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STOCHASTIC ANALYSIS OF A THREE PARALLEL INDENTICAL UNITS SUBJECT TO TWO PHASE REPAIR

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ABSTRACT

This paper deals with the stochastic analysis of a system having three parallel identical operating units with two types of repair. Whenever any operating unit fails it goes to repair. There is a two phase independent repair facility to repair the failed units on first come first serve basis. Whenever an operating unit fails its goes to phase I repair first. After the completion of first phase repair the unit goes to Phase II repair. After the completion of total repair i.e. Phase I repair and Phase II repair unit works as a new unit. All the failure and repair time distributions are assumed to be negative exponential. Various reliability characteristics are obtained in order to analyze the expected profit earned by the system.

1. INTRODUCTION

Most of the authors in the field of reliability have analyzed the cold standby systems by making any one of the following assumptions about the repair facility:

(1) The repairman is always available in the system.

(2) The repairman is called at the time of need in negligible time.

(3) Two types of repair facilities regular and expert are available in the system to do the repair. Whenever the expert repairman is called to do the repair, he repairs all the units which fail during his stay.

However in most of the electric and electronic systems it is seen that the operator himself repairs all the faults. But in practice all the repairs are not done by one repairman. It is completed by a group of repairmen in two phases. Whenever operating unit fails it goes to Phase I repair and after the completion of Phase I repair it goes to Phase II repair. In phase II repairmen will check the Phase I repair and do those repairs which are not possible in Phase I. However very few works have been reported in this direction.

Quite a few papers (Goyal (1984), Singh et. al. (1988, 1996, 1989)) have dealt with an optimal policy in a two unit standby system with two types of repairmen. Some early works on reliability models have been generalized in one direction only. Chandrakar and Natrajan (1994) has obtained the confidence limits for steady state availability of parallel systems. Murar and Goyal (1984) obtained the optimal policies in a two unit standby system with two types of repairman. Singh and Singh (2007) analyze two unit parallel systems with erlangian repair time. Singh et. al. (1988) analyzed the two duplex unit standby system with two types of repair. Bhardwaj and Malik (2011) studied stochastic modeling and performance analysis of a $2(K)-\text{out}-\text{of}-3(N)$ system with inspection subject to operational restriction. This paper discusses a three parallel identical operating units with two types of repair under the specific assumption about the repair of the failed unit. Whenever operating unit fails it goes to phase I repair. And when the repair of the Phase I is completed unit goes to Phase II repair. Repair of a failed unit is completed when the total repair of phase I & II are completed. Employing regenerative point technique for Markov renewal process we obtain the following measures of reliability:

(1) Steady state transition probabilities and sojourn times

(2) Distribution of time to system failure and its mean time to system failure

(3) Point wise and steady state availability of the system

(4) Expected busy period of the repairmen in repair of failed unit in Phase I & Phase II

(5) Expected profit earned by the system in (0, t] and in steady state.

2. DESCRIPTION OF THE SYSTEM

(1) The system consists of three parallel operating units. Initially all the units are in operation.

(2) Whenever an operating unit fails it goes to phase I repair immediately.

(3) There are two types of repair facilities. Repair completed by type I repair facility is known as Phase I repair and the repairs done by the type II repair is known as Phase II repair. Type I and type II repairs are done on first come first serve basis and after the completion of phase I repair, the phase II repair is started. After total repair, unit works as new.

(4) All the failure & repair time distributions follow negative exponential distribution.

3. SYMBOLS USED FOR STATES OF THE SYSTEM

N_0 : Normal unit is operating.

F_{r1}, F_{r2} : Failed unit is under phase I/phase II repair.

F_{wr1}, F_{wr2} : Failed unit is waiting for phase I /phase II repair.

4. NOTATIONS

λ : Failure rate of operating unit.

μ_1, μ_2 : Phase I/Phase II repair rate of operating unit.

m_i : Mean sojourn time in states $\{S_i; i = 0-10\}$

E : Set of regenerative states $\{S_i; i = 0-10\}$

© : Symbol for ordinary convolution.

