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AN ALGORITHM FOR STOCHASTIC FRACTIONAL FUNCTIONAL TRANSPORTATION MODEL WITH SOME INADMISSIBLE ROUTES

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ABSTRACT

This study analyses a transportation type fractional programming problem with random demands and some or all inadmissible routes. In our model the random demand has not been replaced by its expectation but the probabilistic nature of the problem has been built into the problem formulation itself so that the system has the opportunity to take maximum advantage of the probability distribution of demand.

1. INTRODUCTION

At times there are transportation problems in which one of the sources is unable to ship to one or more of the destinations. When this type of problem occurs, then the problem is said to have an unacceptable or prohibited route or inadmissible route. In the conventional approach the demands are assumed to be fixed constants. However, in real life the demands are usually uncertain and have to be treated as random variables which create considerable complications. Complications are further enhanced if some of the routes are inadmissible i.e. prohibited due to one or the other reasons such as security, road construction, weight limits on bridges, unexpected floods, transportation strikes, bad road conditions and local traffic rules, etc. A very large cost is assigned to each of such routes, which are not available. To block the allocation to a cell with a prohibited route, we can cross out that cell.

Mou *et al.* (2013) considered a transportation problem, in which, the truck times and transportation costs are assumed as uncertain variables. Gabrel et al (2011) obtained a robust version of the location transportation problem with an uncertain demand using a 2-stage formulation. The obtained robust formulation is a convex (not linear) program, and they applied a cutting plane algorithm to exactly solve the problem.

In theoretical aspect, Gao (2009) proved some properties of continuous uncertain measure, and You (2009) gave some convergence theorems on uncertain sequences. Yang (2011) studied moment inequality of uncertain variable. Liu (2008) proposed uncertain process and uncertain differential equation, and Liu (2009) introduced uncertain calculus. Besides, Li and Liu (2009) proposed uncertain logic and Liu (2010) introduced uncertain inference to deal with uncertain knowledge. In practical aspect, uncertainty programming was proposed by Liu (2009) as a type of mathematical programming involving uncertain variables. It has been used to model system reliability design, project scheduling problem, vehicle routing problem, facility location problem, and so on.

So far, many researchers have done a lot of significant work in uncertain demand but none of the author except Currin (1986) has used prohibited routes. Only Shakeel (2008) and Javaid et al (2011) has used uncertainty with prohibited routes.

The purpose of this paper is to study a fractional transportation problem with uncertain demands and some or all prohibited routes. No doubt, the prohibited routes may sometimes cause infeasibility Currin (1986), but this study is concerned with feasible problems only. For dealing with uncertainty of demands, we have used the technique of Dantzig (1963) as applied by Shakeel and Gupta (2011).

2. FORMULATION OF THE PROBLEM

We consider a transportation problem with m sources and n sinks. The mathematical model of the problem considered here is of the following form

Problem P_1:

$$\text{Min} \quad Z = \frac{\sum_i \sum_j l_{ij}\, x_{ij} - \sum_j f_j(r_j, b_j)}{\sum_i \sum_j c_{ij}\, x_{ij}} \tag{2.1}$$

subject to

$$\sum_j x_{ij} = a_i, \qquad i = 1, 2, \cdots, m \tag{2.2}$$

$$\sum_i x_{ij} = d_j, \qquad j = 1, 2, \cdots, n \tag{2.3}$$

$$x_{ij} \geq 0 \qquad (\forall\, i, j) \tag{2.4}$$

where

$$l_{ij}, c_{ij}, r_j, a_i \geq 0 \qquad (\forall\, i, j)$$

and

$a_i =$ is the supply of a single homogeneous product available at origin i.

$x_{ij} =$ is the amount of the product transported from origin i to destination j.

$c_{ij} =$ the per unit transportation cost on the route (i, j).

$f_j =$ are the expected value functions to be determined according to the interpretation of the problem.

l_{ij} = per unit loss due to pilferage etc. on the route (i, j).

r_j = is the revenue received for each unit of demand satisfied at destination j.

$d_j =$ are independent random variables with following probability distribution j.

