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Estimation Of Parameters Of Pearson Type I Family Of Distributions Using Ranked Set Sampling

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ABSTRACT

In this work the ranked set sampling technique has been applied to estimate the best linear unbiased estimator (BLUE) of the location parameter μ and scale parameter σ of Pearson type I family of distributions, when the shape parameters p and q are known. The values of coefficients of ranked set sample in the BLUEs of μ and σ for $p > q$ have been explicitly derived in terms of coefficients of ranked set sample involved in the BLUE of μ and σ for $p < q$. The efficiency comparison of the BLUE of μ and σ is also done using ranked set sample with that of using order statistics.

1. INTRODUCTION

The concept of ranked set sampling (RSS) was first introduce by McIntyre (1952) as a process of improving the precision of the sample mean as an estimator of the population mean. This technique of data collection was introduced for situations where taking the actual measurements on sample observations is difficult as compared to the judgment ranking of them (see, Chen *et al.* (2004).The ranked set sampling technique can be executed as follows:

• Step 1: Randomly draw n simple random samples each of size n from the population of interest.

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• Step 2: Within each of the *n* sets, the sampled items are ranked from lowest to largest according to the variable of interest based on the researcher's judgment or by any negligible cost method that does not require actual quantifications.
• Step 3: From the first set of *n* units, the unit ranked lowest is measured. From the second set of *n* units, the unit ranked second lowest is measured. The process is continued until the n^{th} ranked unit is measured from the n^{th} set. Note that n^2 units are sampled but only n of them are measured with respect to the variable of interest. The above 3 steps describe one cycle of the RSS technique. Then $X_{(1)1}, X_{(2)2}, ..., X_{(n)n}$ form a ranked set sample of size n (see Lam *et al*. (1994), page 724).

The appealing feature of RSS is that, since RSS provides more structured samples than SRS, improved estimation may result from the use of RSS design.

Takahasi and Wakimoto (1968) established a very important statistical foundation for the theory of RSS. They showed that the mean of the RSS is unbiased estimator for the population mean and has higher efficiency than the mean of SRS. Dell and Clutter (1972) studied the effect of ranking error on the procedure. Stokes (1977) studied RSS with concomitant variables. Stokes and Sager (1988) discussed the application of this method to estimate volumes of trees in a forest.RSS was initially introduced for the estimation of mean (see Patil *et al.,*1994) without making assumptions on the underlying model. However, a large number of articles research on parametric methods for RSS focused mainly on BLUE estimators of location and scale parameters (see Bhoj and Ahsanullah, (1996), Fei *et al.*(1994), Lam *et al.,* (1994), and Lam *et al.* (1996).

The RSS has many statistical applications in biological and environmental studies and reliability theory (e.g. Dell and Clutter (1972), Stokes (1977), Stokes (1980), Mode *et al.* (1999); and Barabesi and El-Sharaawi, 2001).

2. ESTIMATION OF LOCATION AND SCALE PARAMETERS OF A DISTRIBUTION USING RANKED SET SAMPLING

In this section we consider the family of all absolutely continuous distributions with location parameter μ and scale parameter σ. Then any distribution belonging to it has a pdf of the form

$$f(x\,;\mu,\sigma)=\frac{1}{\sigma}g\left(\frac{x-\mu}{\sigma}\right),\text{ where } x\in R,\ \mu\in R,\ \sigma>0\,. \tag{2.1}$$

Let $X_{(1)1}, X_{(2)2}, \ldots, X_{(n)n}$ be the ranked set sample drawn from (2.1).Clearly $X_{(1)1}, X_{(2)2}, \ldots, X_{(n)n}$ are independent. Let $\underline{X} = \left(X_{(1)1}, X_{(2)2}, \ldots, X_{(n)n}\right)'$, and then define $\underline{Y} = \left(Y_{(1)1}, Y_{(2)2}, \ldots, Y_{(n)n}\right)'$ as a vector of corresponding RSS arising from $f(x\,;0,1)$. Let $\alpha = \left(\alpha_{1:n}, \alpha_{2:n}, \ldots, \alpha_{n:n}\right)'$ and $V = \left(\left(V_{i,j:n} \right)\right)$ be the vector of means and dispersion matrix of $\underline{Y}$. Clearly, $V = dig\left(V_{1:n}, V_{2:n}, \ldots, V_{n:n}\right)$, where $V_{i:n} = V_{i,i:n}$.Then the best linear unbiased estimator of μ based on ranked set sampling is given by Lam *et al.,* (1994, P.726),

$$\hat{\mu} = \frac{\alpha' \, V^{-1}\left(1\,\alpha' - \alpha \;\, 1'\right) V^{-1}}{\Delta}$$

$$= \frac{\sum_{i=1}^{n} \frac{X_{(i)i}}{V_{i:n}} \sum_{i=1}^{n} \frac{\alpha_{i:n}^{2}}{V_{i:n}} - \sum_{i=1}^{n} \frac{\alpha_{i:n}}{V_{i:n}} \sum_{i=1}^{n} \frac{\alpha_{i:n}}{V_{i:n}} X_{(i)i}}{\sum_{i=1}^{n} \frac{1}{V_{i:n}} \sum_{i=1}^{n} \frac{\alpha_{i:n}^{2}}{V_{i:n}} - \left(\sum_{i=1}^{n} \frac{\alpha_{i:n}}{V_{i:n}}\right)^{2}} \tag{2.2}$$

