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Analysis of a Two Unit Series System with one Standby Having Geometric Distributions of Failure and Repair Times

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ABSTRACT

The paper deals with the cost-benefit analysis of a system composed of two non-identical units A and B arranged in series configuration with an identical cold standby unit corresponding to unit-A. System failure occurs when either unit-A (including its redundancy) or unit-B are in total failure mode. Failure and repair times of the units are independent random variables of discrete nature and follow geometric distributions with different parameters. The various measures of system effectiveness are obtained by using regenerative point technique[1].

1. Introduction

Due to ever increasing needs of modern society, the systems are becoming complex day by day. Many researchers including Gupta and Kumar (1981), Singh and Singh(1989), Yadavalli (2001), Gupta et at. (2009) analyzed the complex redundant system models in the field of reliability theory. Gupta and Kumar (1981) obtained the availability of a complex system with two types of failure and different repair preemptions. Singh and Singh (1989) performed the stochastic analysis of a complex system with two types of repair and patience time for repair. Yadavalli (2001) considered a complex system model consisting of generators, each with main and auxiliary switch boards. Gupta et. al (2009) treated a complex system with two physical conditions (poor and good) of repairman. The common assumption in analyzing all the above system models is

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that the random variables denoting failure, repair and other events are of continuous nature so that they follow any of the continuous distribution.

Few studies by the authors Gupta and Mogha (2000) Gupta and Varshney (2006), Gupta and Varshney (2007) are carried out in the literature of reliability analyzing the system models by taking geometric distributions of failure and repair times. The purpose of the present study is to analyze a complex system under discrete parametric Markov-Chain i.e. the random variables denoting failure and repair times of various units arranged in a specified configuration in the complex system follow geometric distributions with different parameters. The following economic related measures of system effectiveness are obtained by using regenerative point technique-

i) Transition probabilities and mean sojourn times in various states.
ii) Reliability and mean time to system failure.
iii) Point-wise and steady-state availabilities of the system as well as expected up time of the system during a finite interval of time.
iv) Point-wise and steady state probabilities of the busyness of repairman in unit A and B as well as expected busy period of the repairman in the repair of unit A and B during a finite interval of time.
v) Net expected profit earned by the system during a finite interval of time and in steady-state.

2. Justification for Consideration of Geometric Distributions

In real existing situations we observe so many times when a device/system fails at discrete epochs. Actually, discrete failure data arise in several common situations when a device is monitored only once per time period (i.e. an hour, a day) and the observation is the number of time periods successfully completed prior to failure of the device. Similarly, when a piece of equipment operates in cycles and experimenter observes the number of cycles successfully completed prior to failure. In view of these some examples of discrete life times are as follows-

i) In a photocopy machine the bulb inside the machine is lightened every time when a copy is taken and can fail only at the time of its functioning at discrete epochs.
ii) In an on/off switching device, the life time of the switch is a discrete random variable.

iii) An electric fan may break down completing a certain number of cycles's successfully, so that the life time of fan is a discrete random variable.

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v) An electric fan may break down completing a certain number of cycles's successfully, so that the life time of fan is a discrete random variable.

Thus, we see that discrete life time models represent may practical situations of importance. Similarly, the repair time may be considered as discrete random variable on dividing the whole time interval into various small parts of time. A more clear picture in this regard can be visualized as follows;

Let the continuous time period $(0,\infty)$ is divided as 0, 1, 2,…, n,… of equal distance on real line and the probability of failure of a unit during time (i, i+1); i = 0, 1, 2,... is p, then the probability that the unit will fail during (t, t+1) i.e. after passing successfully t intervals of time is given by $p(1-p)^t$; t = 0, 1, 2,…. This is the probability mass function (p.m.f) of geometric distribution. Similarly, if r denotes the probability that a failed unit is repaired during (i, i+1); i = 0, 1, 2,… then the probability that the unit will be repaired during (t, t+1) is given by $r(1-r)^t$; t = 0, 1, 2,… and it represents the p.m.f. of repair time of a failed unit.

