196	0.453198	0.474154	0.099664	0.10421	0.115369
		3.0	0.423244	0.429316	0.446797	0.094586	0.099705	0.109419
		4.0	0.401248	0.408118	0.42409	0.090405	0.096374	0.104443
	2.0	2.0	0.224298	0.228444	0.238965	0.05031	0.05326	0.056897
		3.0	0.212954	0.21714	0.221512	0.047017	0.049927	0.055018
		4.0	0.201088	0.204603	0.208688	0.045507	0.047701	0.0522
	4.0	2.0	0.112364	0.112553	0.119609	0.024579	0.026251	0.029005
		3.0	0.105874	0.107927	0.111554	0.024101	0.024974	0.027345
		4.0	0.100726	0.101568	0.105022	0.022758	0.023533	0.026328
8	1.0	2.0	0.445294	0.45638	0.465305	0.100021	0.10491	0.117171
		3.0	0.429159	0.432017	0.435532	0.09443	0.099583	0.109127
		4.0	0.402052	0.405125	0.419655	0.091487	0.093909	0.104506
	2.0	2.0	0.227388	0.227039	0.237566	0.050665	0.052184	0.05834
		3.0	0.214047	0.215791	0.223301	0.047624	0.050203	0.05509
		4.0	0.200976	0.203682	0.209185	0.045559	0.047058	0.052478
	4.0	2.0	0.112495	0.114568	0.116942	0.025046	0.02626	0.02876
		3.0	0.106734	0.107871	0.112903	0.023661	0.024902	0.02759
		4.0	0.100618	0.101535	0.10201	0.022566	0.023741	0.026115
12	1.0	2.0	0.45443	0.466234	0.474122	0.100718	0.106292	0.117014
		3.0	0.420134	0.430005	0.446309	0.096126	0.099996	0.111539
		4.0	0.404082	0.406177	0.416849	0.089833	0.094846	0.104226
	2.0	2.0	0.224756	0.232225	0.235167	0.050414	0.053371	0.058483
		3.0	0.212241	0.215365	0.221023	0.047134	0.049749	0.05508
		4.0	0.200566	0.202317	0.207435	0.045453	0.046743	0.052319
	4.0	2.0	0.111415	0.115047	0.117129	0.025217	0.026296	0.029139
		3.0	0.104605	0.107334	0.111855	0.023829	0.024922	0.02786
		4.0	0.099434	0.101381	0.10356	0.022585	0.023686	0.026418
16	1.0	2.0	0.45355	0.460161	0.476216	0.100637	0.108551	0.120915
		3.0	0.424737	0.425809	0.444282	0.094885	0.101282	0.111579
		4.0	0.389229	0.397408	0.412419	0.08922	0.095096	0.106168
	2.0	2.0	0.222924	0.228039	0.24014	0.04995	0.054207	0.060668
		3.0	0.206473	0.213065	0.220481	0.047421	0.050551	0.056485
		4.0	0.196482	0.195984	0.207882	0.044992	0.048189	0.053585
	4	2.0	0.113646	0.114541	0.118267	0.02524	0.027119	0.030037
		3.0	0.106134	0.107232	0.109192	0.023883	0.025154	0.028129
		4.0	0.096297	0.09835	0.101961	0.022276	0.02403	0.026673

Table 5 : Prediction intervals in case when τ and ϑ are unknown for m=n=6 when hyperparameters are fixed at a_0=0.5, b_0=1.

