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Construction of Neighbor Designs in Binary Blocks of Some Large Sizes

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ABSTRACT

Neighbor designs are useful to neutralize the neighbor effects. These designs are available in literature for several configurations especially for circular blocks of sizes 3, 4, ... , 10. In this paper, first order neighbor balanced designs are constructed for circular binary blocks of size 11, 12, 13 and 14.

1. Introduction

A design (v, k, λ') in which each pair of distinct adjacent treatments appears λ' times as neighbors is called nearest-neighbor balanced design (NNBD), where v is number of treatments, k is block size and λ' is number of times each pair of distinct treatments appears as neighbors. These designs are useful for the cases where the performance of a treatment is affected by the treatments applied to its neighboring plots. Neighbor designs were initially used by Rees (1967) in serology. He presented a technique used in virus research and constructed neighbor designs for odd v with $\lambda' = 1$. Lawless (1971), Hwang (1973), Misra *et al*. (1991), Azais *et al*. (1993), Ahmed and Akhtar (2008, 2009, 2012, 2015), Akhtar and Ahmed (2009), Shehzad *et al*. (2011) constructed neighbor designs for several cases. For some more references see, Ahmed *et al*. (2011). Ahmed *et al*. (2009), Ahmed *et al*. (2010), Akhtar *et al*. (2010), Ahmed and Akhtar (2011), Ahmed *et al*. (2011), Yasmin *et al*. (2013) and Ahmed and Akhtar (2013)

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presented the construction of individual block size 9, 8, 5, 6, 10, 4 and 7, respectively. Following are some important definitions.

Neighbor Effect: If the response on a given plot is affected by the treatments on neighboring plots as well as by the treatment applied to that plot then there is neighbor effect. According to Azais *et al*. (1993) experiments in agriculture, horticulture, and forestry often show neighbor effects.

Neighbor Balanced Designs: A design in which each pair of non-identical treatments appears same number of times, say λ' in adjacent plots of the same block is called neighbor balanced design of first order or nearest-neighbor balanced design.

Minimal NNBD: If $\lambda' = 1$, then nearest-neighbor balanced design is called minimal and is considered to be economical.

Binary Blocks: Blocks in which each treatment appears at most once are called binary blocks.

Binary Design: A design in which all blocks are binary is called a binary design.

Circular Block: A block formed in a cycle and the treatments allocated to its first and last plots are considered as neighbors is called circular block.

Circular Design: A design with all its blocks circular is called a circular design.

Bailey and Druilhet (2004) showed that a circular neighbor balanced design (where no treatment is nearest neighbor to itself) is universally optimal for total (direct and neighbor) effects under the linear models containing the nearest neighbor effects. Filipiak and Rozanski (2005) showed that circular neighbor balanced designs are universally optimal. In this article, NNBD/ first order neighbor balanced designs are constructed for k = 11, 12, 13 and 14 which will be a positive addition to the present literature. In Section 3, the proposed designs are obtained by developing initial blocks cyclically. Initial blocks are developed through sets of shifts. In Section 2, Method of Cyclic Shifts is described briefly.

2. Method of Cyclic Shifts

Method of cyclic shifts is explained here briefly. For detail, see Ahmed and Akhtar (2009).

Rule I: Let $S = [q_1, q_2, \ldots, q_{k-1}]$ be a set of shifts, where $1 \le q_i \le v-1$. If each element 1, 2, …, v-1 appears an equal number of times, say λ' in a new set of shifts S*, where $S^* = [q_1, q_2, \ldots, q_{k-1}, (q_1+q_2+\ldots+q_{k-1}) \bmod (v), v-(q_1), v-(q_2), \ldots, v-(q_{k-1}), v-(q_1+q_2+\ldots+q_{k-1}) \bmod (v)]$ then initial block $(0, q_1, q_1+q_2, \ldots, (q_1+q_2+\ldots+q_{k-1}) \bmod v)$ provides circular NNBD.

Example 2.1: NNBD is generated for v = 13 and k = 6 by developing the initial block (0, 1, 3, 6, 2, 7) cyclically mod 13. Initial block is obtained from set of shift [1, 2, 3, 9, 5].