3. Model Description and Assumptions

i) The system consists of two non-identical units A and B arranged in series configuration with an identical cold standby unit corresponding to unit-A.

ii) Initially, system starts functioning by the unit-A and unit-B arranged in series. The redundancy of unit-A is kept in cold standby. The replacement of failed unit-A by its redundancy is instantaneous.

iii) Each unit of the system has two modes- Normal (N) and Total failure (F).

iv) A single repairman is always available with the system to repair a failed unit.

v) Unit-B gets priority in repair over the repair of unit-A.

vi) Failure and repair times of units are independent random variables of discrete nature and follow geometric distributions with different parameters.

vii) The system failure occurs when either unit-A (including its redundancy) or unit-B are in total failure mode.

viii) Each repaired unit works as good as new.

4. Notations and States of the System

Notations:

ac^t **:** p.m.f. of failure time of unit-A $(a+c=1)$.

bd^t **:** p.m.f. of failure time of unit-B $(b+d=1)$.

pq^t **:** p.m.f. of repair time of unit-A $(p+q=1)$.

rs^t **:** p.m.f. of repair time of unit-B $(r+s=1)$.

$q_{ij}(t), Q_{ij}(t)$ **:** p.m.f. and c.d.f. of one step or direct transition time (T_{ij}) from state S_i to S_j.

p_{ij} **:** Steady state transition probability from state S_i to S_j.

$$p_{ij} = Q_{ij}(\infty)$$

$Z_i(t)$ **:** Probability that the system sojourns in state S_i at epochs 0, 1, 2,…,up to (t-1).

ψ_i **:** Mean sojourn time in state S_i.

$*, h$ **:** Symbol and dummy variable used in geometric transform e.g.

$$GT\left[q_{ij}(t)\right] = q^*_{ij}(h) = \sum_{t=0}^{\infty} h^t q_{ij}(t)$$

© **:** Symbol for ordinary convolution e.g.

$$A(t) © B(t) = \sum_{u=0}^{t} A(u) B(t-u)$$

Symbols for the states of the system:

$A_O / A_S / A_g$: Unit-A in normal (N) mode and operative/standby/good.

B_O / B_g : Unit-B in normal (N) mode and operative/good.

A_r / A_{wr} : Unit-A in failure (F) mode and under repair/waiting for repair

B_r / B_{wr} : Unit-B in failure (F) mode and under repair/waiting for repair

With the help of above symbols the possible states of the system along with failure and repair rates are shown in the transition diagram (Fig.1)-

TRANSITION DIAGRAM

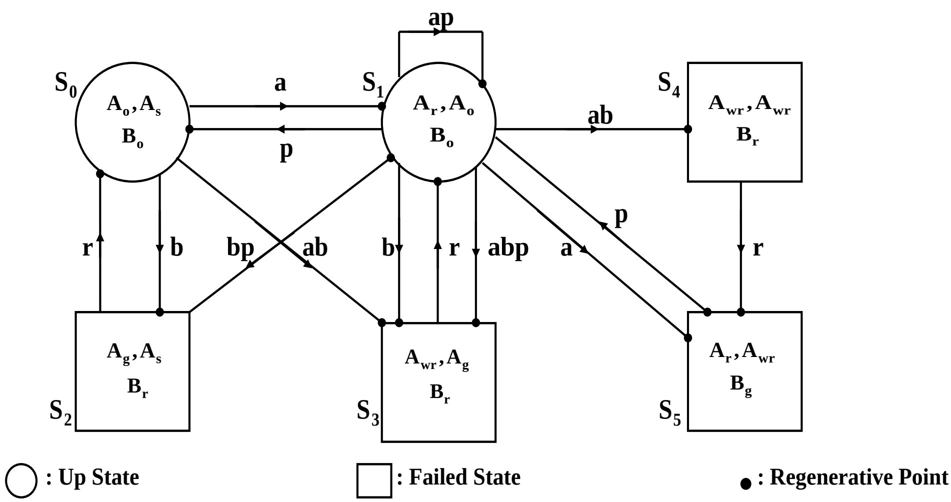

Fig.1

5. Transition Probabilities

Let $Q_{ij}(t)$ be the probability that the system transits from state S_i to S_j during time interval (0, t) i.e., if T_{ij} is the transition time from state S_i to S_j then

$$Q_{ij}(t) = P\left[T_{ij} \le t\right]$$

By using simple probabilistic arguments, we have

$$Q_{01}(t)=\frac{ad}{(1-cd)}\left\{1-(cd)^{t+1}\right\},\ Q_{02}(t)=\frac{bc}{(1-cd)}\left\{1-(cd)^{t+1}\right\}$$