Demand d_j	$d_{1j} <$	$d_{2j} <$	...	$d_{H_j j}$
$p(b_j = b_{hj}) = p_{hj}$	p_{1j}	p_{2j}	...	$p_{H_j j}$
$p(b_j \geq b_{hj})$	$w_{1j} = \sum_{h=1}^{H_j} p_{hj}$	$w_{2j} = \sum_{h=2}^{H_j} p_{hj}$	...	$w_{H_j j} = p_{H_j j}$

(After using the above table, the value of the unknown function $f_j(r_j, d_j)$ so obtained is as follows (Javaid et al (2008, 2011).

$$f_j(r_j, b_j) = \sum_{h=1}^{H_j} r_j w_{hj} \, y_{hj} \tag{2.5}$$

3. DETERMINISTIC EQUIVALENT PROBLEM FORMULATION

Substituting the value of $f_j(r_j, b_j) = \sum_{h=1}^{H_j} r_j w_{hj} \, y_{hj}$ from (2.5) in (2.1), the objective functional becomes as

$$Z = \frac{\sum_i \sum_j l_{ij} \, x_{ij} + \sum_j \sum_h r_j w_{hj} y_{hj}}{\sum_i \sum_j c_{ij} \, x_{ij}} \tag{3.1}$$

If both x_{ij} and y_{hj} are treated as decision variables, the deterministic equivalent to Problem P_1, called the Problem P_2 is:

Problem P_2:

$$\text{Min} \quad Z = \frac{\sum_i \sum_j l_{ij}\, x_{ij} + \sum_j \sum_h r_j w_{hj} y_{hj}}{\sum_i \sum_j c_{ij}\, x_{ij}} = \frac{Z_1}{Z_2}, \tag{3.2}$$

where $(A_{hj} = -r_j w_{hj})$.

Subject to

$$\sum_j x_{ij} = a_i, \qquad i = 1, 2, \cdots, m \tag{3.3}$$

$$\sum_i x_{ij} - \sum_i y_{hj} = 0, \qquad j = 1, 2, \cdots, n \tag{3.4}$$

$$x_{ij}, y_{hj} \geq 0 \qquad \forall\ i, h, j \tag{3.5}$$

$$y_{hj} \leq U_{hj} \qquad \forall\ h, j \tag{3.6}$$

and subject to the additional stipulation y_{hj} as satisfied by Javaid et al (2008, 2011).

Fortunately, it turns out that y_{hj} do not restrict our choice of optimum solution in any way. This can be handled by the theorem as given by Javaid *et al.* (2008).

4. PRELIMINARIES TO THE SOLUTION OF PROBLEM P_4

i) It is assumed that the set of all feasible solutions of Problem P_2 is regular (i.e. non- empty and bounded) and that the denominator of the objective functional is positive for all feasible solution.

ii) Since the deterministic Problem P_2 is a transportation type fractional linear fractional programming problem with upper bound restrictions on some variables.

iii) A global minima to the Problem P_2 exists at a basic feasible solution to the capacitated system.

iv) As none of the constraints in the original system is redundant, a basic feasible solution to the original system shall contain $2(m+n)$ basic variables. For the capacitated system also, a basic feasible solution shall contain $2(m+n)$ basic variables, Dantzig (1963).

The special structure of Problem P_2, permits us to arrange it into an array as shown below, Garwin (1963).

Table 1:

x_{11} l_{11} c_{11}	x_{12} l_{12} c_{12}	……	x_{1n} l_{1n} c_{1n}	a_1
x_{21} l_{21} c_{21}	……	……	……	……
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_{m1} l_{m1} c_{m1}	……	……	……	a_m
y_{11} A_{11}	……	……	y_{1n} A_{1n}	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	
y_{H1} A_{H1}	……	……	y_{Hn} A_{Hn}	
0	0	……	0	

In the above table, there are $(m+H)$ rows where $H = \max . H_j$. Obviously there shall be some empty boxes near the bottom of the table which shall be crossed out. Absence of the row totals for y_{hj}'s in the table indicates that there are no row equations for y_{hj} variables. Besides, to obtain the column equations (3.4), each y_{hj} has to be multiplied by (-1). We have omitted (-1) from y_{hj} boxes for convenience.

