Aligarh Journal of Statistics

Vol. 34 (2014), 55-66

A TWO IDENTICAL UNIT COLD STANDBY SYSTEM WITH SWITCHING DEVICE AND GEOMETRIC FAILURE AND REPAIR TIME DISTRIBUTIONS

Rakesh Gupta and Arti Tyagi

ABSTRACT

The paper deals with cost benefit analysis of a two identical unit standby system model with two possible modes- Normal (N) and Total failure (F). Initially one unit is operative and other is kept into cold standby. A switching device is used to put the repaired unit into operation and it may be perfect or imperfect at the time of need with fixed known probabilities θ and $\bar{\theta}$ respectively $(\theta+\bar{\theta}=1)$. A single repairman is always available with the system to repair the failed switching device and failed unit. The failure and repair times of unit and switching device are taken as independent random variables of discrete nature having geometric distributions with different parameters.

1. INTRODUCTION

Two unit redundant system models have been widely studied in the literature of reliability due to their importance in modern business and industrial systems. Various authors including (Goel et al. (1994, 2006), Nakagawa and Osaki (1975), Sharma and Agarwal (2010), Sharma et. al. (2012), Subramanian and Ravichandan (1978)) have analyzed reparable system models with one or more units under continuous parametric Markov Chain by considering various concepts such as imperfect switch, slow switch, two types of repairmen, repair machine failure, random appearance and disappearance of repairman, two types of failure and two types of repair. Very few authors Gupta and Varshney (2006, 2007) have obtained the reliability characteristics of redundant system models under discrete parametric Markov Chain i.e. the random variables denoting failure and repair times are taken as discrete. As an instance, the life time of a light bulb in a Xerox machine is an example when the bulb operates whenever a paper enters into the machine for Xerox purpose i.e. the bulb function at discrete time epochs and can fail only during its functioning. Goel and Sharma (1986) have analyzed a two unit standby system model with two switching devices. The first switching device is used to carry the failed unit into repair facility whereas second switching device is used to put good unit into operation. Both the switching devices may be found failed at the time of need with known

probabilities. Various measures of system effectiveness are obtained by assuming the failure and repair times as continuous random variables.
The purpose of the paper is to analyze a two identical unit redundant system model with a switching device which is used to put the repaired unit into operation. The system model is analyzed under discrete parametric Markov-Chain i.e. time to failure and time to repair are taken as discrete random variables having geometric distributions with different parameters. The following economic related measures of system effectiveness are obtained by using regenerative point technique

i) Transition probabilities and mean sojourn times in various states.

ii) Reliability and mean time to system failure.

iii) Pointwise and steady-state availabilities of the system as well as expected up time of the system during interval $(0, t)$.

iv) Expected busy period of the repairman during time interval $(0, t)$ in case of failed unit and failed switching device.

v) Net expected profit earned by the system during a finite interval and in steady-state.

2. MODEL DESCRIPTION AND ASSUMPTIONS

i) The system comprises of two identical units having two possible modes-Normal (N) and Total failure (F). Initially one unit is operative and other is kept into cold standby.

ii) A switching device is used to put the repaired unit into operation which may be perfect and imperfect at the time of need with fixed known probabilities θ and $\overline{\theta}$ respectively.

iii) A single repairman is always available with the system.

iv) The priority in repair is given to the repair of switching device over the repair of a failed unit.

v) The repaired unit and switching device work as good as new.

vi) Failure and repair times follow independent geometric distributions with different parameters.