B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}	B_{11}	B_{12}	B_{13}
0	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12	0
3	4	5	6	7	8	9	10	11	12	0	1	2
6	7	8	9	10	11	12	0	1	2	3	4	5
2	3	4	5	6	7	8	9	10	11	12	0	1
7	8	9	10	11	12	0	1	2	3	4	5	6

Rule II: If any set contains k-2 elements with‘t’ then apply rule II. Let S = [q_1, q_2, …, q_{k-2}]t be a set of shifts, where $1 \leq q_i \leq v$-2. If each element 1, 2, …, v-2 appears an equal number of times, say λ' in a new set of shifts S*, where S* = [q_1, q_2, …, q_{k-1}, v-1-(q_1), v-1-(q_2), …, v-1-(q_{k-2})] then initial block (0, q_1, q_1+q_2, …, (q_1+q_2+…+q_{k-2}) mod (v-1)) provides circular NNBD.

Example 2.2: NNBD is generated for v = 12 and k = 11 by developing the initial block (0, 2, 5, 9, 3, 8, 1, 4, 6, 7, ∞) cyclically mod 11 along with augmented block (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10), where ∞ = 11. Initial block and augmented block are obtained through the sets of shifts:

[1, 1, 1, 1, 1, 1, 1, 1, 1, 1](1/11) +[2, 3, 4, 5, 5, 4, 3, 2, 1]t

B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}	B_{11}
0	1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	0	1
5	6	7	8	9	10	0	1	2	3	4
9	10	0	1	2	3	4	5	6	7	8
3	4	5	6	7	8	9	10	0	1	2
8	9	10	0	1	2	3	4	5	6	7
1	2	3	4	5	6	7	8	9	10	0
4	5	6	7	8	9	10	0	1	2	3
6	7	8	9	10	0	1	2	3	4	5
7	8	9	10	0	1	2	3	4	5	6
∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞

In the process of developing initial block, '∞' remains unchanged. '∞' is replaced by v -1 in the final design and augmented block (**B$_{12}$**) is added. The required design in 12 blocks is as follows:

B$_1$	**B$_2$**	**B$_3$**	**B$_4$**	**B$_5$**	**B$_6$**	**B$_7$**	**B$_8$**	**B$_9$**	**B$_{10}$**	**B$_{11}$**	**B$_{12}$**
0	1	2	3	4	5	6	7	8	9	10	0
2	3	4	5	6	7	8	9	10	0	1	1
5	6	7	8	9	10	0	1	2	3	4	2
9	10	0	1	2	3	4	5	6	7	8	3
3	4	5	6	7	8	9	10	0	1	2	4
8	9	10	0	1	2	3	4	5	6	7	5
1	2	3	4	5	6	7	8	9	10	0	6
4	5	6	7	8	9	10	0	1	2	3	7
6	7	8	9	10	0	1	2	3	4	5	8
7	8	9	10	0	1	2	3	4	5	6	9
11	11	11	11	11	11	11	11	11	11	11	10

3. NNBD for k = 11, 12, 13 and 14

In this section, NNBD are constructed in circular binary blocks of size 11, 12, 13 and 14.

3.1 NNBD for $v = 2ik+1$; i integer

Minimal NNBD can be generated for $v = 2ik+1$; i integer and $k = 11$ in iv blocks by developing i initial blocks cyclically mod v.

Example 3.1. NNBD is generated for $v = 45$ and $k = 11$ by developing the following two initial blocks cyclically mod 45.

I_1 = (0, 1, 3, 6, 10, 15, 21, 28, 36, 12, 34),
I_2 = (0, 9, 19, 31, 44, 30, 1, 16, 33, 6, 25)

3.2 NNBD for $v = ik+1$; i odd

NNBD with $\lambda' = 2$ can be generated for $v = ik+1$; i odd and $k = 11$ by developing either i initial blocks cyclically mod v (or i initial blocks (one of these block contains ∞) cyclically mod (v-1) along with (v-1)/11 augmented blocks).

Example 3.2. NNBD is generated for $v = 12$ and $k = 11$ by developing the following initial block cyclically mod 11 along with one augmented block.

I_1 = (0, 2, 5, 9, 3, 8, 1, 4, 6, 7, ∞), where ∞ = 11 and augmented block is (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10)

3.3 NNBD for $v = ik$; i even

NNBD with $\lambda' = 2$ can be generated for $v = ik$; i even and $k = 11$ in $i(v-1)$ blocks by developing i initial blocks (one of these blocks contains ∞) cyclically mod (v-1).