and its variance is

$$V(\hat{\mu}) = \frac{\left(\alpha' \, V^{-1} \, \alpha\right) \sigma^{2}}{\Delta}$$

$$= \frac{\sum_{i=1}^{n} \frac{\alpha_{i:n}^{2}}{V_{i:n}}}{\sum_{i=1}^{n} \frac{1}{V_{i:n}} \sum_{i=1}^{n} \frac{\alpha_{i:1}^{2}}{V_{i:n}} - \left(\sum_{i=1}^{n} \frac{\alpha_{i:n}}{V_{i:n}}\right)^{2}} \sigma^{2}. \tag{2.3}$$

Also the BLUE of σ using RSS is

$$\hat{\sigma} = \frac{1' V^{-1}\left(1\,\alpha' - \alpha\, 1'\right) V^{-1}}{\Delta} \underline{X}$$

$$= \frac{\sum_{i=1}^{n} \frac{X_{(i)i}}{V_{i:n}} \alpha_{i:n} \sum_{i=1}^{n} \frac{1}{V_{i:n}} - \sum_{i=1}^{n} \frac{\alpha_{i:n}}{V_{i:n}} \sum_{i=1}^{n} \frac{X_{(i)i}}{V_{i:n}}}{\sum_{i=1}^{n} \frac{1}{V_{i:n}} \sum_{i=1}^{n} \frac{\alpha_{i:n}^{2}}{V_{i:n}} - \left(\sum_{i=1}^{n} \frac{\alpha_{i:n}}{V_{i:n}}\right)^{2}} \tag{2.4}$$

and its variance

$$V(\hat{\sigma}) = \frac{\left(1'V^{-1}1\right)\sigma^2}{\Delta}$$

$$= \frac{\sum_{i=1}^{n} \frac{1}{V_{i:n}}}{\sum_{i=1}^{n} \frac{1}{V_{i:n}} \sum_{i=1}^{n} \frac{\alpha_{i:n}^2}{V_{i:n}} - \left(\sum_{i=1}^{n} \frac{\alpha_{i:n}}{V_{i:n}}\right)^2} \sigma^2 \tag{2.5}$$

and

$$\operatorname{cov}(\hat{\mu}, \hat{\sigma}) = -\frac{\left(\alpha' V^{-1} 1\right)}{\Delta}\sigma^2$$

$$= -\frac{\sum_{i=1}^{n} \frac{\alpha_{i:n}}{V_{i:n}}}{\sum_{i=1}^{n} \frac{1}{V_{i:n}} \sum_{i=1}^{n} \frac{\alpha_{i:n}^2}{V_{i:n}} - \left(\sum_{i=1}^{n} \frac{\alpha_{i:n}}{V_{i:n}}\right)^2} \sigma^2,$$

$$where \quad \Delta = \left(\alpha' V^{-1}\alpha\right)\left(1'V^{-1}1\right) - \left(\alpha' V^{-1} 1\right)^2. \tag{2.6}$$

3. ESTIMATION OF LOCATION AND SCALE PARAMETERS OF PEARSON TYPE I FAMILY OF DISTRIBUTIONS USING RANKED SET SAMPLING.

The family of distributions with probability density function (pdf) of the form

$$f(x, p, q, \mu, \sigma) = \frac{1}{\sigma\beta(p,q)}\left(\frac{x-\mu}{\sigma}\right)^{p-1}\left[1-\left(\frac{x-\mu}{\sigma}\right)^{q-1}\right], \mu < x < \mu+\sigma$$

$$= 0, \quad otherwise, \tag{3.1}$$

where $\mu \in R, \sigma > 0, p > 0, q > 0$ and $\beta(p,q)$ is the usual complete beta function is called Pearson Type I family of distributions (see Johnson *et al.,* 1995, p.210). A distribution defined by the pdf given in (3.1) is also called as generalized beta distribution. For convenience, we may write GBD (p, q, μ, σ) to denote the distribution defined in (3.1). If we put $q = 1$ in (3.1), the obtained distribution is called Power function distribution.

If X has a distribution defined by (3.1), then $Y = \left(\frac{X-\mu}{\sigma}\right)$ follows the well-known standard beta distribution with pdf given by

$$g(y; p, q) = \frac{1}{\beta(p,q)} y^{p-1}(1-y)^{q-1}, 0 < y < 1$$
$$= 0, \quad otherwise. \tag{3.2}$$

For convenience, we may write BD (p, q) to denote the distribution defined in (3.2). Extensive applications of beta distribution are seen in the available literature. For example see, Harrop - Williams (1989), Kopper and Grover (1992). Though the problem of estimating the parameters in (3.1) is discussed extensively in Johnson *et al.,* (1995), one may not get explicit solution for maximum likelihood estimators from likelihood equation (3.1).Moreover the procedure involved in obtaining estimators by the method of moments is cumbersome (see, Elderton and Johnson (1969). In most of the real life situations only small samples could be reliable from a population and in such situations one cannot say much on reliability of estimators obtained by the method of moments or maximum likelihood procedure. These estimators are not even unbiased. In the available literature not any good finite sample estimators are seen, derived and their properties analyzed for the parameters involved in (3.1).However, Sajeevkumar *et al.,* (2007) derived the BLUE of μ and σ involved in (3.1) using order statistics. Hence, there is necessity to derive reasonably good finite sample estimators of the parameters involved in (3.1) using ranked set sampling.