3. NOTATIONS AND STATES OF THE SYSTEM

a) Notations

p : Constant failure rate of a unit, so that the p.m.f. of failure time of the unit is pq^{x}; $x = 0,1,2,....$ $(q = 1 - p)$.

r	:	Constant repair rate of a unit, so that the p.m.f. of repair time of the unit is rs^{x}; $x = 0,1,2,....\ (s = 1 - r)$.
a	:	Constant repair rate of the switching device, so that the p.m.f. of repair time of the switching device is ab^{x}; $x = 0,1,2,....$ $(b = 1 - a)$.
$q_{ij}(\bullet), Q_{ij}(\bullet)$	:	P.m.f. and c.d.f. of one step or direct transition time from state S_i to S_j.
$p_{ij}(\bullet)$	:	Steady state transition probability from state S_i to S_j. $p_{ij} = Q_{ij}(\infty)$
$Z_i(t)$	:	Probability that the system sojourn in state S_i at epochs $0,1,2,....,\ (t-1)$.
ψ_i	:	Mean sojourn time in state S_i.
$*, h$	:	Symbol and dummy variable used in geometric transform e. g. $GT\left[q_{ij}(t)\right] = q_{ij}^{*}(h) = \sum_{t=0}^{\infty} h^{t} q_{ij}(t)$

b) Symbols for the states of the systems

N_O / N_S	:	Unit is in normal (N) mode and operative/standby.
N_g	:	Unit is in normal (N) mode and good.
F_r / F_w	:	Unit is in total failure (F) mode and under repair/waits for repair.
T_r	:	Switching device is under repair.

With the help of above symbols the possible states of the system are-

$S_0 \equiv (N_O, N_S)$,

$S_1 \equiv (F_r, N_O)$,

$$S_2 \equiv \begin{pmatrix} N_g, F_w \\ T_r \end{pmatrix},$$

$$S_3 \equiv (F_r, F_w)$$

The transition diagram of the system model is shown in fig. 1

TRANSITION DIAGRAM

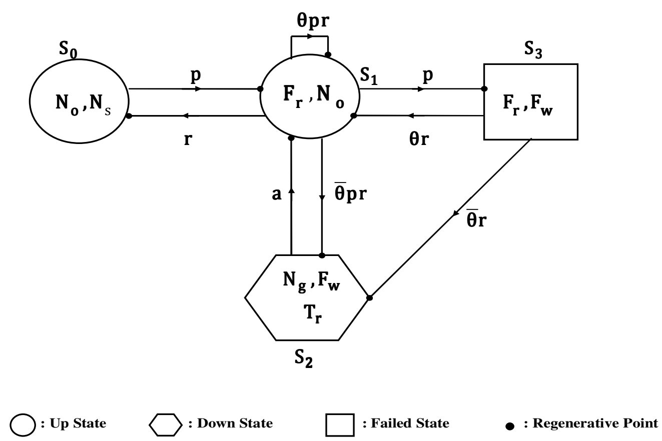

Fig.1

4. TRANSITION PROBABILITIES

Let $Q_{ij}(t)$ be the probability that the system transits from state S_i to S_j during time interval $(0,t)$ i.e., if T_{ij} is the transition time from state S_i to S_j then

$$Q_{ij}(t) = P\left[T_{ij} \le t\right]$$

By using simple probabilistic arguments we have

$$Q_{01}(t) = 1 - q^{t+1}, \tag{1}$$

$$Q_{10}(t) = \frac{qr}{1-qs}\left[1-(qs)^{t+1}\right] \tag{2}$$

$$Q_{11}(t) = \frac{\theta pr}{1-qs}\left[1-(qs)^{t+1}\right], \tag{3}$$

$$Q_{12}(t) = \frac{\bar{\theta} pr}{1-qs}\left[1-(qs)^{t+1}\right], \tag{4}$$

$$Q_{13}(t) = \frac{ps}{1-qs}\left[1-(qs)^{t+1}\right], \tag{5}$$

$$Q_{21}(t) = 1-b^{t+1}, \tag{6}$$

$$Q_{31}(t) = \theta\left[1-s^{t+1}\right], \tag{7}$$

$$Q_{32}(t) = \overline{\theta}\left[1-s^{t+1}\right] \tag{8}$$

The steady state transition probabilities from state S_i to S_j can be obtained from (1-8) by taking $t \to \infty$, as follows:

