Aligarh Journal of Statistics

Vol. 33 (2013), 119-128

ON EXTENDED ALTERNATIVE HYPER-POISSON DISTRIBUTION

C. Satheesh Kumar and B. Unnikrishnan Nair

ABSTRACT

Here we introduce an extended version of the alternative hyper-Poisson distribution of Kumar and Nair (Statistica, 2012a) and study some of its important properties. The maximum likelihood estimation of the parameters of this extended version is also discussed and demonstrated its usefulness with the help of some real life data sets. For S3 (2015), 119-128

ON EXTENDED ALTERNATIVE HYPER-POISSON DISTRIBUTION

C. Satheesh Kumar and B. Unnikrishnan Nair

ABSTRACT

Here we introduce an extended version of the alternative hyper-Poisson

distribution of Kum **EXAMPLE ALLER ATTAINTMENT**
 ABSTRA
 ABSTRA
 ABSTRA
 P
 P
 **EXECUTE ALLE ALLE ATTAINMENT ATTAINMENT ATTAINMENT ATTAINT AND properties. The maximum likelihood estimation

1. INTRODI**
 hyper-Poisson di ED ALTERNATIVE HYPER-POISSON DIS
 C. Satheesh Kumar and B. Unnikrishnan N:
 ABSTRACT
 C. Satheesh Kumar and B. Unnikrishnan N:
 ABSTRACT
 C. Kumar and Nair (Statistica, 2012a) and study some cemaximum likeliho **ENDED ALTERNATIVE HYPER-POISS**
 C. Satheesh Kumar and B. Unnikr:
 ABSTRACT

introduce an extended version of the all

in of Kumar and Nair (Statistica, 2012a) and studies

is. The maximum likelihood estimation of the **TENDED ALTERNATIVE HYPER-POISSON DISTRIBUTION**

C. Satheesh Kumar and B. Unnikrishnan Nair
 ABSTRACT

we introduce an extended version of the alternative hyper-Poisson

tion of Kumar and Nair (Statistica, 2012a) and st

1. INTRODUCTION

$$
f(x) = \frac{\Gamma(x)}{W(1; x;_{n})} \frac{x}{\Gamma(x+x)},
$$
\n(1.1)

C. Satheesh Kumar and B. Unnikrishnan Nair

C. Satheesh Kumar and B. Unnikrishnan Nair

Here we introduce an extended version of the alternative hyper-Poisson

distribution of Kumar and Nair (Statistica, 2012a) and stud C. Satheesh Kumar and B. Unnikrishnan Nair

MBSTRACT

Here we introduce an extended version of the alternative hyper-Poisson

distribution of Kumar and Nair (Statistica, 2012a) and study some of its important

properties. (Mathai and Haubold, (2008) or Slater, (1960)). When $x = 1$, the hyper-Poisson distribution reduces to Poisson distribution and when x is a positive integer, the distribution is known as the displaced Poisson distribution considered by Staff (1964). Bardwell and Crow (1964) termed the distribution as sub-Poisson when $x < 1$ and super–Poisson when $x > 1$. Various methods of estimation of the parameters of the distribution were discussed in Bardwell and Crow (1964) and Crow and Bardwell (1965). Some queuing theory with hyper-Poisson arrivals has been worked out by Nisida (1962). Roohi and Ahmad (2003a) attempted estimation of the parameters of the hyper-Poisson distribution using negative moments. Roohi and Ahmad (2003b) derived expressions for ascending factorial moments and further obtained certain recurrence relations for negative moments and ascending factorial moments of the hyper-Poisson distribution. Kemp (2002) developed q-analogue of the distribution and Ahmad (2007) introduced and studied Conway-Maxwell hyper-Poisson distribution. Kumar and Nair