In this work our aim is to derive the BLUEs of μ and σ involved in (3.1) using ranked set sampling technique. Let $X_{(1)1}, X_{(2)2}, ..., X_{(n)n}$ are the ranked set sample arising from (3.1) and let $Y_{(1)1}, Y_{(2)2}, ..., Y_{(n)n}$ are the ranked set sample from (3.2). Define $\underline{Y} = (Y_{(1)1}, Y_{(2)2}, ..., Y_{(n)n})'$ as a vector of corresponding observations in an RSS arising from (3.2). Let $\alpha = (\alpha_{1:n}, \alpha_{2:n}, ..., \alpha_{n:n})'$ and $V = ((V_{i,j:n}))$ be the vector of means and dispersion matrix of Y. Clearly $V = dig(V_{1:n}, V_{2:n}, ..., V_{n:n})$, where $V_{i:n} = V_{i,i:n}$. Then the BLUEs of μ and σ based on ranked set sampling are given in (2.2) to (2.5). We have evaluated the BLUE of μ and σ of GBD (p, q, μ, σ)

using RSS for $n = 5(5)10$, $p = 2(0.5)4$, $q = 2(0.5)4$ with $p \le q$ are given in the Table 1 and Table 2.

It may be noted that the required estimators $\hat{\mu}$ and $\hat{\sigma}$ for $\boldsymbol{p > q}$ can be obtained from Table 1 and Table 2 itself. The way in which the coefficients of the order statistics in $\hat{\mu}$ and $\hat{\sigma}$ for $p > q$ can be determined from those in Table 1 and Table 2 becomes clear from the results that we prove in the next Section. All the computational works involved in this paper were done using 'Mathcad '.

4. RELATIONSHIP BETWEEN THE COEFFICIENTS OF RSS IN THE BLUES $\hat{\mu}$ AND $\hat{\sigma}$ OF GBD (P, Q, μ , σ) FOR $p > q$ WITH THOSE OF $p < q$

Let $X_{(1)1}, X_{(2)2},...,X_{(n)n}$ be the ranked set sampling arising from (3.1) and let $Y_{(1)1}, Y_{(2)2},...,Y_{(n)n}$ be the ranked set sampling arising from (3.2). Let $\underline{X} = \left(X_{(1)1}, X_{(2)2},...,X_{(n)n}\right)'$, and then define $\underline{Y} = \left(Y_{(1)1}, Y_{(2)2},...,Y_{(n)n}\right)'$ as a vector of corresponding observations from RSS arising from (3.2). Let $\alpha = \left(\alpha_{1:n}, \alpha_{2:n},...,\alpha_{n:n}\right)'$ and $V = \left(\left(V_{i,j:n}\right)\right)$ be the vector of means and dispersion matrix of Y. Clearly, $V = dig\left(V_{1:n}, V_{2:n},...,V_{n:n}\right)$, where $V_{i:n} = V_{i,i::n}$. For convenience we write $\hat{\mu}_{(p,q)}$ and $\hat{\sigma}_{(p,q)}$ to denote the BLUE of μ and σ involved in (3.1).Then we have

$$\hat{\mu}_{(p,q)} = -\frac{\alpha' V^{-1}(1\alpha' - \alpha 1')V^{-1}}{\Delta}\underline{X}, \tag{4.1}$$

$$\hat{\sigma}_{(p,q)} = -\frac{1' V^{-1}(1\alpha' - \alpha 1')V^{-1}}{\Delta}\underline{X}, \tag{4.2}$$

$$Var\left(\hat{\mu}_{(p,q)}\right) = \frac{\left(\alpha' V^{-1}\alpha\right)\sigma^2}{\Delta}, \tag{4.3}$$

$$Var\left(\hat{\sigma}_{(p,q)}\right) = \frac{\left(1' V^{-1} 1\right)\sigma^2}{\Delta} \tag{4.4}$$

and

$$\operatorname{cov}\left(\hat{\mu}_{(p,q)}, \hat{\sigma}_{(p,q)}\right) = -\frac{\left(\alpha' V^{-1} 1\right)\sigma^2}{\Delta} \tag{4.5}$$

where $\Delta = \left(\alpha' V^{-1}\alpha\right)\left(1' V^{-1} 1\right) - \left(\alpha' V^{-1} 1\right) - \left(\alpha' V^{-1} 1\right)^2$ and 1 is a column vector of n ones.