$$p_{01} = 1,$$

$$p_{10} = \frac{qr}{1-qs},$$

$$p_{11} = \frac{\theta pr}{1-qs},$$

$$p_{12} = \frac{\overline{\theta} pr}{1-qs}$$

$$p_{13} = \frac{ps}{1-qs},$$

$$p_{21} = 1,$$

$$p_{31} = \theta,$$

$$p_{32} = \overline{\theta}$$

We observe that the following relations hold-

$$p_{01} = p_{21} = 1, \tag{9}$$

$$p_{10} + p_{11} + p_{12} + p_{13} = 1, \tag{10}$$

$$p_{31} + p_{32} = 1 \tag{11}$$

5. MEAN SOJOURN TIMES

Let ψ_i be the sojourn time in state $S_i \,(i = 0,1,2,3)$ then mean sojourn time in state S_i is given by

$$\psi_i = \sum_{t=1}^{\infty} P[T \geq t]$$

In particular,

$$\psi_0 = \frac{q}{p}, \tag{12}$$

$$\psi_1 = \frac{qs}{1-qs}, \tag{13}$$

$$\psi_2 = \frac{b}{a}, \tag{14}$$

$$\psi_3 = \frac{s}{r} \tag{15}$$

6. ANALYSIS OF RELIABILITY AND MTSF

Let $R_i(t)$ be the probability that the system does not fail at epochs $0,1,2,...,(t-1)$ when it is initially started from up state S_i. To determine it, we regard the failed state S_2 and S_3 as observing state. By using the definition of $R_i(t)$ and simple probabilistic arguments, the following recurrence relations can be easily developed.

$$R_0(t) = Z_0(t) + q_{01}(t-1)©R_1(t-1) \tag{16}$$

$$R_1(t) = Z_1(t) + q_{10}(t-1)©R_0(t-1) + q_{11}(t-1)©R_1(t-1) + q_{12}(t-1)©R_2(t-1) \tag{17}$$

$$R_2(t) = Z_2(t) + q_{21}(t-1)©R_1(t-1) \tag{18}$$

Where,

$$Z_0(t) = q^t,$$

$$Z_1(t) = q^t s^t,$$

$$Z_2(t) = b^t$$

Taking geometric transform of (16-18) and simplifying the resulting set of algebraic equations for $R_0^*(h)$ we get

$$R_0^*(h)=\frac{\left(1-hq_{11}^*-h^2q_{21}^*q_{12}^*\right)Z_0^*+hq_{01}^*Z_1^*+h^2q_{01}^*q_{12}^*Z_2^*}{1-hq_{11}^*-h^2q_{12}^*q_{21}^*-h^2q_{10}^*q_{01}^*} \tag{19}$$

Collecting the coefficient of h^t from expression (19), we can get the reliability of the system $R_0(t)$. The MTSF is given by-

$$E(T)=\lim_{h\to 1}\sum_{t=1}^{\infty}h^tR(t)=\frac{(p_{10}+p_{13})\psi_0+\psi_1+p_{12}\psi_2-p_{13}}{p_{13}} \tag{20}$$

7. AVAILABILITY ANALYSIS

Let $A_i(t)$ be the probability that the system is up at epoch t-1, when it initially started from state S_i. Then by using simple probabilistic arguments, the following recurrence relations can be easily developed for $A_i(t)$

$$A_0(t)=Z_0(t)+q_{01}(t-1)©A_1(t-1) \tag{21}$$

$$A_1(t)=Z_1(t)+q_{10}(t-1)©A_0(t-1)+q_{11}(t-1)©A_1(t-1)$$
$$+q_{12}(t-1)©A_2(t-1)+q_{13}(t-1)©A_3(t-1) \tag{22}$$

$$A_2(t)=q_{21}(t-1)©A_1(t-1) \tag{23}$$

$$A_3(t)=q_{31}(t-1)©A_1(t-1)+q_{32}(t-1)©A_2(t-1) \tag{24}$$

On taking geometric transform of (21-24) and simplifying the resulting equations we get-

$$A_0^*(h)=\frac{N_1(h)}{D_1(h)} \tag{25}$$

Where,

$$N_1(h)=Z_0^*\left[1-hq_{11}^*-hq_{13}^*\left(hq_{31}^*+h^2q_{32}^*q_{21}^*\right)-h^2q_{12}^*q_{21}^*\right]+hq_{01}^*Z_1^*$$

$$D_1(h)=1-hq_{11}^*-hq_{13}^*\left(hq_{31}^*+h^2q_{32}^*q_{21}^*\right)-h^2q_{12}^*q_{21}^*-h^2q_{01}^*q_{10}^*$$