$$Q_{03}(t)=\frac{ab}{(1-cd)}\left\{1-(cd)^{t+1}\right\},\ Q_{10}(t)=\frac{cdp}{(1-cdq)}\left\{1-(cdq)^{t+1}\right\}$$

$$Q_{11}(t)=\frac{adp}{(1-cdq)}\left\{1-(cdq)^{t+1}\right\},\ Q_{12}(t)=\frac{bcp}{(1-cdq)}\left\{1-(cdq)^{t+1}\right\}$$

$$Q_{13}(t)=\frac{b(cq+ap)}{(1-cdq)}\left\{1-(cdq)^{t+1}\right\},\ Q_{14}(t)=\frac{abq}{(1-cdq)}\left\{1-(cdq)^{t+1}\right\}$$

$$Q_{15}(t)=\frac{adq}{(1-cdq)}\left\{1-(cdq)^{t+1}\right\}\ Q_{20}(t)=Q_{31}(t)=Q_{45}(t)=\left(1-s^{t+1}\right)$$

$$Q_{51}(t)=\left(1-q^{t+1}\right) \qquad (5.1-5.11)$$

As an illustration $Q_{01}(t)$ is obtained as the probability that the operating unit-A fails at the time u with rate a and unit-B doesn't fail with rate b up to time u. This probability is given by $ac^{u}d^{u+1}$. As u takes values 0, 1, 2,…, t. Therefore

$$Q_{01}(t)=\sum_{u=0}^{t}ac^{u}d^{u+1}=ad\sum_{u=0}^{t}(cd)^{u}=\frac{ad}{(1-cd)}\left\{1-(cd)^{t+1}\right\}$$

The steady state transition probabilities from state S_i to S_j can be obtained from $(5.1-5.11)$ by taking $t\to\infty$, as follows:

$$p_{01}=\frac{ad}{(1-cd)},\qquad p_{02}=\frac{bc}{(1-cd)},\qquad p_{03}=\frac{ab}{(1-cd)},$$

$$p_{10}=\frac{cdp}{(1-cdq)},\qquad p_{11}=\frac{adp}{(1-cdq)},\qquad p_{12}=\frac{bcp}{(1-cdq)}$$

$$p_{13}=\frac{b(cq+ap)}{(1-cdq)},\qquad p_{14}=\frac{abq}{(1-cdq)},\qquad p_{15}=\frac{adq}{(1-cdq)}$$

$$p_{20}=p_{31}=p_{45}=p_{51}=1$$

We observe that the following relations hold-

$$p_{01}+p_{02}+p_{03}=1,\qquad p_{10}+p_{11}+p_{12}+p_{13}+p_{14}+p_{15}=1$$

$$p_{20}=p_{31}=p_{45}=p_{51}=1$$

$$(5.12-5.14)$$

6. Mean Sojourn Times

Let ψ_i be the sojourn time in state S_i (i =0, 1, 2, 3, 4, 5) then mean sojourn time in state S_i is given by

$$\psi_i = \sum_{t=1}^{\infty} P\left[T \geq t\right]$$

In particular,

$$\psi_0 = \frac{cd}{(1-cd)}, \qquad \psi_1 = \frac{cdq}{(1-cdq)}, \qquad \psi_2 = \psi_3 = \psi_4 = \frac{s}{r}$$

$$\psi_5 = \frac{q}{p} \qquad (6.1-6.4)$$

As an illustration ψ_0 is obtained as the probability that neither the operating unit-A and nor the operating unit–B at epoch u and is given by $c^{u+1}d^{u+1}$. As u varies $0,1,2,\ldots,\infty$. Therefore,

$$\psi_0 = \sum_{u=0}^{\infty} c^{u+1}d^{u+1} = \frac{cd}{(1-cd)}$$

7. Methodology For Developing Equations

In order to obtain various interesting measures of system effectiveness we develop the recurrence relations for reliability, availability and busy period of repairman as follows-

a) Reliability of the system-

Here we define $R_i(t)$ as the probability that the system does not fail up to t epochs 0, 1, 2,…,(t-1) when it is initially started from up state S_i. To determine it, we regard the failed states S_2, S_3, S_4 and S_5 as absorbing states. Now, the expressions for $R_i(t)$; i =0, 1 we have the following set of convolution equations.