l	τ	c_0	ϑ =0.5				ϑ =2.0			
			d_0=0.5	d_0=1.0	d_0=2.0	d_0=3.0	d_0=0.5	d_0=1.0	d_0=2.0	d_0=3.0
2	0.5	2.0	6.95391	7.23282	8.31669	10.5588	1.42823	1.57483	1.90505	2.27777
		3.0	4.28715	4.54354	5.10018	6.90627	1.12745	1.23117	1.46966	1.74699
		4.0	3.10286	3.12596	3.48365	5.35045	0.93160	1.01734	1.20103	1.40970
	1.0	2.0	3.72554	4.09322	4.77209	6.16856	0.48456	0.57254	0.77445	0.99549
		3.0	2.60764	2.84564	3.05976	4.28751	0.40119	0.47361	0.61968	0.79019
		4.0	1.87416	2.04744	2.31245	3.06325	0.33849	0.39362	0.52170	0.66501
	2.0	2.0	2.13783	2.24409	2.79970	3.41258	0.20767	0.14402	0.13158	0.22580
		3.0	1.54376	1.60085	1.91373	2.55734	0.03119	0.03809	0.09534	0.19289
		4.0	1.17426	1.14839	1.33348	1.81913	0.15610	0.09333	0.08741	0.16315
	4.0	2.0	1.08235	1.14238	1.33601	1.66818	4.02542	3.09332	1.62442	0.81804
		3.0	0.82714	0.73057	0.99202	1.11649	0.00056	0.00291	0.00382	0.01326
		4.0	0.61719	0.66003	0.76396	0.89479	2.95076	2.13356	1.08730	0.57603
4	0.5	2.0	9.40787	9.56240	11.1654	14.0259	1.57160	1.75174	2.13330	2.58013
		3.0	5.67120	5.75143	5.76514	8.62259	1.22045	1.35046	1.63402	1.91534
		4.0	3.37940	3.48355	4.06119	5.46208	0.98370	1.09317	1.29994	1.52801
	1.0	2.0	4.56065	5.21781	5.76076	7.92455	0.51485	0.62453	0.85733	1.11481
		3.0	2.97922	3.22213	3.56845	4.36560	0.40215	0.48935	0.68163	0.87209
		4.0	2.08593	2.07668	2.38031	3.40277	0.32658	0.41216	0.55388	0.69991
	2.0	2.0	2.31314	2.78697	3.09971	3.51486	0.18115	0.13013	0.09403	0.16838
		3.0	1.62198	1.83350	2.09129	2.34375	0.06321	0.09275	0.05194	0.12734
		4.0	1.01485	1.12792	1.28405	1.51306	0.11780	0.09644	0.06142	0.09835
	4.0	2.0	0.92293	0.81173	0.84106	1.18094	2.88684	2.17589	1.04636	0.56508
		3.0	0.58330	0.49090	0.55294	0.73923	0.16607	0.21540	0.10080	0.18657
		4.0	0.42121	0.35369	0.48485	0.59185	1.97158	1.46553	0.75271	0.42230

Table 6 : Prediction intervals in case when τ and ϑ are unknown for m=n=20 when hyperparameters are fixed at a_0=0.5, b_0=1.