$[s]$: Laplace-Stieltjes transforms

A_∞ : Study state availability

$\hat{A}_\infty$ $\hat{A}_\infty$: Consistent asymptotic estimator (CAN) of A_∞

$W_i(t)$: Probability that system is in Phase I repair in regenerative state $S_i(i = 1, 3-6, 8)$ /in Phase II repair in regenerative state $S_i(i = 2, 4, 6-10)$ respectively and has no transition till time t.

$B_i^j(t)$: Probability that repairman is busy in Phase I repair / Phase II repair respectively in regenerative state $S_i(i = 0-10)$ for $j = 1, 2$ respectively.

Possible transitions between states are shown in Fig. 1.

5. TRANSITION PROBABILITIES AND SOJOURN TIMES

Let $T_0(\equiv 0), T_1, T_2, T_3, \ldots\ldots\ldots.$ denote the epochs at which the system enters any state $S_i \in E$.

Let X_n *d*enote the state visited at epoch T_n+, *i*.e. just after the transition at T_n. Then

$$Q_{ij}(t) = P[X_{n+1} - T_n \le t, X_n = S_i]$$

The transition probability matrix is given by

$$P = (p_{ij}) = [Q_{ij}(\infty) = Q(\infty)]$$

$$p_{01} = 1, p_{12} = \frac{\mu_1}{A_1}, p_{13} = \frac{2\lambda}{A_1} \qquad \text{where } A_1 = 2\lambda + \mu_1$$

$$p_{20} = \frac{\mu_2}{A_2}, p_{24} = \frac{2\lambda}{A_2} \qquad \text{where } A_2 = 2\lambda + \mu_2, p_{34} = \frac{\mu_1}{A_3},$$

$$p_{35} = \frac{\lambda}{A_3} \qquad \text{where } A_3 = \lambda + \mu_1, p_{41} = \frac{\mu_2}{A_4}, p_{46} = \frac{\lambda}{A_4},$$

$$p_{47} = \frac{\mu_1}{A_4} \qquad \text{where } A_4 = \lambda + \mu_1 + \mu_2, p_{56} = 1 p_{63} = \frac{\mu_2}{A_6},$$

$$p_{68} = \frac{\mu_1}{A_6} \qquad \text{where } A_6 = \mu_1 + \mu_2, p_{72} = \frac{\mu_2}{A_7}$$

$$p_{78} = \frac{\lambda}{A_7} \qquad \text{where } A_7 = \lambda + \mu_2$$

$$p_{84} = \frac{\mu_2}{A_8}, p_{89} = \frac{\mu_1}{A_8} \qquad \text{where } A_8 = \mu_1 + \mu_2, p_{9,10} = 1$$

$$p_{10,2} = \frac{\mu_2}{A_{10}}, p_{10,8} = \frac{\lambda}{A_{10}} \qquad \text{where } A_{10} = \lambda + \mu_2 \tag{1}$$

6. SOJOURN TIMES

Mean sojourn time m_i in S_i is defined as

$$\left.\begin{aligned}
m_0 &= \frac{1}{3\lambda} = \frac{1}{A_0}, \\
m_1 &= \frac{1}{A_1}, \\
m_2 &= \frac{1}{A_3}, \\
m_4 &= \frac{1}{A_4}, \\
m_5 &= \frac{1}{\mu_1} = \frac{1}{A_5}, \\
m_6 &= \frac{1}{A_6}, \\
m_7 &= \frac{1}{A_7}, \\
m_8 &= \frac{1}{A_8}, \\
m_9 &= \frac{1}{A_9}, \\
m_{10} &= \frac{1}{A_{10}}
\end{aligned}\right\} \tag{2}$$

7. MEAN TIME TO SYSTEM FAILURE (MTSF)

Let T_i be the random variable depicting time to system failure when system starts from state $S_i \in E(i = 0-4, 7, 10)$ and

$$\pi_i(t) = P[T_i \le t] \tag{3}$$

To calculate the distribution function $\pi_i(t)$, we regard the failed states S_5, S_6, S_8 and S_9 as absorbing states.