Example 3.3. NNBD is generated for $v = 44$ and $k = 11$ by developing the following four initial blocks cyclically mod 43.

I_1 = (0, 1, 3, 6, 10, 15, 21, 28, 36, 2, 13), I_2 = (0, 1, 3, 6, 10, 15, 21, 28, 36, 2, 13), I_3 = (0, 10, 20, 32, 1, 15, 30, 3, 21, 38, 14), I_4 = (0, 15, 31, 5, 23, 42, 19, 39, 17, 38, ∞)

3.4 NNBD for $v = ik$; i odd

Minimal NNBD can be generated for $v = ik$; i odd and $k = 11$ by developing (i-1)/2 initial blocks cyclically mod v along with (v-3)/2 augmented blocks.

Example 3.4. NNBD is generated for $v = 33$ and $k = 11$ by developing the following initial blocks cyclically mod 33 along with 15 augmented blocks.

(0, 1, 3, 7, 12, 19, 27, 4, 15, 2, 16)

With augmented blocks

(0, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30), (1, 2, 7, 10, 11, 16, 19, 22, 25, 28, 31),
(2, 3, 8, 11, 12, 17, 20, 21, 26, 29, 32), (0, 6, 12, 18, 24, 30, 3, 9, 15, 21, 27),
(1, 7, 13, 19, 25, 31, 4, 10, 16, 22, 28), (2, 8, 14, 20, 26, 32, 5, 11, 17, 23, 29),
(0, 9, 18, 27, 3, 12, 21, 30, 6, 15, 24), (1, 10, 19, 28, 4, 13, 22, 31, 7, 16, 25),
(2, 11, 20, 29, 5, 14, 23, 32, 8, 17, 26), (0, 12, 24, 3, 15, 27, 6, 18, 30, 9, 21),
(1, 13, 25, 4, 16, 28, 7, 19, 31, 10, 22), (2, 14, 26, 5, 17, 29, 8, 20, 32, 11, 23),
(0, 15, 30, 12, 27, 9, 24, 6, 21, 3, 18), (1, 16, 31, 13, 28, 10, 25, 7, 22, 4, 19),
(2, 16, 31, 13, 28, 10, 25, 7, 22, 4, 19)

3.5 NNBD for $v = 2ik + 1$; i integer

Theorem 3.5.1. Minimal NNBD can be generated for $v = 2ik+1$; i integer and k =12 by developing the following i initial blocks mod v.

I_j = (0, v -(12 j -11), 1, v -(12 j -10),2, v -(12 j -9), 3, 12 j -2, 24 j -5, 12 j -3, 24 j -4, 12 j -4); j = 1, 2, …, i.

Proof. Combined set of forward and backward differences between neighboring elements takes all the values from 1 to v-1 once. It is, therefore, NNBD with $\lambda' = 1$.

Example 3.5. NNBD is generated for $v = 49$ and $k = 12$ by developing the following two initial blocks cyclically mod 49.

$I_1 =$ (0, 48, 1, 47, 2, 46, 3, 10, 19, 9, 20, 8),
$I_2 =$ (0, 36, 1, 35, 2, 34, 3, 22, 43, 21, 44, 20)

3.6 NNBD for $v = ik + 1$; $i = 2s+1$; s integer

Theorem 3.6.1. NNBD can be generated for $v = ik + 1$; $i = 2s+1$; s integer and $k = 12$ with $\lambda' = 2$ by developing the following i initial blocks mod v.

$I_j =$ (0, v -(12 j -11), 1, v -(12 j -10),2, v -(12 j -9), 3, 12 j -2, 24 j -5, 12 j -3, 24 j -4, 12 j -4); $I_{j+z} =$ (0, v -(12 j -11),1, v -(12 j -10),2, v -(12 j -9),3, 12 j -2, 24 j -5, 12 j -3, 24 j -4, 12 j -4); $j = 1, 2, \ldots, s$.

$I_i =$ (0,(v +11)/2,1, (v +9)/2, 2, (v +7)/2, 3, (v -5)/2, (v -6), (v -3)/2, (v -3), (v -1)/2)

Proof. Combined set of forward and backward differences between neighboring elements takes all the values from 1 to v -1 twice. So it is NNBD with $\lambda' = 2$.