5. INITIAL BASIC FEASIBLE SOLUTION

To start with, we fix the demands d_j's approximately equal to their expected values such that $\sum_{j=1}^{n} d_j = \sum_{i=1}^{m} a_i$ and also such that for all j except $j = j^*$, each d_j falls at the upper end of one of the intervals y_{hj} into which d_j has been divided, i.e., $d_j = \sum_{h=1}^{h'_j} U_{hj}$ for some $h'_j \le H_i$ and for all j except $j = j^*$.

With these fixed demands the upper portion of the Table 1 resembles a $m \times n$ standard transportation problem for which an initial basic feasible solution with $(m+n-1)$ basic variables is obtained by any of the several available methods. Now, in each of the columns, the values of the non basic y_{hj}'s are entered at their upper bounds in turn $h = 1, 2, \cdots$ until we have entered enough non basic y_{hj}'s so that their sum over h is equal to d_j. Obviously, we shall never have to enter y_{hj} below its upper bound except in column $j = j^*$, where the last nonzero entry will be $y_{hj^*} \le U_{hj^*}$. This last entry and the $(m+n-1)$ basic x_{ij}'s found earlier constitute the required initial basic feasible solution with $(m+n)$ basic variables. In case the last non zero entry in column j^* is also at its upper bound, then we take the last y_{hj} entry of any column as our $(m+n)-$th basic variable.

6. SIMPLEX MULTIPLIERS AND OPTIMALITY CRITERIA

Let the simplex multipliers corresponding to the objective function

$$Z_1 = \sum_i \sum_j l_{ij}\, x_{ij} + \sum_j \sum_h A_{hj} y_{hj} \text{ be } u_i \text{ and } v_j \quad (\forall\ i, j = 1, 2, \cdots, n)$$

and corresponding to the objective function

$$Z_2 = \sum_i \sum_j c_{ij}\, x_{ij} \;\; Z_2 \text{ be } \text{«}_i \text{ and } \text{³}_i \quad (\forall\ i, j = 1, 2, \cdots, m+n)$$

These are determined by solving the following equations.

$$\left.\begin{aligned} l_{ij} + u_i + v_j &= 0 \quad \text{for basic } x_{ij} \\ A_{hj} - v_j &= 0 \quad \text{for basic } y_{hj} \end{aligned}\right\} \qquad (6.1)$$

$$\left.\begin{aligned} c_{ij} + \text{«}_i + \text{³}_j &= 0 \quad \text{for basic } x_{ij} \\ - v_j &= 0 \quad \text{for basic } y_{hj} \end{aligned}\right\} \qquad (6.2)$$

Each of the system (6.1) and (6.2) have $2(m+n)$ linear equations in as many unknowns u_i, v_j, «_i and ³_j and can be easily solved. Let the relative cost coefficients corresponding to the variables x_{ij} and y_{hj} be l'_{ij} and A'_{hj} for Z_1 and C'_{ij} and B'_{hj} for Z_2.

These are determined by solving the following equations

$$\left.\begin{aligned} l'_{ij} &= l_{ij} + u_i + v_j \quad \text{for non basic } x_{ij} \\ A'_{hj} &= A_{hj} - v_j \quad \text{for non basic } y_{hj} \end{aligned}\right\} \tag{6.3}$$

$$\left.\begin{aligned} C'_{ij} &= c_{ij} + \text{«}_i + \text{³}_j \quad \text{for non basic } x_{ij} \\ U'_{hj} &= -v_j \quad \text{for non basic } y_{hj} \end{aligned}\right\} \tag{6.4}$$

The relative cost coefficients for basic variables and the values of the non basic x_{ij}'s are zero. As regards the values of non basic y_{hj}'s - some are zero and others at upper bounds.