l	τ	c_0	ϑ =0.5			ϑ =2.0		
			d_0=0.5	d_0=10	d_0=2.0	d_0=0.5	d_0=1.0	d_0=2.0
4	1.0	2.0	0.52694	0.50525	0.52389	0.10377	0.11191	0.12040
		3.0	0.46947	0.47516	0.48977	0.09954	0.10448	0.11579
		4.0	0.44454	0.44974	0.45716	0.09574	0.09943	0.10917
	2.0	2.0	0.30352	0.31415	0.32134	0.05135	0.04522	0.03328
		3.0	0.29190	0.29101	0.30040	0.00080	0.00118	0.00236
		4.0	0.27365	0.27077	0.28378	0.04702	0.04403	0.02857
	4.0	2.0	0.14528	0.14315	0.15574	1.07013	0.98489	0.83630
		3.0	0.13017	0.13571	0.14441	0.00182	0.00053	0.00146
		4.0	0.12496	0.12890	0.13480	0.96338	0.86641	0.74824
8	1.0	2.0	0.49490	0.50402	0.52891	0.10481	0.11051	0.12137
		3.0	0.46352	0.47795	0.49338	0.10041	0.10536	0.11607
		4.0	0.44204	0.44918	0.46581	0.09534	0.09998	0.11047
	2.0	2.0	0.29706	0.30993	0.31904	0.05206	0.04457	0.03313
		3.0	0.27759	0.27881	0.30488	0.00442	0.00270	0.00109
		4.0	0.26403	0.27170	0.28682	0.05205	0.03513	0.02823
	4.0	2.0	0.12688	0.12965	0.14169	1.05190	0.95048	0.79949
		3.0	0.11367	0.12417	0.13362	0.00664	0.00446	0.00211
		4.0	0.11199	0.11301	0.13062	0.93174	0.84313	0.73739
12	1.0	2.0	0.50546	0.50973	0.52964	0.10532	0.11116	0.12303
		3.0	0.46525	0.47239	0.49140	0.10025	0.10592	0.11522
		4.0	0.42933	0.44900	0.46565	0.09451	0.09997	0.11053
	2.0	2.0	0.28701	0.29595	0.30241	0.05091	0.04179	0.03423
		3.0	0.26303	0.27801	0.28610	0.01744	0.02056	0.01480
		4.0	0.25122	0.25806	0.26728	0.03826	0.03845	0.02528
	4.0	2.0	0.09465	0.10066	0.10520	0.96560	0.86766	0.74833
		3.0	0.08614	0.09150	0.10262	0.02275	0.02156	0.01012
		4.0	0.07437	0.08505	0.09129	0.85060	0.77955	0.66969
16	1.0	2.0	0.50948	0.51743	0.54193	0.10775	0.11341	0.12772
		3.0	0.46892	0.47046	0.49465	0.10029	0.10550	0.11913
		4.0	0.43808	0.44150	0.45817	0.09427	0.09941	0.11349
	2.0	2.0	0.20475	0.21118	0.23090	0.07323	0.08352	0.05690
		3.0	0.18088	0.18808	0.21067	0.07284	0.05975	0.05367
		4.0	0.15575	0.16097	0.18674	0.06505	0.06537	0.04252
	4.0	2.0	0.13261	0.11379	0.10837	0.71716	0.78154	0.48872
		3.0	0.11899	0.11703	0.09211	0.02241	0.00777	0.08663
		4.0	0.09708	0.08942	0.08761	0.66133	0.61651	0.43978

Table 7 : Prediction intervals in case when τ and ϑ are unknown for m=n=6 when hyperparameters are fixed at c_0=4.0, d_0=0.5.

l	τ	a_0	ϑ =0.5				ϑ =2.0			
			b_0=0.5	b_0=1.0	b_0=2.0	b_0=3.0	b_0=0.5	b_0=1.0	b_0=2.0	b_0=3.0
2	0.5	2.0	2.89923	3.67808	4.56569	4.54574	0.86989	0.90243	0.66485	0.29367
		3.0	2.85579	3.63365	4.06994	4.40978	0.85753	0.89541	0.66677	0.31116
		4.0	2.70825	3.47267	4.21462	4.41737	0.83630	0.88249	0.66737	0.30594
	1.0	2.0	1.76666	2.23910	2.42854	2.43962	0.45241	0.33697	0.30809	2.94867
		3.0	1.77028	2.19781	2.55068	2.35374	0.44307	0.33008	0.29456	2.89736
		4.0	1.71682	2.10410	2.39688	2.05740	0.44042	0.32873	0.31173	2.84030
	2.0	2.0	1.08842	1.18242	1.22549	2.10063	0.17004	0.15594	5.59664	32.2652 0
		3.0	1.07617	1.09714	1.16706	2.17885	0.16163	0.15206	5.50171	30.9033
		4.0	1.04078	1.12065	1.22529	2.18518	0.16219	0.15415	5.53248	29.8935
	4.0	2.0	0.59330	0.61632	2.37780	11.6053	0.07365	2.78058	51.8639	239.409
		3.0	0.57636	0.60494	2.56295	11.7049	0.07628	2.75093	50.0863	237.652
		4.0	0.57230	0.59932	2.71474	13.1023	0.07895	2.72058	48.3788	235.002
4	0.5	2.0	3.15878	3.93152	3.95519	3.70790	0.93643	0.97216	0.65047	0.17820
		3.0	3.16161	3.71684	4.10240	3.56338	0.92564	0.96248	0.65483	0.18298
		4.0	2.97815	3.64155	3.64102	3.50379	0.92036	0.94810	0.64955	0.17476
	1.0	2.0	1.88752	1.98395	1.61382	1.27626	0.49038	0.32031	0.29494	2.33874
		3.0	1.92816	1.93296	1.66081	1.31953	0.47910	0.31686	0.28352	2.16819
		4.0	1.84333	2.05281	1.51384	1.39888	0.47372	0.32363	0.28015	2.27032
	2.0	2.0	0.99000	0.86457	0.77980	1.55826	0.16383	0.15216	3.87679	21.8476
		3.0	0.96594	0.83242	0.82123	1.50368	0.16044	0.14519	3.55415	21.0422
		4.0	0.98708	0.70654	0.74195	1.65006	0.16005	0.18348	4.07462	23.1010
	4.0	2.0	0.41313	0.38909	2.12880	7.66565	0.07275	1.99216	36.4821	178.391
		3.0	0.40469	0.34711	1.95703	8.08074	0.08594	1.83492	35.4190	179.387
		4.0	0.40414	0.34756	2.07894	7.42498	0.08690	2.22407	35.7629	167.478