Let $Z_{(1)1}, Z_{(2)2}, \ldots, Z_{(n)n}$ be the RSS random sample of size n arising from $f(x;q,p,\mu,\sigma)$,where $f(x;q,p,\mu,\sigma)$ is obtained by inter changing p and q in (3.1) .Clearly $\left(\left(\frac{Z_{(1)1}-\mu}{\sigma}\right),\left(\frac{Z_{(2)2}-\mu}{\sigma}\right),\left(\frac{Z_{(3)3}-\mu}{\sigma}\right),\ldots,\left(\frac{Z_{(n)n}-\mu}{\sigma}\right)\right)$ distributed identically as $\left(\left(1-Y_{(n)n}\right),\left(1-Y_{(n-1)n}\right),\cdots,\left(1-Y_{(1)n}\right)\right)$.

Consequently we have

$$E\left(Z_{(r)n}\right) = \sigma\left(1-\alpha_{(n-r+1)n}\right) + \mu$$

and

$$Var\left(Z_{(r)n}\right) = Var\left(Y_{(n-r+1)n}\right).$$

Let $Z = \left(Z_{(1)1}, Z_{(2)2}, \cdots, Z_{(n)n}\right)$, then

$E(Z) = \sigma(1-J\alpha) + 1\mu$, where J is the $n \times n$ matrix given by

$$J = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 1 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}.$$

Clearly we have $J = J^{-1} = J'$ and $J1 = 1$ .If we write $D(X) = \sigma^2 V$, then $D(Z) = JVJ\sigma^2$. If we write $\hat{\mu}_{(q,p)}$ and $\hat{\sigma}_{(q,p)}$ to denote the BLUEs of μ and σ involved in $f(x;q,p,\mu,\sigma)$, then

$$\hat{\mu}_{(q,p)} = -\frac{(1-J\alpha)' JV^{-1}J\left(1(1-J\alpha)' - (1-J\alpha)1'\right)JV^{-1}J}{\Delta'} \underline{\mathrm{X}}. \tag{4.6}$$

$$\begin{aligned}
\Delta' &= (1-J\alpha)'\left(JV^{-1}J\right)(1-J\alpha)\left(1'JV^{-1}J1\right) - \left((1-J\alpha)'\left(JV^{-1}J\right)1\right)^2 \\
&= \left(1'V^{-1}J - \alpha'V^{-1}J\right)\left(1'V^{-1}1 - J\alpha 1'V^{-1}1\right) - \left(\left(1'V^{-1}1 - \alpha'V^{-1}1\right)^2\right) \\
&= \left(1'V^{-1}J1'V^{-1}1 - \alpha^1 V^{-1}J1V^{-1}1\right) - \left(1'V^{-1}\alpha\right)\left(1'V^{-1}1\right) + \\
&\quad \left(\alpha'V^{-1}\alpha\right)\left(1'V^{-1}1\right) - \left(1'V^{-1}1\right)^2 - \left(\alpha'V^{-1}1\right)^2 + 2\left(1'V^{-1}1\right)\left(\alpha'V^{-1}1\right) \\
&= \left(\alpha'V^{-1}\alpha\right)\left(1'V^{-1}1\right)\left(\alpha V^{-1}1\right)^2 .
\end{aligned} \tag{4.7}$$

Thus using (4.6) and (4.7) we have $\Delta' = \Delta$.
Further simplifying the numerator of (4.6) using (4.7) the BLUE $\hat{\mu}_{(q,p)}$ reduces to

$$\hat{\mu}_{(q,p)} = \frac{1'V^{-1}(1\alpha' - \alpha 1')V^{-1}}{\Delta}(JZ) - \frac{\alpha' V^{-1}(1\alpha^1 - \alpha 1')V^{-1}}{\Delta}(JZ). \tag{4.8}$$

The BLUE of $\sigma^*_{(q,p)}$ of σ given by

$$\hat{\sigma}_{(q,p)} = \frac{1'(JVJ^{-1})[1-(1-J\alpha)^1 - (1-J\alpha)1'](JVJ)^{-1}}{\Delta}Z. \tag{4.9}$$

Then on simplifying (4.9) we get

$$\hat{\sigma}_{(q,p)} = -\frac{1'V^{-1}(1\alpha^1 - \alpha 1')V^{-1}}{\Delta}(JZ), \tag{4.10}$$

$$\begin{aligned} Var(\hat{\mu}_{(q,p)}) &= \frac{(1-J\alpha)'(JV^{-1}J)(1-J\alpha)}{\Delta}\sigma^2 \\ &= \frac{1V^{-1}1 - 2\alpha' V^{-1}1 + \alpha' V^{-1}\alpha}{\Delta}\sigma^2, \end{aligned} \tag{4.11}$$

$$\begin{aligned} Var(\hat{\sigma}_{(q,p)}) &= \frac{1'JV^{-1}J1}{\Delta}\sigma^2 \\ &= \frac{1'V^{-1}1}{\Delta}\sigma^2 \end{aligned} \tag{4.12}$$

and

$$\operatorname{cov}(\hat{\mu}_{(q,p)}, \hat{\sigma}_{(q,p)}) = \frac{(\alpha' V^{-1}1 - 1'V^{-1}1)}{\Delta}\sigma^2, \tag{4.13}$$

where $\Delta = (\alpha' V^{-1}\alpha)(1'V^{-1}1) - (\alpha' V^{-1}1)^2$ and 1 being a column vector of n ones. Since in (4.8), $J(Z) = (Z_{(n)n}, Z_{(n-1)n-1}, ..., Z_{(1)1})$, we note that the coefficients of $Z_{(r)n}$ in $\hat{\mu}_{(q,p)}$ is equal to the sum of the coefficients of $X_{(n-r+1)n}$ in $\hat{\mu}_{(p,q)}$ and $\hat{\sigma}_{(p,q)}$ and the coefficients of $Z_{(r)n}$ in $\hat{\sigma}_{(p,q)}$ is equal to negative value of the coefficient of $X_{(n-r+1)n}$ in $\hat{\sigma}_{(p,q)}$. Then we have proved the following theorem.