To obtain $\pi_0(t)$, we consider the possible transitions from S_0. Thus

$$\pi_0(t) = Q_{01}(t)[s]\pi_1(t) \tag{4}$$

$$\pi_1(t) = Q_{12}(t)[s]\pi_2(t) + Q_{13}(t)[s]\pi_3(t) \tag{5}$$

$$\pi_2(t) = Q_{20}(t)[s]\pi_0(t) + Q_{24}(t)[s]\pi_4(t) \tag{6}$$

$$\pi_3(t) = Q_{34}(t)[s]\pi_4(t) + Q_{35}(t) \tag{7}$$

$$\pi_4(t) = Q_{41}(t)[s]\pi_1(t) + Q_{46}(t) + Q_{47}(t)[s]\pi_7(t) \tag{8}$$

$$\pi_7(t) = Q_{72}(t)[s]\pi_2(t) + Q_{78}(t) \tag{9}$$

$$\pi_{10}(t) = Q_{10,2}(t)[s]\pi_2(t) + Q_{10,8}(t) \tag{10}$$

Since $\tilde{\pi}(0)$ is a proper *cdf* so after taking Laplace-Stieltjes transforms of equations (4-10), the solution for $\tilde{\pi}(0)$, i.e. mean time to system failure when the system starts from S_0, can be written in the following

$$E(T_0) = -\frac{d}{ds}\tilde{\pi}_0(s)\Big|_{s=0} = \frac{D_1^{'}(0) - N_1^{'}(0)}{D_1(0)} \tag{11}$$

where

$$D_1(0) = 1 - p_{12}p_{20} - p_{12}p_{24}p_{41} - p_{24}p_{47}p_{72} - p_{13}p_{34}p_{41} - p_{13}p_{20}p_{34}p_{72}$$

$$N_1(0) = p_{12}p_{24}p_{46} + p_{12}p_{24}p_{47}p_{78} + p_{13}p_{35} + p_{13}p_{34}p_{47}p_{78} - p_{13}p_{24}p_{35}p_{47}p_{72} \tag{12}$$

and

$$\left.\begin{aligned} D_1^{'}(0)-N_1^{'}(0)=[&p_{12}p_{20}-p_{12}p_{24}p_{46}-p_{12}p_{24}p_{47}p_{78}-p_{13}p_{35}\\ &+p_{13}p_{20}p_{34}p_{47}p_{72}-p_{13}p_{34}p_{46}+p_{13}p_{34}p_{47}p_{78}\\ &+p_{13}p_{24}p_{35}p_{47}p_{47}p_{72}]m_0+[p_{24}(1-p_{41})-p_{34}p_{46}]m_1\\ &+[p_{12}+p_{13}p_{34}p_{47}p_{72}]m_2+[p_{20}p_{47}p_{72}+p_{13}p_{72}]m_3\\ &+[p_{12}p_{24}+p_{13}p_{34}]m_4+[p_{24}p_{47}+p_{13}p_{20}p_{34}p_{72}]m_7 \end{aligned}\right\} \quad (13)$$

$$MTSF=E(T_0)=\int \pi_0(t)dt=\lim_{s\to 0}\tilde{\pi}_0(s) \quad (14)$$

Using equation (12-13) in equation (11) we get the required result of MTSF.