Table 8 : Prediction intervals in case when τ and ϑ are unknown for m=n=20 when hyperparameters are fixed at c_0=4.0, d_0=0.5.

l	τ	a_0	ϑ =0.5			ϑ =2.0		
			b_0=0.5	b_0=1.0	b_0=2.0	b_0=0.5	b_0=1.0	b_0=2.0
4	1.0	2.0	0.32119	0.43494	0.53778	0.11873	0.09407	0.10027
		3.0	0.32017	0.43810	0.54210	0.11851	0.09367	0.09988
		4.0	0.31677	0.43570	0.54206	0.11905	0.09493	0.10490
	2.0	2.0	0.22345	0.26653	0.23852	0.04748	0.05125	1.87926
		3.0	0.21760	0.27064	0.24356	0.04776	0.05066	1.92943
		4.0	0.21751	0.26814	0.24720	0.04734	0.05034	1.92786
	4.0	2.0	0.13495	0.12478	0.30862	0.02335	0.97099	14.66358
		3.0	0.13631	0.12497	0.29593	0.02464	0.94535	14.54831
		4.0	0.13328	0.12277	0.30150	0.02654	0.93695	15.02869
8	1.0	2.0	0.32073	0.42925	0.53076	0.12007	0.09574	0.09921
		3.0	0.31871	0.44007	0.53596	0.11930	0.09520	0.10138
		4.0	0.31966	0.43291	0.52470	0.11905	0.09484	0.10071
	2.0	2.0	0.21829	0.26363	0.23248	0.04766	0.05476	1.88354
		3.0	0.22037	0.26869	0.22676	0.04668	0.05267	1.86468
		4.0	0.22080	0.26621	0.21828	0.04746	0.05460	1.86430
	4.0	2.0	0.13174	0.11414	0.28343	0.03041	0.92715	14.42667
		3.0	0.13005	0.11243	0.24576	0.02512	0.92574	14.43300
		4.0	0.12972	0.11051	0.27357	0.02164	0.93458	14.30841
12	1.0	2.0	0.32579	0.44422	0.49581	0.11972	0.09401	0.11915
		3.0	0.32672	0.43941	0.50235	0.12048	0.09322	0.14279
		4.0	0.32564	0.43153	0.49150	0.11963	0.09346	0.13302
	2.0	2.0	0.22019	0.25027	0.16479	0.04746	0.05986	1.64937
		3.0	0.21929	0.25046	0.15221	0.04722	0.07770	1.70250
		4.0	0.21435	0.25420	0.16180	0.04751	0.05565	1.71430
	4.0	2.0	0.12488	0.08247	0.19056	0.03452	0.81954	13.42364
		3.0	0.12556	0.08555	0.26569	0.03817	0.82815	13.74385
		4.0	0.12272	0.08331	0.32852	0.02706	0.83784	13.51744
16	1.0	2.0	0.33771	0.43528	0.30544	0.12805	0.09338	0.10183
		3.0	0.34035	0.43466	0.30460	0.12724	0.09235	0.11441
		4.0	0.33509	0.43128	0.31314	0.12679	0.09369	0.12327
	2.0	2.0	0.21121	0.16015	0.20468	0.04624	0.03383	1.14430
		3.0	0.21515	0.15189	0.15949	0.04729	0.05318	1.57977
		4.0	0.21203	0.15384	0.18171	0.04662	0.04590	2.19337
	4.0	2.0	0.07692	0.09852	0.47418	0.02876	0.57736	10.88102
		3.0	0.07672	0.08459	0.58430	0.02633	0.88034	10.88151
		4.0	0.07696	0.07358	0.49681	0.02704	0.92697	11.93747