(2011, 2012b) developed and studied certain modified form of the *HPD* . Kumar and Nair (2012a) considered a new version of hyper-Poisson distribution *C. Satheesh Kumar and B. Unnikrishnan Nair*
(2011, 2012b) developed and studied certain modified form of the *HPD*.
Kumar and Nair (2012a) considered a new version of hyper-Poisson distribution
namely 'the alternative hy () (12 %) C. Satheesh Kumar and

() (12 %) developed and studied certain modified

r and Nair (2012a) considered a new version of hyper

y'the alternative hyper-Poisson distribution (*AHPD*)

(y) = $\frac{x^y}{(x)_y}$ w(1+ y; 2b) developed and studied
Nair (2012a) considered a near
alternative hyper-Poisson dia
 $\frac{y}{(x)}$ w(1+ y;x + y; -_n)
2,..., x > 0, n > 0
 $a(a + 1)...(a + k - 1) = \Gamma(a + k)$
... and (a)₀ = 1. An interes *C. Satheesh Kumar and B.*

1, 2012b) developed and studied certain modified form

ar and Nair (2012a) considered a new version of hyper-Po

ely `the alternative hyper-Poisson distribution (*AHPD*)' wi
 $g(y) = \frac{y}{(x)}w(1 + y$ *C. Satheesh Kumar and B. Unnikrishi*
developed and studied certain modified form of the
(2012a) considered a new version of hyper-Poisson distribution
(*AHPD*)' with (*p.m.f*
 $w(1 + y; x + y; -$ _n)
 $x > 0$, $n > 0$
+1)...($a + k -$ C. Satheesh Kumar and B. Unnikrishnan Nair

112b) developed and studied certain modified form of the *HPD*.

Ind Nair (2012a) considered a new version of hyper-Poisson distribution

the alternative hyper-Poisson distribut 120

C. Satheesh Kumar and B. Unnikrishnan Nair

(2011, 2012b) developed and studied certain modified form of the *HPD*.

Kumar and Nair (2012a) considered a new version of hyper-Poisson distribution

namely 'the alternat

$$
g(y) = \frac{y}{(x)_y} W(1 + y; x + y; -x)
$$
 (1.2)

for $y = 0, 1, 2, \dots, x > 0$, ≤ 0

under- dispersed when $x < 1$ and over dispersed when $x > 1$.

for 11, 2012b) developed and studied certain modified form of the *HPD*.

Kumar and Nair (2012a) considered a new version of hyper-Poisson distribution

mamely 'the alternative hyper-Poisson distribution (*AHPD*)' with (Through this paper, we obtain an extended version of the alternative hyper- Poisson distribution which we call "the extended alternative hyper-Poisson distribution" or in short "the *EAHPD* ". In section 2 we establish that the *EAHPD* possess a random sum structure and it is shown that both the Hermite and generalized Hermite distributions are special cases of the *AHPD* . In section 3, we derive the explicit expressions for its probability mass function, mean and variance. Certain recurrence relations for probabilities, raw moments and factorial moments are also obtained in the section. The maximum likelihood estimation of the parameters of the *EAHPD* has been discussed in section 4 and section 5 contains some numerical illustrations for emphasizing the usefulness of the model. *AHPD* with $\pi = 0, 1, 2, ..., X > 0$, $\pi > 0$

And $(a)_k = a(a+1)...(a+k-1) = \Gamma(a+k)/\Gamma(a)$,
 π or $k = 1, 2, ...$ and $(a)_0 = 1$. An interesting property of the *AHPD* is that it is
 π inder-dispersed when $x < 1$ and over dispersed when *f* = 0, 1, and (*a*), θ = 0, θ , θ = 0, θ , θ = 1, An interesting property of the *AHPD* is that it is $k = 1, 2,...$ and (*a*)₀ = 1. An interesting property of the *AHPD* is that it is θ -1; cased when $x >$ for $k = 1, 2, ...$ and $(a)_0 = 1$. An interesting property of the *AHPD* is that it is
under-dispersed when $x < 1$ and over dispersed when $x > 1$.
Through this paper, we obtain an extended version of the alternative hyper-Pois en $x < 1$ and over dispersed when $x > 1$.