Example 3.6. NNBD is generated for $v = 37$ and $k = 12$ by developing the following three initial blocks cyclically mod 37.

$I_1 =$ (0, 36, 1, 35, 2, 34, 3, 10, 19, 9, 20, 8),
$I_2 =$ (0, 36, 1, 35, 2, 34, 3, 10, 19, 9, 20, 8),
$I_3 =$ (0, 24, 1, 23, 2, 22, 3, 16, 31, 17, 34, 18)

3.7 NNBD for $v = ik$; i (>1) integer

NNBD can be generated with $\lambda' = 2$ for $v = ik$; i(>1) integer and $k = 12$ by developing i initial blocks (one of these blocks contains ∞) cyclically mod (v -1).

Example 3.7. NNBD is generated for $v = 36$ and $k = 12$ by developing the following three initial blocks cyclically mod 35.

$I_1 = I_2 =$ (0, 3, 4, 6, 10, 15, 21, 28, 1, 27, 2, 13),
$I_3 =$ (0, 12, 27, 6, 22, 4, 16, 30, 10, 26, 8, ∞)

3.8 NNBD when HCF of v -1 and k is 4; (v -1)/4 even

NNBD can be generated with $\lambda' = 3$, $k = 12$; (v -1)/4 is even when HCF of v -1 and k is 4 by developing (v -1)/8 initial blocks cyclically mod v.

Example 3.8. NNBD is generated for $v = 17$ and $k = 12$ by developing the following two initial blocks cyclically mod 17.

$I_1 =$ (0, 7, 9, 10, 15, 4, 5, 13, 16, 2, 6, 8),
$I_2 =$ (0, 1, 3, 6, 10, 15, 4, 11, 2, 8, 12, 7)

3.9 NNBD for $v = 2ik + 1$; i integer

Minimal NNBD can be generated for $v = 2ik+1$; i integer and $k = 13$ in iv blocks by developing i initial blocks cyclically mod v.

Example 3.9. NNBD is generated for $v = 53$ and $k = 13$ by developing the following two initial blocks cyclically mod 53.

I_1 = (0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 2, 13, 25),
I_2 = (0, 13, 28, 42, 5, 22, 4, 23, 43, 11, 33, 3, 27)

3.10 NNBD for *v* = *ik* +1; *i* odd

NNBD with $\lambda' = 2$ can be generated for $v = ik + 1$, i odd and $k = 13$ in iv blocks by developing either i initial blocks cyclically mod v (or i initial blocks (one of these blocks contains ∞) cyclically mod (v -1) along with (v -1)/13 augmented blocks).

Example 3.10(a). NNBD is generated for $v = 40$ and $k = 13$ by developing the following three initial blocks cyclically mod 40.

$I_1 = I_2$ = (0, 1, 3, 6, 10, 15, 21, 28, 36, 5, 23, 33, 12),
I_3 = (0, 11, 22, 35, 8, 23, 37, 12, 26, 10, 27, 4, 20)

Example 3.10(b). NNBD is generated for $v = 66$ and $k = 13$ by developing the following five initial blocks cyclically mod 65 along with five augmented blocks.

$I_1 = I_2$ = (0, 1, 3, 6, 10, 16, 23, 31, 40, 50, 61, 8, 21),
$I_3 = I_4$ = (0, 51, 1, 17, 34, 52, 7, 26, 48, 6, 30, 55, 29),
I_5 = (0, 27, 55, 20, 51, 18, 45, 8, 38, 4, 36, 41, ∞), where ∞ = 65

Augmented blocks

(0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60),
(1, 6, 11, 16, 21, 26, 31, 36, 41, 46, 51, 56, 61),
(2, 7, 12, 17, 22, 27, 32, 37, 42, 47, 52, 57, 62),
(3, 8, 13, 18, 23, 28, 33, 38, 43, 48, 53, 58, 63),
(4, 9, 14, 19, 24, 29, 34, 39, 44, 49, 54, 59, 64)

3.11 NNBD for *v* = *ik*; *i* even

NNBD with $\lambda' = 2$ can be generated for $v = ik$; i even and $k = 13$ in $i(v\text{-}1)$ blocks by developing i initial blocks (one of these blocks contains ∞) cyclically mod (v - 1).

Example 3.11. NNBD is generated for $v = 52$ and $k = 13$ by developing the following four initial blocks cyclically mod 51.