Theorem 4.1

Let $X_{(1)1}, X_{(2)2}, ..., X_{(n)n}$ be the ranked set sampling arising from $f(x; p, q, \mu, \sigma)$. Let $Z_{(1)1}, Z_{(2)2}, ..., Z_{(n)n}$ be the RSS random sample of size *n* arising from $f(x; q, p, \mu, \sigma)$ then the coefficient of $Z_{(r)n}$ in the BLUE of $\hat{\mu}_{(q,p)}$ of μ in $f(x; q, p, \mu, \sigma)$ is equal to the sum of the coefficients of $X_{(n-r+1)n-r+1}$ in $\hat{\mu}_{(q,p)}$ and $\hat{\sigma}_{(q,p)}$ of the parameters of μ and σ involved in $f(x; p, q, \mu, \sigma)$ and the coefficients of $Z_{(r)n}$ in the BLUE of $\hat{\sigma}_{(q,p)}$ of σ in $f(x; p, q, \mu, \sigma)$ is equal to the $-ve$ values of the coefficient of $X_{(n-r+1)n-r+1}$ in $\hat{\sigma}_{(p,q)}$ of σ involved in $f(x; p, q, \mu, \sigma)$. Further

$Var(\hat{\mu}_{(q,p)}) = Var(\hat{\mu}_{(p,q)}) + 2Cov(\hat{\mu}_{(p,q)}, \hat{\sigma}_{(p,q)}) + Var(\hat{\sigma}_{(p,q)})$,

$Var(\hat{\sigma}_{(p,q)}) = Var(\hat{\sigma}_{(q,p)})$, $Cov(\hat{\mu}_{(q,p)}, \hat{\sigma}_{(q,p)}) = -Cov(\hat{\mu}_{(p,q)}, \hat{\sigma}_{(p,q)}) - Var(\hat{\sigma}_{(p,q)})$.

5. EFFICIENCY COMPARISON OF THE BLUE OF μ AND σ USING RANKED SET SAMPLE WITH THAT OF ORDER STATISTICS.

To compare our estimator defined in Section 3 using RSS we take the BLUE of μ and σ using order statistics derived by Sajeevkumar *et al.,* (2007). Let $X_{1:n}, X_{2:n}, ..., X_{n:n}$ be the order statistics of size n taken from (3.1) and let $Y_{1:n}, Y_{2:n}, ..., Y_{n:n}$ be the order statistics of size n taken from (3.2).Also let $\underline{X} = (X_{1:n}, X_{2:n}, ..., X_{n:n})'$ and $\underline{Y} = (Y_{1:n}, Y_{2:n}, ..., Y_{n:n})'$.

Also let $\xi = (\xi_{1:n}, \xi_{2:n}, \ldots, \xi_{n:n})'$ and $B = ((b_{r,s:n}))$ be the vector of means and dispersion matrix of Y. Then the BLUE and its variance of μ and σ are given by see Sajeevkumar *et al.,* (2007) are as follows

$$\mu^*_{(p,q)} = \frac{\xi' B^{-1}(1\xi' - \xi 1')B^{-1}}{\Delta} X, \tag{5.1}$$

$$\sigma^*_{(p,q)} = \frac{1' B^{-1}(1\xi' - \xi 1')B^{-1}}{\Delta} X, \tag{5.2}$$

$$Var(\mu^*_{(p,q)}) = \frac{(\xi' B^{-1} \xi)\sigma^2}{\Delta}, \tag{5.3}$$

$$Var\left(\sigma^{*}_{(p,q)}\right)=\frac{\left(1'B^{-1}1\right)\sigma^{2}}{\Delta}, \tag{5.4}$$

and

$$Cov\left(\mu^{*}_{(p,q)},\sigma^{*}_{(p,q)}\right)=-\frac{\left(\xi'B^{-1}1\right)\sigma^{2}}{\Delta}, \tag{5.5}$$

where $\Delta=\left(\xi'B^{-1}\xi\right)\left(1'B^{-1}1\right)-\left(\xi'B^{-1}1\right)^{2}$ and 1 is a column vector of n ones.

Efficiency of our estimator using RSS related to the BLUE of μ and σ using order statistics is also made and is given in Table 1 and Table 2. We have evaluated Var($\hat{\mu}$), Var($\hat{\sigma}$) and tabulated the values of $\frac{Var(\mu^{*})}{\sigma^{2}}, \frac{Var(\sigma^{*})}{\sigma^{2}}, \frac{Var(\hat{\mu})}{\sigma^{2}}, \frac{Var(\hat{\sigma})}{\sigma^{2}}$, and the efficiency $e_{1}\left(\hat{\mu}/\mu^{*}\right)$ of $\hat{\mu}$ relative to μ^{*} and the efficiency $e_{1}\left(\hat{\sigma}/\sigma^{*}\right)$ of $\hat{\sigma}$ relative to σ^{*} for $n=5\,(\,5\,)10$, $p=2(0.5)4$, $q=2(0.5)4$ with $p \le q$ are given in the Table 1 and Table 2.