$$R_0(t) = c^t d^t + \sum_{u=0}^{t-1} q_{01}(u) R_1(t-1-u)$$

$$= Z_0(t) + q_{01}(t-1) © R_1(t-1)$$

Similarly,

$$R_1(t) = Z_1(t) + q_{10}(t-1) © R_0(t-1) + q_{11}(t-1) © R_1(t-1) \quad (7.1-7.2)$$

where,

$$Z_1(t) = c^t d^t q^t$$

b) Availability of the system-

Let $A_i(t)$ be the probability that the system is up at epoch (t-1), when it initially starts from state S_i. By using simple probabilistic arguments, as in case of reliability the following recurrence relations can be easily developed for $A_i(t)$; i = 0 to 5.

$$A_0(t) = Z_0(t) + q_{01}(t-1) © A_1(t-1) + q_{02}(t-1) © A_2(t-1)$$
$$+ q_{03}(t-1) © A_3(t-1)$$
$$A_1(t) = Z_1(t) + q_{10}(t-1) © A_0(t-1) + q_{11}(t-1) © A_1(t-1)$$
$$+ q_{12}(t-1) © A_2(t-1) + q_{13}(t-1) © A_3(t-1)$$
$$+ q_{14}(t-1) © A_4(t-1) + q_{15}(t-1) © A_5(t-1)$$
$$A_2(t) = q_{20}(t-1) © A_0(t-1)$$
$$A_3(t) = q_{31}(t-1) © A_1(t-1)$$
$$A_4(t) = q_{45}(t-1) © A_5(t-1)$$
$$A_5(t) = q_{51}(t-1) © A_1(t-1) \quad (7.3-7.8)$$

where,

The values of $Z_i(t)$; i = 0, 1 are same as given in section 7(a).

c) Busy period of repairman-

Let $B_i^a(t)$ and $B_i^b(t)$ be the respective probabilities that the repairman is busy at epoch (t-1) in the repair of unit A and B, when system initially starts from S_i. Using simple probabilistic arguments as in case of reliability, the recurrence relations for $B_i^j(t)$; i =0 to 5 can be easily developed as below. The dichotomous variable δ takes values 1 and 0 respectively for j = a and b.

$$B_0^j(t) = q_{01}(t-1) © B_1^j(t-1) + q_{02}(t-1) © B_2^j(t-1)$$

$$+q_{03}(t-1)©B_3^j(t-1)$$

$$B_1^j(t)=\delta Z_1(t)+q_{10}(t-1)©B_0^j(t-1)+q_{11}(t-1)©B_1^j(t-1)$$
$$+q_{12}(t-1)©B_2^j(t-1)+q_{13}(t-1)©B_3^j(t-1)$$
$$+q_{14}(t-1)©B_4^j(t-1)+q_{15}(t-1)©B_5^j(t-1)$$

$$B_2^j(t)=(1-\delta)Z_2(t)+q_{20}(t-1)©B_0^j(t-1)$$

$$B_3^j(t)=(1-\delta)Z_3(t)+q_{31}(t-1)©B_1^j(t-1)$$

$$B_4^j(t)=(1-\delta)Z_4(t)+q_{45}(t-1)©B_5^j(t-1)$$

$$B_5^j(t)=\delta Z_5(t)+q_{51}(t-1)©B_1^j(t-1) \qquad (7.9-7.14)$$

where,

$Z_1(t)$ has the same values as in section 7(a) and

$Z_2(t)=Z_3(t)=Z_4(t)=s^t$, $\qquad Z_5(t)=q^t$.