$I_1 = I_2$ = (0, 1, 3, 6, 10, 16, 21, 28, 36, 45, 4, 15, 27),
I_3 = (0, 13, 27, 43, 7, 24, 42, 10, 30, 1, 22, 45, 19),
I_4 = (0, 13, 27, 43, 7, 24, 42, 11, 33, 3, 26, 1, ∞), where ∞ = 51

3.12 NNBD for *v* = *ik*; *i* odd

Minimal NNBD can be generated for $v = ik$; i odd and $k = 13$ by developing (i-1)/2 initial blocks cyclically mod v along with (v -3)/2 augmented blocks.

Example 3.12. NNBD is generated for $v = 39$ and $k = 13$ by developing the following initial block cyclically mod 39 along with 18 augmented blocks.

(0, 1, 5, 7, 2, 9, 17, 27, 38, 12, 26, 3, 20), Augmented blocks

(0, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36),
(1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37),
(2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38),
(0, 6, 12, 18, 24, 30, 36, 3, 9, 15, 21, 27, 33),
(1, 7, 13, 19, 25, 31, 37, 4, 10, 16, 22, 28, 34),
(2, 8, 14, 20, 26, 32, 38, 5, 11, 17, 23, 29, 35),
(0, 9, 18, 27, 36, 6, 15, 24, 33, 3, 12, 21, 30),
(1, 10, 19, 28, 37, 7, 16, 25, 34, 4, 13, 22, 31),
(2, 11, 20, 29, 38, 8, 17, 26, 35, 5, 14, 23, 32),
(0, 12, 24, 36, 9, 21, 33, 6, 18, 30, 3, 15, 27),
(1, 13, 25, 37, 10, 22, 34, 7, 19, 31, 4, 16, 28),
(2, 14, 26, 38, 11, 23, 35, 8, 20, 32, 5, 17, 29)
(0, 15, 30, 6, 21, 36, 12, 27, 3, 18, 33, 9, 24),
(1, 16, 31, 7, 22, 37, 13, 28, 4, 19, 34, 10, 25),
(2, 17, 32, 8, 23, 38, 14, 29, 5, 20, 35, 11, 26),
(0, 18, 36, 15, 33, 12, 30, 9, 27, 6, 24, 3, 21),
(1, 19, 37, 16, 34, 13, 31, 10, 28, 7, 25, 4, 22),
(2, 20, 38, 17, 35, 14, 32, 11, 29, 8, 26, 5, 23)

3.13 NNBD for $v = ik$; i(>1) integer

NNBD with $\lambda' = 2$ can be generated for $v = ik$; i(>1) integer and $k = 14$ by developing i initial blocks (one of these blocks contains ∞) cyclically mod (v -1).

Example 3.13. NNBD is generated for $v = 42$ and $k = 14$ by developing the following three initial blocks cyclically mod (v-1).

$I_1 = I_2 = (0, 1, 3, 6, 10, 15, 21, 28, 36, 4, 14, 25, 37, 13)$,
$I_3 = (0, 14, 29, 4, 23, 2, 16, 34, 8, 24, 1, 20, 40, \infty)$

3.14 NNBD for $v = 2ik + 1$; i integer

Minimal NNBD can be generated for $v = 2ik + 1$; i integer and $k = 14$ by developing i initial blocks cyclically mod v.

Example 3.14. NNBD is generated for $v = 57$ and $k = 14$ by developing the following two initial blocks cyclically mod 57.

$I_1 = (0, 1, 3, 6, 10, 15, 22, 28, 36, 45, 55, 9, 21, 34)$
$I_2 = (0, 43, 1, 17, 34, 16, 35, 55, 19, 41, 8, 33, 2, 29)$

3.15 NNBD for $v = ik+1$; i (>1) odd

NNBD with $\lambda' = 2$ can be generated for $v = ik + 1$; i(>1) odd and $k = 14$ by developing i initial blocks cyclically mod v.

Example 3.15. NNBD is generated for v = 43 and k = 14 by developing the following three initial blocks cyclically mod 43.

$$I_1 = I_2 = (0, 1, 3, 6, 10, 15, 21, 28, 36, 2, 12, 23, 35, 13)$$
$$I_3 = (0, 19, 33, 5, 21, 40, 14, 37, 8, 23, 39, 13, 38, 20)$$

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