CONCLUSION

In this work we found that BLUE of μ and σ using ranked set sampling is much better than the BLUE of μ and σ using order statistics suggested by Sajeevkumar *et al*., (2007).

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REFERENCES

Barabesi, L. and El –Sharaawi A. (2001). The efficiency of ranked set sampling for parameter estimation. Statistics and Probability Letters, **53**,189-199.

Bhoj, D.S., Ahsanullah, M. (1996).Estimation of parameters of the generalized geometric distribution using ranked set sampling, Biometrics,**52**,685-694.

Chen, Z; Bai, A. and Sinha, B.K. (2004).Ranked set sampling: Theory and applications; New York. Springer.

Dell, T, R & Clutter, J.L. (1972). Ranked set sampling Theory with order statistics Background. Biometrics, **28**,545-555.

Elderton, W.P and Johnson, N.L (1969).Systems of frequency curves. Cambridge University Press, Cambridge.

Fei, H., Sinha, B.K and Wu, Z. (1994). Estimation of Parameters in two-parameter Weibull and extreme- value distributions using ranked set sampling, Journal of Statistical Research **28**,149-161.

Harrop-Williams, K. (1989).Random nature of soil porosity and related properties. J. Engg. Mech. **115**, 1123-1129.

Johnson, N.L., Kotz, S. and Balakrishnan, N. (1995).Continuous univariate Distributions Second edition, John Wiley and sons, New York.

Koppes, L.J and Grover, N.B. (1992).Relationship between size of parent at cell division and relative size of progeny in Escherchia coli. Archives microbiology, **157**, 402-405.

Lam, K., Sinha, B.K. and Wu, Z. (1994).Estimation of a two parameter exponential distribution using ranked set sampling, Annals of Institute of Statistical mathematics, **46**, 723-736.

Lam, K., Sinha, B.K. and Wu, Z. (1996). Estimation of location, scale and quantiles of a logistic distribution using a ranked set sample, Statistical Theory and applications: Papers in Honor of Herbert, A. David,(Eds. Nagaraja, H.N., Sen, P.K., Morrison, D.F), Springer, New York,187-197.

McIntyre, G.A. (1952).A method for unbiased selective sampling using ranked sets, Australian Journal of Agricultural Research, **3**, 385-390.

Mode, N., Conquest, L. and Marker, D. (1999). Ranked set sampling for ecological research: Accounting for the total cost of sampling. Environmetrics **10**,179-194.

Patil, G.P., Sinha, B.K., Taillie, C. (1994). Ranked set sampling, In: Patil, G.P., Rao, C. R. ed., Handbook of statistics, Vol.12, Amsterdam: North- Holland, 167-200.

Sajeevkumar, N.K. and Thomas, P.Y. and Philip Samuel (2007).Estimation of location and scale parameters of Pearson type I family of distributions, Journal of the Indian society of agricultural statistics, 61(I),2007: 42-50.

Stokes, S.L. (1980).Estimation of variance using judgment ordered ranked set samples. Biometrics **36**, 35-42.

Stokes and Sager, T. (1988). Characterisation of ranked set sample with application to estimating distribution functions. Journal of the American Statistical Association, **83**, 374-381.

Stokes, S.L. (1977). Ranked set sampling with concomitant variables. Communications in statistics- Theory and Methods, **6**, 1207-1211.

Takahasi, K and Wakimoto, K. (1968).On unbiased estimates of the population mean based on the sample stratified by means of ordering. Annals of the Institute of Statistical Mathematics **20**, 1-31.