References

Ahmadi, M. V., & Doostparast, M. (2019) : Pareto analysis for the lifetime performance index of products on the basis of progressively first-failure-censored batches under balanced symmetric and asymmetric loss functions. *Journal of Applied Statistics*, **46(7)**, 1196-1227.

Aitchison, J, & Dunsmore I.R. (1975) : *Statistical Prediction Analysis*. Cambridge University Press.

Ali Mousa, M. A. M. (2003) : Bayesian prediction based on Pareto doubly censored data. *Statistics*, **37(1)**, 65-72.

Arnold, B. C. (2015) : *Pareto distribution*. John Wiley & Sons, Ltd.

Arnold, B. C., & Press, S. J. (1989) : Bayesian estimation and prediction for Pareto data. *Journal of the American Statistical Association*, **84(408)**, 1079-1084.

Dunsmore, I. R., & Amin, Z. H. (1998) : Some prediction problems concerning samples from the Pareto distribution. *Communications in Statistics-Theory and Methods*, **27(5)**, 1221-1238.

El-Din, M. M. M., Kotb, M. S., Abd-Elfattah, E. F., & Newer, H. A. (2017) : Bayesian inference and prediction of the Pareto distribution based on ordered ranked set sampling. *Communications in Statistics-Theory and Methods*, **46(13)**, 6264-6279.

Hsu, B. M., Shu, M. H., & Chen, B. S. (2011) : Evaluating lifetime performance for the Pareto model with censored and imprecise information. *Journal of Statistical Computation and Simulation*, **81(12)**, 1817-1833.

Raqab, M. Z., Asgharzadeh, A., & Valiollahi, R. (2010) : Prediction for Pareto distribution based on progressively Type-II censored samples. *Computational Statistics & Data Analysis*, **54(7)**, 1732-1743.

Shafay, A. R., Balakrishnan, N., & Ahmadi, J. (2017) : Bayesian prediction of order statistics with fixed and random sample sizes based on k-record values from Pareto distribution. *Communications in Statistics-Theory and Methods*, **46(2)**, 721-735.

Shastri, V. & Pamari, S.K. (2014) : Bayesian prediction limits for exponential failures when the observations are multiply type II censored, Research and Reviews: *Journal of Statistics, Special Issue on Recent Statistical Methodologies and Applications*, 1-14.

Shastri, V., Ranjan, R & Pal, D.S. (2020) : Bayesian prediction limits for inverse Weibull distribution when observations are mid type II censored, To appear in *J. Stat. Appl. Probab.*

Smadi, M. M., Ansari, S. I., & Jaradat, A. A. (2019) : Reliability estimation for the randomly censored pareto distribution. *International Journal of Electrical & Computer Engineering (2088-8708)*, *9*.

Soliman, A. A. (2000) : Bayes prediction in a Pareto lifetime model with random sample size. *Journal of the Royal Statistical Society: Series D (The Statistician)*, **49(1)**, 51-62.

Upadhyay, S. K., & Shastri, V. (1997) : Bayes results for classical Pareto distribution via Gibbs sampler, with doubly-censored observations. *IEEE transactions on reliability*, **46(1)**, 56-59.

Upadhyay, S. K., Singh, U., & Shastri, V. (1996) : Estimation of exponential parameters under multiply type II censoring. *Communications in Statistics-Simulation and Computation*, **25(3)**, 801-815.