The steady-state availability of the system is given by

$$A_0=\lim_{t\to\infty}A_0(t)=\lim_{h\to 1}(1-h)\frac{N_1(h)}{D_1(h)}$$

Now since $D_1(h)$ at $h=1$ is zero, therefore by applying L hospital rule we get

$$A_0 = -\frac{N_1(h)}{D_1'(h)} \tag{26}$$

Where,

$$N_1(1) = p_{10}\psi_0 + \psi_1$$

$$D_1(1) = -\left[p_{10}\psi_0 + \psi_1 + (p_{13}p_{32} + p_{12})\psi_2 + p_{13}\psi_3\right]$$

Now, the expected up time of the system up to epoch t is given by

$$\mu_{up}(t) = \sum_{x=0}^{t-1} A_0(x)$$

So that,

$$\mu_{up}^*(h) = A_0^*(h)/(1-h) \tag{27}$$

8. BUSY PERIOD ANALYSIS

Let $B_i^U(t)$ and $B_i^S(t)$ be the respective probabilities that the repairman is busy at epoch $(t-1)$ in the repair of a failed unit and repair of a failed switching device when system initially starts from S_i. Using simple probabilistic arguments as in case of availability analysis, the recurrence relations for $B_0^U(t)$ and $B_0^S(t)$ are as follows:

$$B_0^j(t) = q_{01}(t-1) © B_1^j(t-1) \tag{28}$$

$$B_1^j(t) = \delta Z_1(t) + q_{10}(t-1) © B_0^j(t-1) + q_{11}(t-1) © B_1^j(t-1)$$
$$+ q_{12}(t-1) © B_2^j(t-1) + q_{13}(t-1) © B_3^j(t-1) \tag{29}$$

$$B_2^j(t) = (1-\delta)Z_2(t) + q_{21}(t-1) © B_1^j(t-1) + q_{23}(t-1) © B_3^j(t-1) \tag{30}$$

$$B_3^j(t) = \delta Z_3(t) + q_{31}(t-1) © B_1^j(t-1) \tag{31}$$

Where $Z_1(t)$ and $Z_2(t)$ are same as given in reliability analysis and $Z_3(t) = s^t$.

Also δ is dichotomous variable which takes two values 1 and 0 respectively in case of repair of a failed unit and failed switching device. Further, we take $j = U$ and S respectively for the repair of a unit and switching device.

On taking geometric transforms of relations (28-31) and simplifying the resulting equations we get

i) For $j = U$

$$B_0^{U*}(h) = \frac{N_2(h)}{D_1(h)} \tag{32}$$

ii) For $j = S$

$$B_0^{S*}(h) = \frac{N_3(h)}{D_1(h)} \tag{33}$$

Where,

$$N_2(h) = hq_{01}^* Z_1^* + h^2 q_{01}^* q_{13}^* Z_3^*$$

$$N_3(h) = Z_2^* \left[hq_{01}^* \left(h^2 q_{13}^* q_{32}^* + hq_{12}^* \right) \right]$$

and $D_1(h)$ is same as in case of availability analysis.

In the long run the respective probabilities that the repairman is busy in the repair of failed unit and failed switching device are given by

$$B_0^U = \lim_{t\to\infty} B_o^U(t)$$

and

$$B_0^S = \lim_{t\to\infty} B_o^S(t)$$

But $D_1(h)$ at $h = 1$ is zero, therefore by applying L Hospital rule, we get

$$B_0^U = -\frac{N_2(1)}{D_1'(1)} \text{ and } B_0^S = -\frac{N_3(1)}{D_1'(1)} \tag{34}$$

Where,

$$N_2(1) = \psi_1 + p_{13}\psi_3$$

$$N_3(1) = (p_{13}p_{32} + p_{12})\psi_2$$

and $D_1'(1)$ is same as in availability analysis.