we obtain an extended version of the alternative hyper-Poisson

which we call "the extended alternative hyper-Poisson

short "the *EAHPD*". In section 2 we establish that the

fr () (1) . *^m Q t t t* (2.2) Poisson distribution which we call "the extended alternative hyper-Poisson

distribution" or in short "the *EAHPD*". In section 2 we establish that the
 EAHPD possess a random sum structure and it is shown that both the and generalized Hermite distributions are special cases of the *AHPD*. In section

3, we derive the explicit expressions for its probabilities, raw moments and

actionial moments are also obtained in the section. The maxi AHPD. In section
unction, mean and
aw moments and
ximum likelihood
ed in section 4 and
ing the usefulness
 $(p.g.f.)$
(2.1)
distributed random
a positive integer.
(2.2)
 $^{-1}$. Suppose that
 $S_y = \sum_{n=0}^{y} Z_n$. Then syrive the expense to the product variance. Certain recurrence relations for probabile
factorial moments are also obtained in the section.
estimation of the parameters of the *EAHPD* has been
section 5 contains some numer

2. GENESIS AND SPECIAL CASES

Consider a non-negative integer valued random variable *y* following the

$$
G(t) = w[1; x; \, (t-1)].
$$
\n(2.1)

variables, where Y_n has the following $p.g.f.,$ in which m is a positive integer.

$$
Q(t) = \Gamma t + (1 - \Gamma)t^m. \tag{2.2}
$$

0 $=\sum_{n=0}^{Y} Z_n$. Then *y* following the
 $(p.g.f.)$

(2.1)

stributed random

positive integer.

(2.2)

¹. Suppose that
 $y = \sum_{n=0}^{Y} Z_n$. Then frain infinition of the parameters of the *EAHPD* has been
ination of the parameters of the *EAHPD* has been
on 5 contains some numerical illustrations for ϵ
e model.
2. **GENESIS AND SPECIAL (**
sider a non-negative int contains some numerical illustrations for emphasizing
del.
2. **GENESIS AND SPECIAL CASES**
a non-negative integer valued random variable y
with $p.m.f.$ (1.2) and probability generating function (p ,
=w[1; X ;, (t - 1)].

$$
= G(Q(t))
$$

= $W[1;X;_{n} (t-1) +_{n} (t^{m} - 1)].$ (2.3)

Godifieralive hyper-poisson distribution
 $G(Q(t))$
 $= W[1; X;_{n_1}(t-1) +_{n_2}(t^m - 1)].$
 Godifierality (*to time p.g.f.* (2.3) as 'the distribution' or in short 'the *EAHPD*'. Cleast the *AHPD*. When $X = 1$ the EAHPD with *Led alternative hyper-poisson distribution* 121
 $= G(Q(t))$
 $= \mathbb{W} [1; x;_{s1}(t-1) +_{s2}(t^m - 1)]$. (2.3)

i.e. a distribution with p.g.f. (2.3) as 'the extended alternative hyper-

distribution of the *EAHPD* '. Clearly, *EAH* On extended alternative hyper-poisson distribution 121

= $G(Q(t))$

= $W[1; X;_{s1}(t-1) +_{s2}(t^m-1)]$. (2.3) as 'the extended alternative hyper-

Poisson distribution' or in short 'the *EAHPD* '. Clearly, *EAHPD* with *m* = 1 o $x_2 = 0$ is the *AHPD*. When $x = 1$ the EAHPD with p.g.f. (2.3) reduces to the *p g f* . . . of generalized Hermite distribution of Gupta and Jain (1974) and when *On extended alternative hyper-poisson distribution* 121
 $= G(Q(t))$
 $= W[1; x;_{*1}(t-1) +_{*2}(t^m - 1)].$ (2.3)