8. Analysis of Characteristics

a) Reliability and MTSF-

Taking geometric transforms of $(7.1-7.2)$ and simplifying the resulting set of algebraic equations for $R_0^*(h)$, we get

$$R_0^*(h)=\frac{N_1(h)}{D_1(h)} \qquad (8.1)$$

where,

$$N_1(h)=\left(1-hq_{11}^*\right)Z_0^*+hq_{01}^*Z_1^*$$

$$D_1(h)=1-hq_{11}^*-h^2q_{01}^*q_{10}^*$$

Collecting the coefficient of h^t from expression (8.1), we can get the reliability of the system $R_0(t)$. The MTSF is given by-

$$E(T)=\lim_{h\to 1}R_0^*(h)=\frac{N_1(1)}{D_1(1)}-1 \qquad (8.2)$$

$$N_1(1)=(1-p_{11})\psi_0+p_{01}\psi_1$$

$$D_1(1)=1-p_{11}-p_{10}p_{10}$$

b) Availability Analysis-

On taking geometric transform of $(7.3-7.8)$ and simplifying the resulting equations, we get

$$A_0^*(h)=\frac{N_2(h)}{D_2(h)} \qquad (8.3)$$

where,

$$N_2(h)=\left[1-hq_{11}^*-h^2q_{13}^*q_{31}^*-hq_{51}^*\left(hq_{15}^*+h^2q_{14}^*q_{45}^*\right)\right]Z_0^*$$
$$+\left(hq_{01}^*+h^2q_{03}^*q_{31}^*\right)Z_1^*$$

$$D_2(h)=\left[1-hq_{11}^*-h^2q_{13}^*q_{31}^*-hq_{51}^*\left(hq_{15}^*+h^2q_{14}^*q_{45}^*\right)\right]-hq_{10}^*\left(hq_{01}^*\right.$$
$$\left.+h^2q_{03}^*q_{31}^*\right)-hq_{20}^*\left[hq_{12}^*\left(hq_{01}^*+h^2q_{03}^*q_{31}^*\right)\right.$$
$$\left.+hq_{02}^*\left\{1-hq_{11}^*-h^2q_{13}^*q_{31}^*-hq_{51}^*\left(hq_{15}^*+h^2q_{14}^*q_{45}^*\right)\right\}\right]$$

The steady state availability of the system is given by-

$$A_0=\lim_{t\to\infty}A_0(t)=\lim_{h\to 1}(1-h)\frac{N_2(h)}{D_2(h)}$$

As $D_2(h)$ at h=1 is zero, therefore by applying L. Hospital rule, we get

$$A_0=-\frac{N_2(1)}{D_2'(1)} \qquad (8.4)$$

where,

$$N_2(1)=(p_{10}+p_{12})\psi_0+(1-p_{02})\psi_1$$

and

$$D_2'(1)=(p_{10}+p_{12})\psi_0+(1-p_{02})\psi_1+\left[(p_{12}+p_{10}p_{02})+(1-p_{02})(p_{13}+p_{14})\right.$$
$$\left.+p_{03}(p_{10}+p_{12})\right]\psi_2+(p_{14}+p_{15})(1-p_{02})\psi_5$$

Now the expected uptime of the system upto epoch (t-1) is given by

$$\mu_{up}(t)=\sum_{x=0}^{t-1}A_0(x)$$

so that

$$\mu_{up}^{*}(h) = A_0^{*}(h)/(1-h) \quad (8.5)$$

c) Busy Period Analysis-

On taking geometric transforms of $(7.9-7.14)$ and simplifying the resulting equations for j = a and b, we get

$$B_0^{a*}(h) = \frac{N_3(h)}{D_2(h)} \quad \text{and} \quad B_0^{b*}(h) = \frac{N_4(h)}{D_2(h)} \quad (8.6-8.7)$$

where,

$$N_3(h) = \left(hq_{01}^{*} + h^2 q_{03}^{*} q_{31}^{*}\right) Z_1^{*} + \left[\left(hq_{01}^{*} + h^2 q_{03}^{*} q_{31}^{*}\right)\left(hq_{15}^{*} + h^2 q_{14}^{*} q_{45}^{*}\right)\right] Z_5^{*}$$

$$N_4(h) = \left[1 - hq_{11}^{*} - hq_{51}^{*}\left(hq_{15}^{*} + h^2 q_{14}^{*} q_{45}^{*}\right)\right]\left(hq_{02}^{*} Z_2^{*} + hq_{03}^{*} Z_3^{*}\right)$$
$$+ h^2 q_{01}^{*} q_{13}^{*} Z_3^{*} + \left[hq_{12}^{*}\left(hq_{01}^{*} + h^2 q_{03}^{*} q_{31}^{*}\right) - h^3 q_{02}^{*} q_{13}^{*} q_{31}^{*}\right] Z_2^{*}$$
$$+ hq_{14}^{*} Z_4^{*}\left(hq_{01}^{*} + h^2 q_{03}^{*} q_{31}^{*}\right)$$

and $D_2(h)$ is same as in availability analysis.