Now the expected busy period of the repairman in repair of a failed unit and failed switching device up to epoch $(t-1)$ are respectively given by-

$$\mu_b^U(t)=\sum_{x=0}^{t-1}B_0^U(x),$$

$$\mu_b^S(t)=\sum_{x=0}^{t-1}B_0^S(x)$$

So that,

$$\mu_b^{U*}(h)=\frac{B_0^{U*}(h)}{(1-h)}, \tag{35}$$

$$\mu_b^{S*}(h)=\frac{B_0^{S*}(h)}{(1-h)} \tag{36}$$

9. COST BENEFIT ANALYSIS

We are now in the position to obtain the net expected profit incurred up to epoch $(t-1)$ by considering the characteristics obtained in earlier section.

Let us consider,

K_0 = revenue per-unit time by the system when it is operative.

K_1 = cost per-unit time when repairman is busy in the repairing failed unit.

K_2 = cost per-unit time when repairman is busy in the repairing failed switching device.

Then, the net expected profit incurred up to epoch $(t-1)$ given by

$$P(t)=K_0\mu_{up}(t)-K_1\mu_b^U(t)-K_2\mu_b^S(t) \tag{37}$$

The expected profit per unit time in steady state is given by-

$$P=\lim_{t\to\infty}\frac{P(t)}{t}$$

$$=K_0\lim_{h\to 1}(1-h)^2\frac{A_0^*(h)}{(1-h)}-K_1\lim_{h\to 1}(1-h)^2\frac{B_0^{U*}(h)}{(1-h)}-K_2\lim_{h\to 1}(1-h)^2\frac{B_0^{S*}(h)}{(1-h)}$$

$$=K_0A_0-K_1B_0^U-K_2B_0^S \tag{38}$$

10. GRAPHICAL REPRESENTATION

The curves for MTSF and profit function have been drawn for different values of parameters. Fig. 2 depicts the variations in MTSF with respect to repair rate of a failed unit (r) for different values of failure rate of the unit and probability of perfect switching device (θ). From the curves we observe that expected life of the system increases uniformly as the values of r and θ increase and decreases with the increase in the values of p.

Similarly, fig. 3 reveals the variations in profit (P) with respect to r for varying values of p and θ, when the values of other parameters are kept fix as $a=0.6, K_o=250, K_1=150$, and $K_2=200$. From this figure it is clearly observed from the dotted curves that the system is profitable only if repair rate r is greater than 0.227, 0.335 and 0.490 respectively for $p=0.3, 0.4$ and 0.5 for fixed value of $\theta=0.3$. From smooth cures, we conclude that the system is profitable only if r is greater than 0.208, 0.290 and 0.387 respectively for $p=0.3, 0.4$ and 0.5 for fixed value of $\theta=0.9$.

REFERENCES

Goel, L. R. and S. C. Sharma (1986): Stochastic analysis of two unit standby system with two switching devices. *Microelectron Reliability*, **26**(**5**), 859-861.

Goel, L. R., P. K. Tyagi and R. Gupta (1994): Analysis of a stsndby system with dependent repair time and slow switching device. *Microelectron Reliability*, **34**(**2**), 383-386.

Gupta, R. and Gaurav Varshney (2007): A two identical unit parallel system with Geometric failure and repair time distributions. *J. of Comb. Inf. System Sci.*, **32**(**1-4**), 127–136.

Gupta, R. and Gaurav Varshney (2006): A two non–identical unit parallel system with Geometric failure and repair time distributions. IAPQR *Trans*. **31**(**2**), 127–139.

Gupta, R., P. K. Tyagi and Ram Kishan (1996): A two-unit System with correlated failures and repairs and random appearance and disappearance of repairman. *Int. J. of Systems Science*, **27**(**6**), 561-566.

Murari, K. and V. Goel (1983): Reliability of a system with two types of repair facilities. *Microelectron. Reliability*, **23**, 1015-1025.