8. AVAILABILITY ANALYSIS

Let $M_i(t)$ be the probability that the system is up initially in regenerative state $S_i(i=1-10)$ has no transition till time t, then by probabilistic arguments, we have:

$$\left.\begin{aligned} &M_0(t)=e^{-3\lambda t},\\ &M_1(t)=e^{-(2\lambda+\mu_1)t},\\ &M_3(t)=e^{-(\lambda+\mu_1)t}\\ &M_4(t)=e^{--(\lambda+\mu_1+\mu_2)t},\\ &M_7(t)=e^{--(\lambda+\mu_2)t},\\ &M_{10}(t)=e^{--(\lambda+\mu_2)t} \end{aligned}\right\} \quad (15)$$

Recursive relations giving the point wise availability $A_i(t)$ are:

$$A_0(t)=q_{01}(t)©A_1(t)+M_0(t) \quad (16)$$

$$A_1(t)=q_{12}(t)©A_2(t)+q_{13}(t)©A_3(t)+M_1(t) \quad (17)$$

$$A_2(t)=q_{20}(t)©A_0(t)+q_{24}(t)©A_4(t)+M_2(t) \quad (18)$$

$$A_3(t)=q_{34}(t)©A_4(t)+q_{35}(t)©A_5(t)+M_3(t) \quad (19)$$

$$A_4(t)=q_{41}(t)©A_1(t)+q_{46}(t)©A_6(t)+q_{47}(t)©A_7(t)+M_4(t) \quad (20)$$

$$A_5(t)=q_{56}(t)©A_6(t) \quad (21)$$

$$A_6(t)=q_{68}(t)©A_8(t) \quad (22)$$

$$A_7(t)=q_{72}(t)©A_2(t)+q_{78}(t)©A_8(t)+M_7(t) \quad (23)$$

$$A_8(t) = q_{89}(t) © A_9(t) \tag{24}$$

$$A_9(t) = q_{9,10}(t) © A_{10}(t) \tag{25}$$

$$A_{10,2}(t) = q_{10,2}(t) © A_2(t) + q_{10,8}(t) © A_8(t) + M_{10}(t) \tag{26}$$

Using above the steady state availability of the system when the system starts from $S_i \in E$ is obtained as follows:

$$A_0(\infty) = \lim_{s \to 0} \overset{*}{A}_0(s) = \frac{N_2(0)}{D_2'(0)} \tag{27}$$

where

$$\left.\begin{aligned} N_2(0) = m_0[(1 - p_{89}, p_{10,8})(1 - p_{24}p_{47}p_{72} - p_{12}p_{24}p_{41} - p_{13}p_{34}p_{41} \\ - p_{13}p_{35}p_{89}p_{10,2}) - (1 - p_{72})(p_{12}p_{24}p_{89}p_{10,2} + p_{13}p_{89}p_{10,2})] \\ + m_1(1 - p_{89}p_{10,8})(1 - p_{24}p_{47}p_{72}) + m_2[p_{13}p_{34}p_{47}p_{72}(1 - p_{89}p_{10,8}) \\ - p_{12}p_{89}p_{10,8}] + m_3p_{13}(1 - p_{89}p_{10,8}) \\ + m_4(1 - p_{89}p_{10,8})(p_{12}p_{24} - p_{13}p_{34}) \\ + m_7(1 - p_{89}p_{10,8})(p_{12}p_{24}p_{47} - p_{13}p_{34}p_{47}) + m_8p_{34}p_{89}(1 - p_{72}) \end{aligned}\right\} \tag{28}$$

$$\left.\begin{aligned} D_2'(0) = m_0[(p_{12}p_{20} + p_{13}p_{20}p_{47}p_{12})(1 - p_{89}p_{9,10})] + m_1(1 - p_{89}p_{10,8}) \\ + m_2[(p_{12} + p_{72})(1 - p_{89}p_{10,8}) + m_3p_{13}(1 - p_{89}p_{10,8}) \\ + m_4p_{24}p_{72}(1 - p_{89}p_{10,8}) + m_6p_{89}p_{10,2} + m_7p_{89}p_{10,8} + m_8p_{10,2} \\ + m_9p_{78} + m_{10}p_{10}p_{13}p_{20}p_{47} \end{aligned}\right\} \tag{29}$$