We define a distribution with *p.g.f.* (2.3) as 'the extended alternative hyper-

Poisson distribution' or in sh distribution of Kemp and Kemp (1965). On extended alternative hyper-poisson distribution
 $= G(Q(t))$
 $= w[1(x;_{x+1}(t-1) +_{x+2}(t^m-1))]$. (2.3)

We define a distribution vith $p.g.f.$ (2.3) as 'the extended alternative hyper-

Poisson distribution or in short 'the *EAH* native hyper-poisson distribution

(1))

(1))

(1))

(1) $\binom{n}{1}(t-1) + \binom{n}{2}(t^m-1)$].

(1) Tribution with p.g.f. (2.3) as

ion' or in short 'the EAHPD'.

(1PD. When $x = 1$ the EAHPD

(1) Tribution of (2),

(1) the EAHPD *alternative hyper-poisson distribution*
 $G(Q(t))$
 $V[1; x;_{n_1}(t-1) +_{n_2}(t^m-1)].$

a distribution with $p.g.f.$ (2.3) as 'the

ribution' or in short 'the *EAHPD*'. Cle

le *AHPD*. When $x = 1$ the EAHPD wi

eneralized Hermite $G(Q(t))$
 $= w[1/x, \frac{1}{x}, \frac{1}{y}(-1) + \frac{1}{x}, \frac{y}{y}(-1)]$. (2.3)

We define a distribution with $p_x g_x f$. (2.3) as 'the extended alternative hyper-

Poisson distribution of in short the *PAHPD* \cdot Checkly, *EAHPD* with $m = 1$ **Result 3.1** The probability mass function (. .) $\sum_{k=0}^{\infty} 1$ is $\sum_{k=0}^{\infty} 1$ is the *AHPD* \cdot Clearly, *EAHPD* with $m = 1$ or $\nu_2 = 0$ is the *AHPD*. When $x = 1$ the *EAHPD* with p_xg , f , of generalized He *P* define a distribution with p,g,f . (2.3) as 'the extended alternative hyper-
 p ois the *PAHPD* V. Clearly, *EAHPD* with $m = 1$ or
 p,g,f , of the *HPD* When $x = 1$ the *EAHPD* with p_g,f . (2.3) reduces to the
 p,g,f , *n* m m and m a latter than the EAHPD with p.g.f. (2.3) reduces to the ized Hermite distribution of Gupta and Jain (1974) and when
i, the *EAHPD* reduces to the *p.g.f.* (2.3) reduces to the *p.g.f.* of the Hermite mp *i* the *AHPD*. When *x* = 1 the EAHPD with p.g.f. (2.3) reduces to the *f* generalized Hermite distribution of Gupta and Jain (1974) and when *m* = 2, the *EAHPD* reduces to the *p.g.f.* of the Hermite on of Kemp and Ke e *AHPD*. When $x = 1$ the EAHPD with p.g.f. (2.3) reduces to the
eneralized Hermite distribution of Gupta and Jain (1974) and when
 $m = 2$, the *EAHPD* reduces to the $p.g. f$, of the Hermite
of Kemp and Kemp (1965).
3. **SOM**

3. SOME IMPORTANT PROPERTIES

In this section we obtain some important properties of the *EAHPD* . Let *U* be a