It can be easily shown that for a given basic feasible solution (x_{ij}, y_{hj}) of the Problem P$_3$, the value of the objective function Z is

$$Z = \frac{\sum_{i=1}^{m+n}\sum_{j=1}^{m+n} l'_{ij}x_{ij} + \sum_{j=1}^{m+n}\sum_{h=1}^{H_j} A'_{hj}y_{hj} - \sum_{i=1}^{m} u_i(a_i)}{\sum_{i=1}^{m+n}\sum_{j=1}^{m+n} C'_{ij}x_{ij} + \sum_{j=1}^{m+n}\sum_{h=1}^{H_j} B'_{hj}y_{hj} - \sum_{i=1}^{m} \text{«}_i(a_i)} = \frac{Z_1}{Z_2} \tag{6.5}$$

But the relative cost coefficients for basic variables and also the values of the non basic x_{ij} are zero, but as regards the values of non basic y_{hj}'s - some are zero and the others are at their upper bounds. Hence,

$$Z = \frac{\sum_{i=1}^{m+n}{}^{*}\sum_{h=1}^{H_j}{}^{*} A'_{hj}U_{hj} - \sum_{i=1}^{m} u_i(a_i)}{\sum_{i=1}^{m+n}{}^{*}\sum_{j=1}^{H_j}{}^{*} B'_{hj}U_{hj} - \sum_{i=1}^{m} \text{«}_i(a_i)} = \frac{Z_1}{Z_2}, \tag{6.6}$$

where $\sum^{*}$ indicates the sum over those non basic y_{hj} which are at their upper bounds. Now if the value of any one of the non basic variables x_{st} or y_{rt} is changed to

$$\hat{x}_{st} = (x_{st} + \text{¯}) \quad \text{or} \quad \hat{y}_{rt} = (y_{rt} \pm \text{¯})$$

The improved value of Z can be obtained by Javaid *et al*. (2008, 2011) as follows

Thus, the current solution is optimum iff

$$\left.\begin{aligned} D_{ij} &\ge 0 \quad (\forall \text{ non basic } x_{ij}) \\ D'_{hj} &\ge 0 \quad (\forall \text{ non basic } y_{ij} \text{ at zero level}) \\ D'_{hj} &\le 0 \quad (\forall \text{ non basic } y_{hj} \text{ at upper bound}) \end{aligned}\right\} \tag{6.7}$$

where we define the following

$$\begin{cases} D_{ij} = l'_{ij} Z_2 - C'_{ij} Z_1 \\ D'_{hj} = A'_{hj} Z_2 - B'_{hj} Z_1 \end{cases}$$

If any of the optimality criteria (6.7) is violated, the current solution can be improved. The non basic variable which violates (6.7) most severely is selected to enter the basis. The values of the new basic variables are found by applying the usual θ-adjustments. It should, however, be kept in mind that the coefficient of each y_{hj} in column equations (4.8) is (-1). The variable to leave the basis is the one that becomes either zero or equal to its upper bound. If two or more basic variables reach zero or their upper bounds simultaneously then only one of them becomes non basic. Should it happen that the entering variable itself attains upper or lower bound (zero) without simultaneously making any of the basic variables zero or equal to its upper bounds, the set of basic variables remains unaltered; only their values are changed to allow the so-called entering variable to be fixed at its upper or lower bound.

7. ALGORITHM OF THE DETERMINISTIC PROBLEM

The step-by-step computational algorithm for determining the optimum solution is given as follows:

Step 1- First of all calculates initial/improved basic feasible solution and records them in a working table.

Step 2- Then obtain the values of simplex multipliers $(u_i, v_j,$ «$_i$ and $v_j)$ and the relative cost coefficients from the given equations (6.1), (6.2), (6.3) and (6.4).

Step 3- Calculate the value of the objective function $Z = Z_1 / Z_2$ by the equation (4.5).

Step 4- Then for the non-basic variables, calculate D_{ij} and D'_{hj} and test whether the solution is optimum or not. If yes, the process terminates and if not, proceed to find the D_{ij} (or D'_{hj}) which violates the optimality criteria (6.7), most severely.

Step 5- Find the entering variable as the one who's corresponding D_{ij} (or D'_{hj}) violates the optimality criteria most severely.

Step 6- Apply ¯ – adjustments and determine the outgoing variable (if any) and find the maximum value ¯ .

Step 7- Go to step 1.

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