9. BUSY PERIOD ANALYSIS

(a) Expected busy period analysis of the repairman busy in Phase I repair in $(0,t)$

As $W_i(i = 1, 3-6, 8)$ be the probability that the system is in first type of repair in regenerative state $S_i(i = 1-10)$, has no transition till time t t, then by probabilistic arguments, we have:

$$\left.\begin{aligned} W_1(t) = e^{-(2\lambda+\mu_1)t}, W_3(t) = e^{-(\lambda+\mu_1)t}, W_4(t) = e^{-(\lambda+\mu_1+\mu_2)t} \\ W_5(t) = e^{-\mu_1 t}, W_6(t) = e^{-(\mu_1+\mu_2)t}, W_8(t) = e^{-(\mu_1+\mu_2)t} \end{aligned}\right\} \tag{30}$$

We define $B_1^1(t)$, the probability that repairman is busy in Phase I repair at epoch t starting from state $S_i \in E, (i = 0-10)$. By probabilistic arguments

$$B_0^1(t) = q_{01}(t)©B_1^1(t) \tag{31}$$

$$B_1^1(t) = q_{12}(t)©B_2^1(t) + q_{13}(t)©B_3^1(t) + W_1(t) \tag{32}$$

$$B_2^1(t) = q_{20}(t)©B_0^1(t) + q_{24}(t)©B_4^1(t) \tag{33}$$

$$B_3^1(t) = q_{34}(t)©B_4^1(t) + q_{35}(t)©B_5^1(t) + W_3(t) \tag{34}$$

$$B_4^1(t) = q_{41}(t)©B_1^1(t) + q_{46}(t)©B_6^1(t) + q_{47}(t)©B_7^1(t) + W_4(t) \tag{35}$$

$$B_5^1(t) = q_{56}(t)©B_6^1(t) + W_5(t) \tag{36}$$

$$B_6^1(t) = q_{68}(t)©B_8^1(t) + W_6(t) \tag{37}$$

$$B_7^1(t) = q_{72}(t)©B_2^1(t) + q_{78}(t)©B_8^1(t) \tag{38}$$

$$B_8^1(t) = q_{89}(t)©B_9^1(t) + W_8(t) \tag{39}$$

$$B_9^1(t) = q_{9,10}(t)©B_{10}^1(t) \tag{40}$$

$$B_{10}^1(t) = q_{10,2}(t)©B_2^1(t) + q_{10,8}(t)©B_8^1(t) \tag{41}$$

After developing and solving the recursive equations, in the long run, the fraction of the time for which the system is under Phase I repair is given by:

$$B_0^1(\infty) = \lim_{t\to\infty} sB_0^{1*}(t) = \lim_{s\to 0} sB_0^{1*}(s) = \frac{N_3(0)}{D_2^{'}(0)} \tag{43}$$

where

$$\begin{aligned} N_3(0) = {} & W_1[(1-p_{89}p_{10,8})(1-p_{24}p_{47}p_{72}) - p_{24}p_{89}p_{10,2}(p_{46}-p_{47}p_{78}) \\ & + W_3 p_{13}(1-p_{24}p_{47}p_{72})(1-p_{89}p_{10,8}) + W_4[(p_{12}p_{24}+p_{13}p_{34}) \\ & (1-p_{89}p_{10,8})] + W_5 p_{13}p_{35}(1-p_{89}p_{10,8}) + W_6 p_{13}p_{34}p_{46} \\ & (1-p_{89}p_{10,8}) + W_8[p_{12}p_{47}p_{78}(p_{24}+p_{34}) + p_{12}p_{46}p_{89}] \end{aligned}$$

(b) Expected busy period analysis of the repairman busy in Phase II repair in (0, t]