p.g.f. of generalized Hermite distribution of Gupta and Jain (1974) and when
\n
$$
x = 1
$$
 and $m = 2$, the *EAHPD* reduces to the *p.g.f.* of the Hermite
\ndistribution of Kemp and Kemp (1965).
\n**3. SOME IMPORTANT PROPERTIES**
\nIn this section we obtain some important properties of the *EAHPD*. Let *U* be a
\nrandom variable following the *EAHPD* with *p.g.f.*
\n
$$
H(t) = w[1;x;_{s+1}(t-1) +_{s+2}(t^m-1)].
$$
\n
$$
= \sum_{n=0}^{\infty} h_n(1,x) t^n,
$$
\n(3.1)
\nin which $h_n(1,x) = P(U = n)$, $n = 0,1,2,...$ On expanding (3.1) and equating the
\ncoefficients of t^n , we get the following result.
\n**Result 3.1** The probability mass function $(p.m.f.)$ $h_n(1,x)$ of the *EAHPD* with
\n $p.g.f.$ (3.1) is the following, in which $u = n - (m-1)k$.
\n
$$
h_n(1,x) = \sum_{k=0}^{\left[\frac{n}{m}\right]} \frac{u!}{(x)_0} w[1+u;x+u;-(x_1+x_2)] \frac{x^{n-mk} - k^k}{(n-mk)!k!},
$$
\n(3.2)
\nfor $n = 0, 1, 2, ..., (a)_r = a(a+1)...(a+r-1)$, for $r \ge 1$, such that $(a)_0 = 1$ and
\n
$$
[k]
$$
 denote the integer part of *k*. Further we obtain the following results.
\n**Result 3.2** The mean and variance of the *EAHPD* with *p.g.f.* (3.1) are
\n
$$
E(U) = \frac{1}{x} \left(a_1 + m_{n2} \right),
$$
\n
$$
Var(U) = \frac{1}{x} \left(\frac{2}{x+1} - \frac{1}{x} \right) \left(a_1 + m_{n2} \right)^2 + \frac{1}{x} \left(a_1 + m_{n2}^2 \right),
$$
\n(3.3)
\n
$$
Var(U) = \frac{1}{x}
$$

coefficients of t^n , we get the following result.

$$
= \sum_{n=0}^{\infty} h_n(1, x) t^n,
$$
\n(3.1)
\nwhich $h_n(1, x) = P(U = n), n = 0, 1, 2, ...$ On expanding (3.1) and equating the
\nficients of t^n , we get the following result.
\n**ult 3.1** The probability mass function $(p.m.f.) h_n(1, x)$ of the *EAHPD* with
\n $f.$ (3.1) is the following, in which $u = n - (m - 1)k$.
\n
$$
h_n(1, x) = \sum_{k=0}^{\left\lfloor \frac{n}{m} \right\rfloor} \frac{u!}{(x)_u} w[1 + u; x + u; -(x_1 + x_2)] \frac{x_1^{n - mk} x_2^k}{(n - mk)!k!},
$$
\n(3.2)
\n $n = 0, 1, 2, ..., (a)_r = a(a + 1)...(a + r - 1)$, for $r \ge 1$, such that $(a)_0 = 1$ and
\ndenote the integer part of k . Further we obtain the following results.
\n**ult 3.2** The mean and variance of the *EAHPD* with $p.g.f.$ (3.1) are
\n
$$
E(U) = \frac{1}{x} (x_1 + m_{n2}),
$$
\n
$$
Var(U) = \frac{1}{x} \left(\frac{2}{x + 1} - \frac{1}{x} \right) (x_1 + m_{n2})^2 + \frac{1}{x} (x_1 + m_{n2})^2,
$$
\n(3.3)
\nproof is simple and hence omitted.
\n**mark 3.1** Result 3.2 shows that the *EAHPD* is over-dispersed (that is, mean
\nthan variance) when $x > 1$ and under dispersed when $x < 1$ and satisfying
\nfollowing inequality, for $m > 1$ and $x_2 > 0$.

 $\lfloor k \rfloor$ denote the integer part of k . Further we obtain the following results.

$$
E(U) = \frac{1}{\mathsf{x}} \left(\mathsf{x}_1 + \mathsf{m}_{\mathsf{x}_2} \right),\tag{3.3}
$$

$$
Var(U) = \frac{1}{x} \left(\frac{2}{x+1} - \frac{1}{x} \right) \left(\frac{1}{x+1} + m_{n} \right)^2 + \frac{1}{x} \left(\frac{1}{x+1} + m_{n} \right)^2, \tag{3.4}
$$

The proof is simple and hence omitted.