TABLE 1 (n=5)											
P	**q**	Estimator	$\mathbf{X_{1:5}}$	$\mathbf{X_{2:5}}$	$\mathbf{X_{3:5}}$	$\mathbf{X_{4:5}}$	$\mathbf{X_{5:5}}$	$Var(\hat{\mu})/Var(\hat{\sigma})$	$Var(\mu^*)/Var(\sigma^*)$	$\mathbf{e_1/e_2}$	$Cov(\hat{\mu},\hat{\sigma})$
2	2	$\hat{\mu}$	1.0188	0.5245	0.1865	-0.1413	-0.5885	0.0275	0.0320	1.16450	-0.0482
		$\hat{\sigma}$	-1.6070	-0.6660	0	0.6660	1.6070	0.0920	0.0930	1.01090	
2	2.5	$\hat{\mu}$	1.0160	0.4867	0.1455	0.1542	-0.4940	0.0208	0.0230	1.10740	-0.0421
		$\hat{\sigma}$	-1.8150	-0.6580	0.1060	0.7880	1.5790	0.0959	0.0970	1.0115	
2	3	$\hat{\mu}$	1.0155	0.4610	0.1183	0.1607	-0.4341	0.0163	0.0180	1.10430	-0.0376
		$\hat{\sigma}$	-2.0300	-0.6600	0.1990	0.9010	1.5900	0.1013	0.1030	1.0168	
2	3.5	$\hat{\mu}$	1.0163	0.4420	0.9905	-0.1645	-0.3929	0.01315	0.0140	1.06464	-0.0340
		$\hat{\sigma}$	-2.2510	-0.6670	0.2850	0.0100	1.6220	0.1064	0.1080	1.01532	
2	4	$\hat{\mu}$	1.0170	0.4279	0.0849	-0.1667	-0.3630	0.0109	0.0120	1.10701	-0.0311
		$\hat{\sigma}$	-2.4730	-0.6780	0.3670	1.1170	1.6670	0.1092	0.1130	1.03499	
2.5	2.5	$\hat{\mu}$	1.0776	0.5620	0.1555	-0.0406	-0.7545	0.0290	0.0320	1.10380	-0.0535
		$\hat{\sigma}$	-1.7800	-0.7360	0	0.7360	1.7800	0.0890	0.0990	1.11235	
2.5	3	$\hat{\mu}$	1.0772	0.5285	0.1179	-0.0575	-0.6661	0.0234	0.0250	1.06701	0.04861
		$\hat{\sigma}$	-1.9720	-0.7310	0.2010	0.5300	1.9730	0.1010	0.1020	1.0099	
2.5	3.5	$\hat{\mu}$	1.0700	0.5258	0.1352	-0.2030	-0.5280	0.0187	0.0210	1.12299	-0.0418
		$\hat{\sigma}$	-2.0980	-0.7790	0.1660	0.9750	1.7360	0.1036	0.1050	1.01351	

2.5	4	$\hat{\mu}$	1.0693	0.5067	0.1159	-0.2060	-0.4859	0.0157	0.0170	1.08626	-0.0384
		$\hat{\sigma}$	-2.2810	-0.7900	0.2390	1.0700	1.7620	0.1088	0.1090	1.00230	
3	3	$\hat{\mu}$	1.1325	0.6370	0.2036	-0.2335	-0.7397	0.0289	0.0330	1.14187	-0.0529
		$\hat{\sigma}$	-1.8720	-0.8710	0	0.8710	1.8720	0.1018	0.1030	1.01178	
3	3.5	$\hat{\mu}$	1.1273	0.6064	0.1717	-0.2398	-0.6656	0.0242	0.0280	1.15894	-0.0486
		$\hat{\sigma}$	-2.0240	-0.8740	0.0750	0.9620	-1.8610	0.1046	0.1060	1.01338	
3	4	$\hat{\mu}$	1.1240	0.5827	0.1481	-0.2434	-0.6114	0.0205	0.0220	1.07317	-0.0449
		$\hat{\sigma}$	-2.1790	-0.8830	0.1430	1.0480	1.8710	0.1063	0.1080	1.01599	
3.5	3.5	$\hat{\mu}$	1.1868	0.6847	0.2088	-0.2751	-0.8051	0.0294	0.0330	1.12360	-0.0544
		$\hat{\sigma}$	-1.9920	-0.9600	0	0.9600	1.9920	0.1059	0.1070	1.01039	
3.5	4	$\hat{\mu}$	1.1810	0.6560	0.1803	-0.2791	-0.7382	0.0252	0.0280	1.11067	-0.0506
		$\hat{\sigma}$	-2.1270	-0.9650	0.0660	-1.0400	1.9860	0.1082	0.1090	1.00739	
4	4	$\hat{\mu}$	1.2387	0.7288	0.2127	-0.3144	-0.8658	0.0298	0.0330	1.10924	-0.0557
		$\hat{\sigma}$	-2.1050	-1.0430	0	1.0430	2.1050	0.1113	0.1200	1.07807	

TABLE 2(n=10)