Nakagawa, T. and S. Osaki (1975): Stochastic behavior of a two unit standby redundant system with imperfect switch over. IEEE *Transactions Reliability*, **R-24**, 193-196.

Sharma, S. C. and Shinjita Agarwal (2010): Effectiveness analysis of a system with a special warranty scheme. *Stat. Methodol.*, **7**, 498-510.

Sharma, S. C., R. Gupta and Vikas Saxena (2012): A two-unit parallel system with dependent failure rates and correlated working and rest time of repairman. *Int. J. Agricultural and Statistical Sciences*, **8**(**1**), 145-155.

Subramanian, R. and N. Ravichandan (1978): On a two-unit standby redundant system with imperfect switchover, *Microelectron Reliability*, **17**, 585-586.

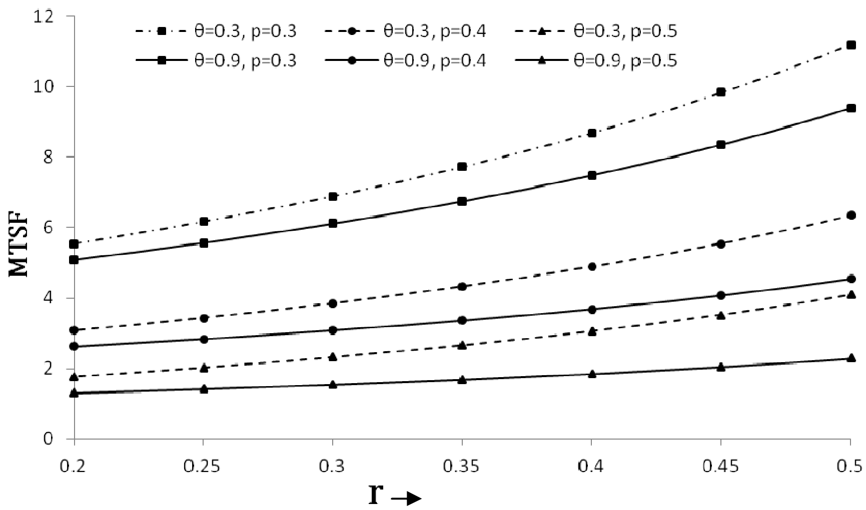

Fig. 2

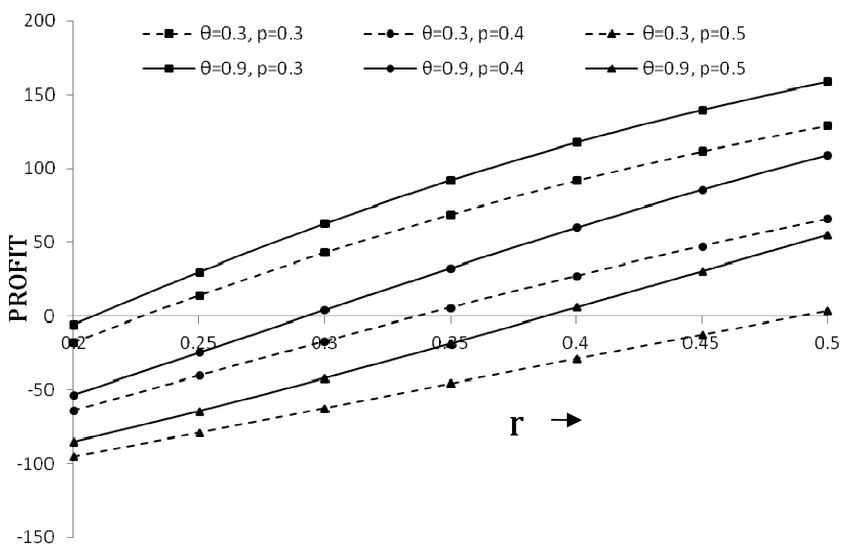

Fig. 3

Revised: 04.04.2013

Revised: 29.08.2013

Rakesh Gupta and Arti Tyagi

Department of Statistics
Ch. Charan Singh University
Meerut-250004

email: prgheadstats@yahoo.in