Remark 3.1 Result 3.2 shows that the *EAHPD* is over-dispersed (that is, mean less than variance) when $x > 1$ and under dispersed when $x < 1$ and satisfying the following inequality, for $m > 1$ and $_{n_2} > 0$.

$$
\frac{(1-x)}{x(x+1)}\binom{m}{x+1} + m_{n-2}^2 > m(m-1)_{n-2}
$$

C. Satheesh Kumar and $1 + m_{n_2}^2 > m(m-1)_{n_2}^2$
recurrence relation for the probabilities $h_r(1, 3.1)$, for $n \ge (m-1)$ is C. Satheesh Kumar and
 $\frac{(1-x)}{(x+1)}\binom{n}{1} + m_{n/2}^2 > m(m-1)_{n/2}$
 lt 3.3 A recurrence relation for the probabilities $h_r(1, p.g.f. (3.1),$ for $n \ge (m-1)$ is
 $n+1)h_{n+1}(x^*) = d_0 \binom{n}{1}h_n(x^*+1) + m_{n/2}h_{n-m+1}(x^*+1)$, $\frac{(1-x)}{(x + 1)} (\binom{n}{1} + m_{n/2})^2 > m(m-1)_{n/2}$
 t 3.3 A recurrence relation for
 $p.g.f.$ (3.1), for $n \ge (m-1)$ is
 $n+1)h_{n+1}(x^*) = d_0 \left\{ \binom{n}{n} h_n(x^* + 1) + \binom{n}{n+1} h_n(x^*) \right\}$ *C. Satheesh Kumar and B. Unnikrishnan Nair*
 $m_{n_2}^2$ $> m(m-1)_{n_2}$

urrence relation for the probabilities $h_r(1,x)$ of the *EAHPD*

, for $n \ge (m-1)$ is
 $= d_0 \left\{ {}_{n_1}h_n(x^* + 1) + m_{n_2}h_{n-m+1}(x^* + 1) \right\},$ (3.5) *C. Satheesh Kumar and B. Unnikrishnan*
 $\frac{x}{x+1}$ ($_{n1} + m_{n2}$)² > $m(m-1)_{n2}$
 3 A recurrence relation for the probabilities $h_r(1,x)$ of the *EAF*
 f. (3.1), for $n \ge (m-1)$ is *C. Satheesh K*
 $\frac{(1-x)}{x(x+1)} (n_1 + m_{n_2})^2 > m(m-1)_{n_2}$
 alt 3.3 A recurrence relation for the probabili
 p.g.f. (3.1), for $n \ge (m-1)$ is
 $(n+1)h_{n+1}(x^*) = d_0 \left\{ n_1 h_n(x^* + 1) + m_{n_2} h_{n-m+1}(x^*) \right\}$ C. Satheesh Kumar and B. Unnikrishnan Nair
 $+m_{n_2}^2 > m(m-1)_{n_2}^2$

ecurrence relation for the probabilities $h_r(1,x)$ of the *EAHPD*

1), for $n \ge (m-1)$ is **Result 3.3** A recurrence relation for the probabilities $h_r(1,x)$ of the *EAHPD*
with $p.g. f$. (3.1), for $n \ge (m-1)$ is
($n+1)h_{n+1}(x^*) = d_0 \left\{ \prod_{r=1}^k h_r(x^*+1) + m_{r,2}h_{n-m+1}(x^*+1) \right\},$
(3.5)
in which $x^* = (1,x), x^* + 1 = (2,x+1$ *C. Satheesh Kumar and B. Unnikrishnan Nair*
 $\frac{(1-x)}{x(x+1)}\binom{1}{r+1} + m_{r,2}^2 > m(m-1)_{r,2}$
 Result 3.3 A recurrence relation for the probabilities $h_r(1,x)$ of the *EAHPD*

with *p.g.f.* (3.1), for $n \ge (m-1)$ is
 $(n+1)h_{$ C. Satheesh Kumar and B. Unnikrishnan
 $+m_{n_2}^2 > m(m-1)_{n_2}$

currence relation for the probabilities $h_r(1,x)$ of the EAI

(1), for $n \ge (m-1)$ is
 $\binom{*}{n_1} h_n(x^* + 1) + m_{n_2} h_{n-m+1}(x^* + 1)$,