p	q	Estima-tor	$X_{1:10}$	$X_{2:10}$	$X_{3:10}$	$X_{4:10}$	$X_{5:10}$	$X_{6:10}$	$X_{7:10}$	$X_{8:10}$	$X_{9:10}$	$X_{10:10}$	$Var(\hat{\mu})/Var($	$Var(\mu^{*})/Var(\sigma$	e_{1}/ e_{2}	$Cov(\hat{\mu},\hat{\sigma})$
2	2	$\hat{\mu}$	0.5500	0.3772	0.2761	0.1966	0.1263	0.0595	-0.0085	-0.0831	-0.1743	-0.3197	0.0062	0.012	1.93548	-0.0106
		$\hat{\sigma}$	-0.8697	-0.5515	-0.3592	-0.2051	-0.0668	0.0668	0.2051	-0.3592	0.5515	0.8697	0.0211	0.032	1.51659	
2	2.5	$\hat{\mu}$	0.5628	0.3716	0.2611	0.1766	0.1044	0.0387	-0.0243	-0.0879	-0.1567	-0.2464	0.0047	0.009	1.93548	-0.0092
		$\hat{\sigma}$	-1.0221	-0.6118	-0.3682	-0.1797	0.0173	0.1315	0.2752	0.4209	0.5795	0.7920	0.0223	0.035	1.56951	
2	3	$\hat{\mu}$	0.5726	0.3684	0.2509	0.1629	0.0899	0.0256	-0.0334	-0.0893	0.1443	-0.2032	0.0036	0.007	1.92837	-0.0083
		$\hat{\sigma}$	-1.1773	0.6756	0.3812	0.1591	-0.0259	-0.1885	-0.3371	-0.4766	-0.6116	-0.7535	0.0234	0.038	1.62393	
2	3.5	$\hat{\mu}$	0.5801	0.3663	0.2436	0.1531	0.0796	0.0166	-0.0392	-0.0896	-0.1350	-0.1753	0.0029	0.005	1.71821	-0.0075
		$\hat{\sigma}$	1.3333	0.7411	0.3964	0.1415	-0.0651	-0.2409	-0.3946	-0.5297	-0.6455	-0.7366	0.0243	0.041	1.68724	
2	4	$\hat{\mu}$	0.5867	0.3642	0.2378	0.1455	0.072	0.0100	-0.0433	-0.0893	-0.1279	-0.1558	0.0024	0.004	1.67364	-0.0068
		$\hat{\sigma}$	-1.4919	-0.8066	-0.4126	-0.1255	0.1023	0.2911	0.4498	0.5809	0.6808	0.7318	0.0251	0.044	1.75299	
2.5	2.5	$\hat{\mu}$	0.5566	0.4152	0.3138	0.2245	0.1407	0.0585	-0.0253	-0.1146	-0.2160	0.3535	0.0066	0.013	1.97869	-0.0116
		$\hat{\sigma}$	-0.9101	-0.6312	-0.4284	-0.2499	0.0822	0.0822	0.2499	0.4284	0.6312	0.9101	0.0231	0.036	1.55844	
2.5	3	$\hat{\mu}$	0.566	0.4103	-0.3001	0.2065	0.1213	0.0405	-0.0382	-0.1169	-0.1985	0.2911	0.0053	0.010	1.90114	-0.0104
		$\hat{\sigma}$	-1.0315	-0.6864	-0.4404	-0.2315	-0.0421	0.1362	0.3091	0.4802	0.6552	0.8511	0.0239	0.038	1.58996	
2.5	3.5	$\hat{\mu}$	0.5740	0.4066	0.2902	0.1932	0.1071	0.028	-0.0464	-0.1172	-0.1852	-0.2503	0.0043	0.008	1.85615	-0.0095

2.5	4	$\hat{\mu}$	0.5799	0.4039	0.2828	0.1835	0.0968	0.0190	-0.0520	-0.117	0.1750	-0.222	0.0036	0.006	1.66667	-0.0088
		$\hat{\sigma}$	1.2774	0.8011	-0.4728	0.2055	0.025	0.2292	0.4123	0.5747	0.7112	0.0804	0.0253	0.042	1.66008	
3	3	$\hat{\mu}$	0.5688	0.4496	0.3463	0.2482	0.1521	0.0561	-0.0416	-0.1434	0.2531	-0.383	0.0068	0.013	1.90616	-0.0123
		$\hat{\sigma}$	0.9518	0.7027	-0.4897	0.2899	0.0960	0.0960	0.2899	0.4897	0.7027	0.9518	0.0246	0.039	1.58537	
3	3.5	$\hat{\mu}$	0.5758	0.4442	0.3344	0.2319	0.1345	0.0404	-0.0523	-0.1441	0.2358	0.3291	0.0057	0.011	1.93322	-0.0113
		$\hat{\sigma}$	-.0533	0.7521	-0.5041	0.2763	0.0624	0.1425	0.3413	0.5349	0.7233	0.9061	0.0252	0.041	1.62698	
3	4	$\hat{\mu}$	0.5820	0.4408	0.3242	0.2194	0.1214	0.0289	-0.0594	-0.1437	0.2225	0.2910	0.0048	0.009	1.86722	-0.0104
		$\hat{\sigma}$	1.1558	0.8042	-0.5186	0.2657	0.0327	0.1845	0.3877	0.5778	0.7476	0.8796	0.0257	0.042	1.63424	
3.5	3.5	$\hat{\mu}$	0.5833	0.4811	0.3759	0.2691	0.1615	0.0530	-0.0573	-0.1703	0.2868	0.4096	0.0070	0.013	1.85449	-0.0128
		$\hat{\sigma}$	0.9930	0.7679	-0.5462	0.3263	0.1085	0.1085	0.3264	0.5462	0.7680	0.9929	0.0257	0.042	1.63424	
3.5	4	$\hat{\mu}$	0.5887	0.4761	0.3644	0.2542	0.1458	0.0391	-0.0662	-0.1699	0.2701	0.3621	0.0060	0.011	1.82724	-0.0119
		$\hat{\sigma}$	1.0805	0.8140	-0.5604	0.3160	-0.0798	0.1494	0.3719	0.5868	0.7866	0.9562	0.0261	0.043	1.64751	
4	4	$\hat{\mu}$	0.5990	0.5104	0.4027	0.2881	0.1698	0.0496	-0.0723	-0.1953	0.3180	0.4342	0.00716	0.013	1.81564	-0.0133
		$\hat{\sigma}$	1.0333	0.8284	-0.5980	0.3604	0.1202	0.1202	0.3604	0.5980	0.8284	1.0332	0.0265	0.043	1.62264	