Proceeding on similar lines as for (a) we obtain equations for $B_i^2(t)$

$$B_0^2(\infty) = \lim_{t\to\infty} sB_0^{1*}(t) = \lim_{s\to 0} sB_0^{2*}(s) = \frac{N_4(0)}{D_2^{'}(0)} \tag{44}$$

where

$$N_4(0) = W_2[(1 - p_{89}p_{10,8})(p_{12} + p_{13}p_{34}p_{47}p_{72}) + W_4(1 - p_{89}p_{10,8})$$
$$(p_{12}p_{24} + p_{13}p_{34}) + W_6 p_{13}(1 - p_{89}p_{10,8}) + W_7 p_{24}p_{47}(1 - p_{10,8})$$
$$+ W_8[p_{12}p_{47}p_{78}(p_{34} + p_{24}) + p_{12}p_{46}p_{89}]$$
$$+ W_9[p_{12}p_{24}p_{46}p_{89} + p_{13}p_{89}(1 - p_{72})]$$
$$+ W_{10}[p_{12}p_{24}p_{89}(p_{78} - p_{46}) - p_{13}p_{89}(1 - p_{72})]$$

10. COST ANALYSIS

The cost benefit analysis of the system can be carried out by considering the expected busy period of the repairman busy in repair of Phase I and Phase II in (0, t] . Therefore,

$G(t)$ = expected revenue earned by the system in (0, t]

-expected repair cost of the repair facility due to Phase I repair in (0, t]

- expected repair cost of the repair facility due to Phase II repair in (0, t]

$$= C_1\mu_{up}(\text{t}) - C_2\mu_b^1(\text{t}) - C_3\mu_b^2(\text{t})$$

where

$$\mu_{up}(\text{t}) = \int_0^t A_0(t)dt \;;\mu_b^1(\text{t}) = \int_0^t B_0^1(t)dt \;;\mu_b^2(\text{t}) = \int_0^t B_0^2(t)dt$$

The expected profit per unit of time in steady state is

$$G = \lim_{t\to\infty}\frac{G(t)}{t} = \lim_{t\to\infty} s^2 G^*(s)$$
$$= C_1\mu_{up}(\text{t}) - C_2\mu_b^1(\text{t}) - C_3\mu_b^2(\text{t}) \quad (45)$$

where C_1 is the revenue per unit up time and C_2, C_3 are the phase I and phase II repair cost per unit time respectively .

11. CONFIDENCE INTERVAL

$$f(x_1) = \lambda e^{-\lambda}, 0 < x_1 < \infty, \lambda > 0 \quad (46)$$

Let $\text{Y}_{\text{in}}, \text{Y}_{\text{i2}}, \ldots\ldots, \text{Y}_{\text{in}}$ (i=1,2) be random sample of times to repair (phase I and phase II) of the unit with respective p.d.f.'s given by

$$f(y_1) = \mu_1 e^{-\mu_1}, 0 < y_1 < \infty, \mu_1 > 0$$
$$f(y_2) = \mu_2 e^{-\mu_2}, 0 < y_2 < \infty, \mu_2 > 0 \quad (47)$$

It is clear that

$$E(\bar{X}_1) = \frac{1}{\lambda} \ ,$$

$$E(\bar{Y}_1) = \frac{1}{\mu_1}$$

and

$$E(\bar{Y}_2) = \frac{1}{\mu_2} \ ,$$

where $\bar{X}_1$ is the sample mean of time to failure of normal units respectively and $\bar{Y}_1, \bar{Y}_2$ are the sample means of time to repair (phase I and phase II) of the failed unit respectively.