(1), $x^* + 1 = (2, x + 1)$ and $d_$ C. Satheesh Kumar and B. Unnikrishnan Nair
 $\frac{(1-x)}{x(x+1)}(s_1 + m_{s_2})^2 > m(m-1)_{s_2}$
 11t 3.3 A recurrence relation for the probabilities $h_r(1,x)$ of the *EAHPD*
 $p.g. f. (3.1)$, for $n \ge (m-1)$ is
 $(n+1)h_{n+1}(x^*) = d_0\{s_1h_n$

$$
(n+1)h_{n+1}(\mathbf{x}^*) = d_0\left\{l_1h_n(\mathbf{x}^*+1) + m_{n-2}h_{n-m+1}(\mathbf{x}^*+1)\right\},\tag{3.5}
$$

 $d_0 = x^{-1}$.

Proof On differentiating (3.1) with respect to *t*, we get the following.

22. *C. Satheesh Kumar and B. Unnikrishnan Nair*
\n
$$
\frac{(1-x)}{x(x+1)}(s_1 + m_{s_2})^2 > m(m-1)_{s_2}
$$
\n**Result 3.3** A recurrence relation for the probabilities $h_r(1,x)$ of the *EAHPD*
\nwith $p.g.f.$ (3.1), for $n \ge (m-1)$ is
\n $(n+1)h_{n+1}(x^*) = d_0\{s_1h_n(x^*+1) + m_{s_2}h_{n-m+1}(x^*+1)\},$ (3.5)
\nin which $x^* = (1,x), x^*+1 = (2,x+1)$ and $d_0 = x^{-1}$.
\n**Proof** On differentiating (3.1) with respect to *t*, we get the following.
\n
$$
\sum_{n=0}^{\infty} (n+1)h_{n+1}(x^*)t^n = \frac{1}{x}(s_1 + m_{s_2}t^{m-1})w [2;x+1;_{s_1}(t-1)+_{s_2}(t^m-1)]
$$
 (3.6)
\nReplacing *x* by *x* + 1 in (3.1) to obtain the following.
\n*w* [2;*x* + 1;_{s_1}(*t* - 1) + _{s_2}(*t*^m - 1)] =
$$
\sum_{n=0}^{\infty} h_n(x^* + 1)t^n,
$$
 (3.7)
\nRelations (3.6) and (3.7) together lead to the following relationship:
\n
$$
\sum_{n=0}^{\infty} (n+1)h_{n+1}(x^*)t^n = \frac{1}{x} \left\{ s_1 \sum_{n=0}^{\infty} h_n(x^* + 1)t^n + m_{s_2} \sum_{n=0}^{\infty} h_n(x^* + 1)t^{m+n-1} \right\}
$$
 (3.8)

Replacing x by $x + 1$ in (3.1) to obtain the following.

$$
W [2; x + 1;_{n=1}(t-1) +_{n=2}(t^{m} - 1)] = \sum_{n=0}^{\infty} h_n (x^{*} + 1)t^{n},
$$
\n(3.7)

Relations (3.6) and (3.7) together lead to the following relationship:

[2; 1; (1) (1)] (1) , *m n n t t h t* * * * 1 1 1 2 0 0 0 1 (1) () (1) (1) *n n m n n n n r n n n h t h t m h t* (3.8) **Result 3.4** For *ⁿ* ¹ , a recurrence relation for factorial moments * () *r* of the * * * ¹ 0 1 0 2 [] () (1) (1) ()! *m m r r r k r d m d k r k* , (3.9) 0 () 1

Now, on equating the coefficients of t^n on both sides of (3.8) we get (3.5).