In the long run, the respective probabilities that the repairman is busy in the repair of units are given by-

$$B_0^{a} = \lim_{t\to\infty} B_0^{a}(t) = \lim_{h\to 1}(1-h)\frac{N_3(h)}{D_2(h)}$$

$$B_0^{b} = \lim_{t\to\infty} B_0^{b}(t) = \lim_{h\to 1}(1-h)\frac{N_4(h)}{D_2(h)}$$

But $D_2(h)$ at h=1 is zero, therefore by applying L. Hospital rule, we get

$$B_0^{a} = -\frac{N_3(1)}{D_2'(1)} \quad \text{and} \quad B_0^{b} = -\frac{N_4(1)}{D_2'(1)}$$

$(8.8-8.9)$

where,

$$N_3(1) = (1-p_{02})\left[\psi_1 + (p_{14} + p_{15})\psi_5\right]$$

$$N_4(1) = \left[(p_{12} + p_{10}p_{02}) + (1-p_{02})(p_{13} + p_{14}) + p_{03}(p_{10} + p_{12})\right]\psi_2$$

and $D_2'(1)$ is same as in availability analysis.

The expected busy periods of the repairman in the repair of units up to epoch (t-1) are respectively given by-

$$\mu_b^a(t)=\sum_{x=0}^{t-1} B_0^a(x), \qquad \text{and} \qquad \mu_b^b(t)=\sum_{x=0}^{t-1} B_0^b(x)$$

So that,

$$\mu_b^{a*}(h)=\frac{B_0^{a*}(h)}{(1-h)}, \qquad \text{and} \qquad \mu_b^{b*}(h)=\frac{B_0^{b*}(h)}{(1-h)}$$

$(8.10-8.11)$

9. Profit Function Analysis

We are now in the position to obtain the net expected profit incurred up to epoch (t-1) by considering the characteristics obtained in earlier sections.

Let us consider,

K_0 =revenue per-unit time by the system when it is operative.

K_1 =cost per-unit time when repairman is busy in the repair of the unit-A.

K_2 = cost per-unit time when repairman is busy in the repair of the unit-B.

Then, the net expected profit incurred up to epoch (t-1) is given by

$$P(t)=K_0\mu_{up}(t)-K_1\mu_b^a(t)-K_2\mu_b^b(t)$$

The expected profit per unit time in steady state is as follows-

$$P=\lim_{t\to\infty}\frac{P(t)}{t}$$

$$=K_0\lim_{h\to1}(1-h)^2\frac{A_0^*(h)}{(1-h)}-K_1\lim_{h\to1}(1-h)^2\frac{B_0^{a*}(h)}{(1-h)}-K_2\lim_{h\to1}(1-h)^2\frac{B_0^{b*}(h)}{(1-h)}$$

$$=K_0A_0-K_1B_0^a-K_2B_0^b$$

10. Conclusion

The curves for MTSF and profit function have been drawn for different values of parameters a, b, p. Fig. 2 depicts the variations in MTSF with respect to failure rate (a) of unit-A for different values of repair rate (p=0.05, 0.07, 0.09) of unit-A and the failure rate (b=0.005, 0.010) of unit-B. From the curves we observe that MTSF decreases uniformly as the values of a increase. It also reveals that the MTSF increases with the increase in p and decreases with the increase in b.

Similarly, Fig. 3 reveals the variations in profit (P) with respect to p for varying values of p and b, when the values of other parameters are kept fixed as r=0.05, K_0=130, K_1=120 and K_2=100. From the curves we observe that profit decreases uniformly as the values of a increase. It also reveals that the profit increases with the increase in p and decreases with the increase in b. From this figure it is clear from the dotted curves that the system is profitable only if failure rate (a) is greater than 0.049, 0.069and 0.089 respectively for p=0.05, 0.07, and 0.09 for fixed value of b=0.005. From smooth curves, we conclude that the system is profitable only if a is greater than 0.046, 0.064 and 0.082 respectively for p=0.05, 0.07, and 0.09 for fixed value of b=0.010.