Let

$$\theta_1 = \frac{1}{\lambda},$$

$$\theta_2 = \frac{1}{\mu_1}$$

and

$$\theta_3 = \frac{1}{\mu_2} \ .$$

It may be noted that $\hat{A}_{\infty}$ is a real valued function in $\bar{X}_i \, (i=1)$ and $\bar{Y}_i \ (i=1,2)$ which is also differentiable. Now consider the following application of multivariate central limit theorem. Suppose $T_1', T_2', T_3', \ldots\ldots\ldots\ldots$ are independent and identically distributed $k-$dimensional random variables such that

$$T_n' = (T_{1n}, T_{2n}, \ldots\ldots\ldots, T_{kn}), n = 1, 2, 3, \ldots.$$

having the first and second order moments

$$E(T_n') = \mu$$

and

$$D(T_n') = \Sigma \, .$$

Define the sequence of random variables

$$T_n' = (\bar{T}_{1n}, \bar{T}_{2n}, \ldots\ldots\ldots, \bar{T}_{kn}), \ n = 1, 2, 3, \ldots.$$

where

$$\bar{T}_{in} = \frac{1}{n}\sum_{j=1}^{n} T_{ij}, \quad i = 1, 2, ..., k \ \ and \ \ j = 1, 2, ..., n$$

Then, $\sqrt{n}(\bar{T} - \mu) \rightarrow N(0, \Sigma) \quad as\, n \rightarrow \infty$

Applying the above multivariate central limit theorem, it readily follows that

$$\sqrt{n}\left[\left(\bar{X}_1, \bar{Y}_1, \bar{Y}_2\right) - \left(\theta_1, \theta_2, \theta_3\right)\right] \xrightarrow{d} N(0\Sigma) \ \ as \ \ n \rightarrow \infty$$

where the dispersion matrix $\Sigma = \left(\left(\sigma_{ij}\right)\right)_{3\times 3}$ .Again from Rao (1974), we have

$$\sqrt{n}\left(\hat{A}_\infty - A_\infty\right) \xrightarrow{d} N\left(0, \sigma^2(\theta)\right) \ \ as \ \ n \rightarrow \infty$$

where

$\theta = (\theta_1, \theta_2, \theta_3)$

and

$$\sigma^2(\theta) = \sum_{i=1}^{3} \left(\frac{\partial A_\infty}{\partial \theta_i}\right)^2 \sigma_{ii} \tag{48}$$

Hence $\hat{A}_\infty$ is a CAN estimator of A_∞ .

Perhaps an estimator of $\hat{\sigma}(\theta)$ is to be used for obtaining interval estimator (confidence interval) for some reliability characteristics.