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Behavior of MTSF with respect to p, r and c

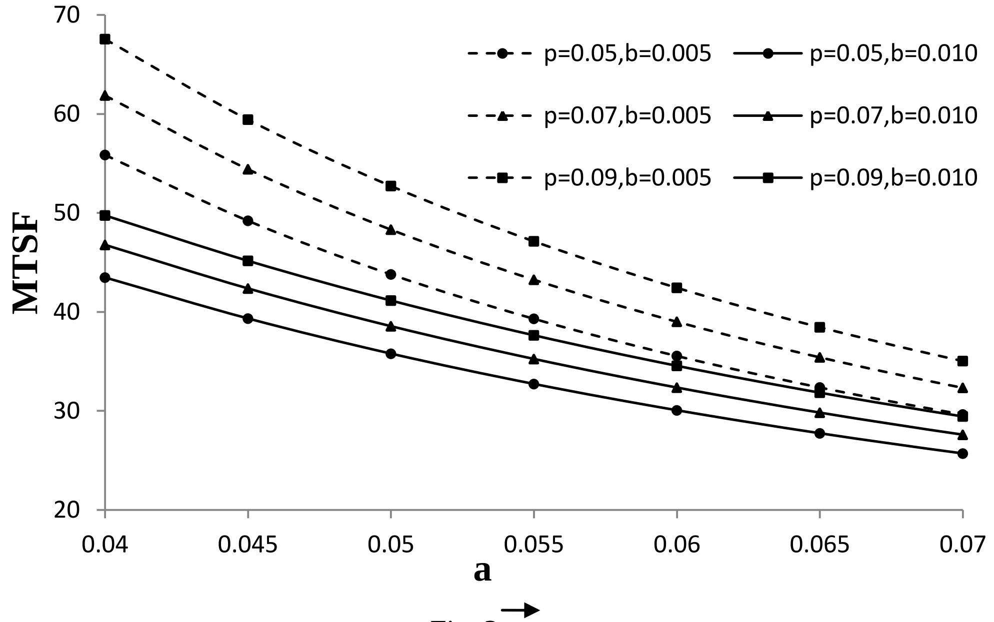

Fig. 2

Behavior of Profit (P) with respect to p, r and c

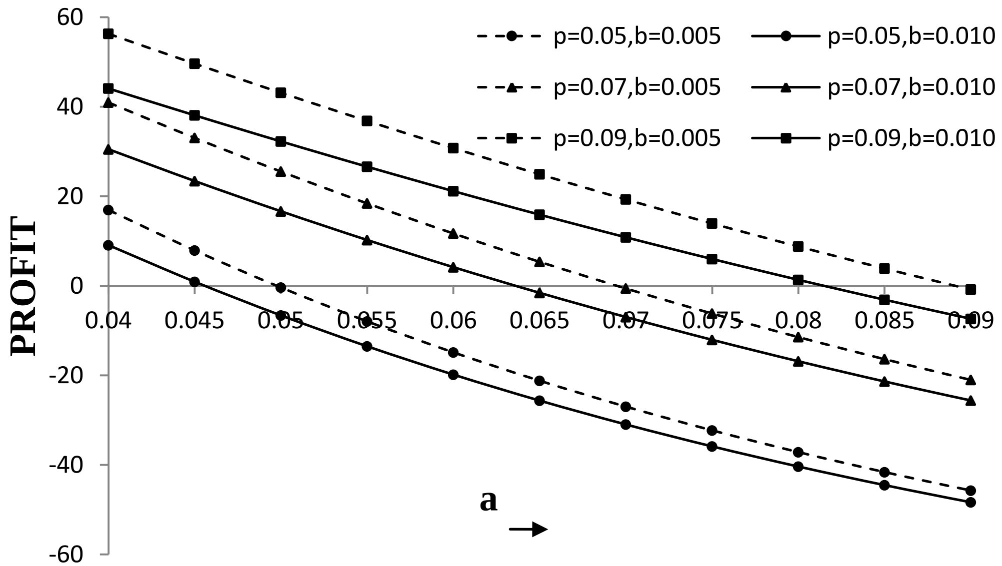

Fig. 3