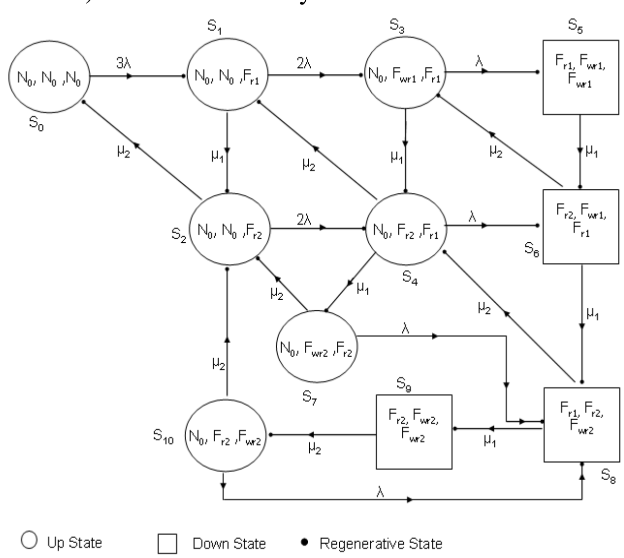

Fig. 1-State transition diagram

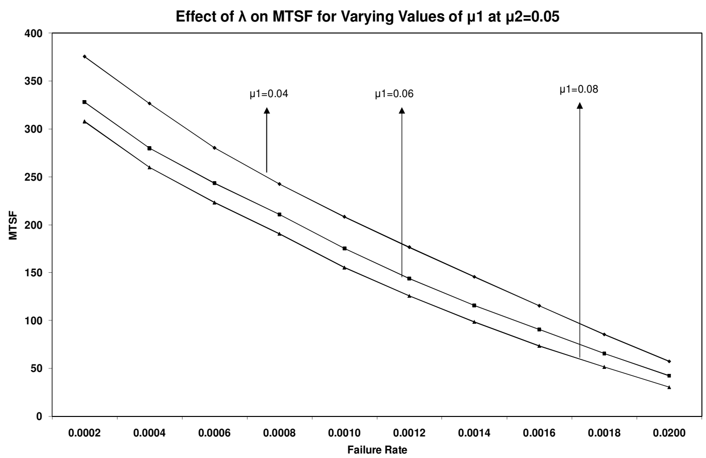
Effect of λ on MTSF for Varying Values of μ1 at μ2=0.05
MTSF
Failure Rate
μ1=0.04
μ1=0.06
μ1=0.08
400
350
300
250
200
150
100
50
0
0.0002
0.0004
0.0006
0.0008
0.0010
0.0012
0.0014
0.0016
0.0018
0.0200

Fig. 2

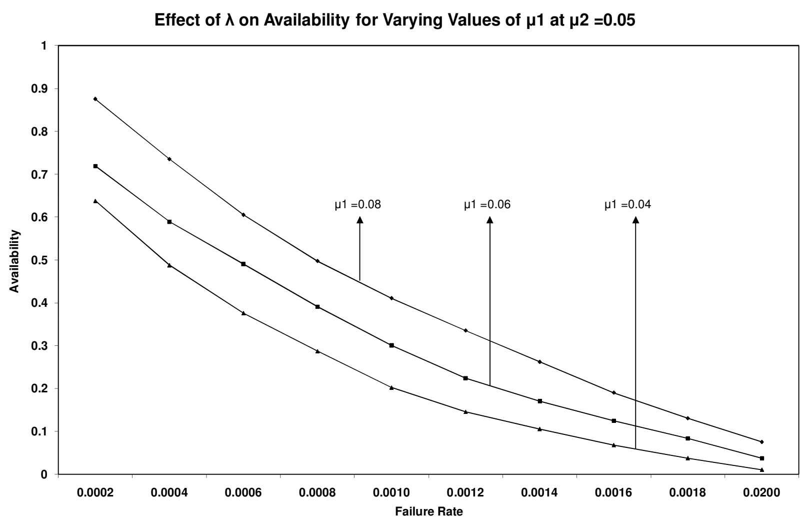
Effect of λ on Availability for Varying Values of μ1 at μ2 =0.05
Availability
Failure Rate
μ1 =0.08
μ1 =0.06
μ1 =0.04
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.0002
0.0004
0.0006
0.0008
0.0010
0.0012
0.0014
0.0016
0.0018
0.0200

Fig. 3

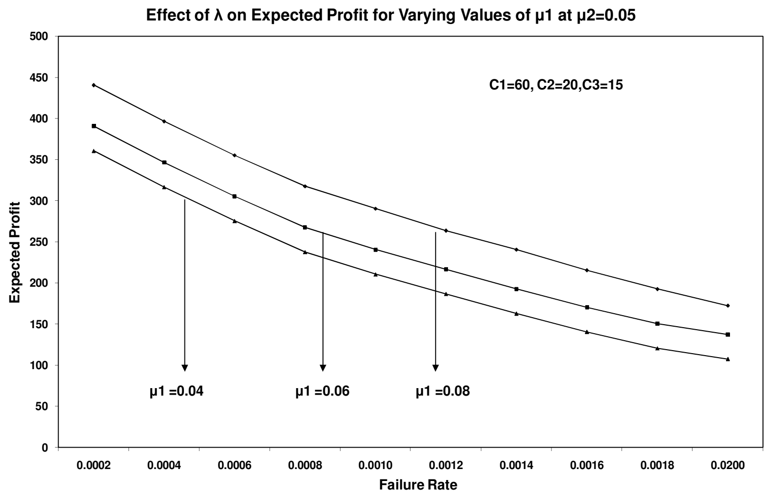

Fig. 4

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