EAHPD is

$$
\widetilde{r}_{[r+1]}(x^*) = d_{0^{\prime\prime}1}\widetilde{r}_{[r]}(x^*+1) + m \, d_{0^{\prime\prime}2} \sum_{k=0}^{m-1} {m-1 \choose k} \frac{r!}{(r-k)!} \widetilde{r}_{[r-k]}(x^*+1) \,, \tag{3.9}
$$

in which $\sim_{[0]}(x^*)=1$

Proof The factorial moment generating function $F_U(t)$ of the *EAHPD* with

$$
\sum_{n=0}^{\infty} (n+1)h_{n+1}(x^*)t^n = \frac{1}{x} (t_1 + m_{n+2}t^{m-1})w [2:x + 1; t_1(t-1) + t_2(t^m-1)]
$$
 (3.6)
Replacing x by x + 1 in (3.1) to obtain the following.

$$
w [2:x + 1; t_1(t-1) + t_2(t^m-1)] = \sum_{n=0}^{\infty} h_n(x^* + 1)t^n,
$$
 (3.7)
Relationship:
$$
\sum_{r=0}^{\infty} (n+1)h_{n+1}(x^*)t^n = \frac{1}{x} \left\{ \int_{x_1}^{\infty} \sum_{n=0}^{\infty} h_n(x^* + 1)t^n + m_{n+2} \sum_{n=0}^{\infty} h_n(x^* + 1)t^{n+n-1} \right\}
$$
 (3.8)
Now, on equating the coefficients of t^n on both sides of (3.8) we get (3.5).
Result 3.4 For $n \ge 1$, a recurrence relation for factorial moments $\sim_{[r]}(x^*)$ of the
EAHPD is

$$
\sim_{[r+1]}(x^*) = d_{0^{n+1}} \sim_{[r]}(x^* + 1) + m d_{0^{n+2}} \sum_{k=0}^{\infty} {m \choose k} \frac{r!}{(r-k)!} \sim_{k} \left\{ (x^*) + 1 \right\},
$$
 (3.9)
in which $\sim_{[0]}(x^*) = 1$
Proof The factorial moment generating function $F_U(t)$ of the *EAHPD* with
 $P_t g_t f_t$. (3.1) is the following

$$
F_U(t) = H(1+t)
$$

$$
= w [1;x; t_1 + t_2(1+t)^m - 1]
$$

$$
= \sum_{r=0}^{\infty} \sim_{[r]}(x^*) \frac{t^n}{n!}
$$
 (3.10)
On differentiating (3.10) with respect to $\cdot t$ to obtain

$$
\frac{1}{x} [t_1 + m_{n+2}(1+t)^{m-1} w [2;x + 1; t_1 + t_{n+2}(1+t)^m - 1] = \sum_{r=0}^{\infty} \
$$

On differentiating (3.10) with respect to ' t ' to obtain

$$
\frac{1}{x} [r_1 + m_{n-2}(1+t)^{m-1}] W [2; x+1;_{n-1}t +_{n-2}(1+t)^m - 1] = \sum_{r=0}^{\infty} \sim [r_1(x^*) \frac{t^{n-1}}{(n-1)!} (3.11)
$$

By using (3.10) with x replaced by $x + 1$ we get the following from (3.11).

On extended alternative hyper-poisson distribution
\nBy using (3.10) with x replaced by x +1 we get the following from (3.11).
\n
$$
\sum_{r=0}^{\infty} \lceil_{r+1} |(x^*) \frac{t^n}{n!} = \frac{1}{x} [I_{r+1} + m_{r+2} (1+t)^{m-1}] \sum_{r=0}^{\infty} \lceil_{r} |(x^*) \frac{t^n}{n!} \rceil
$$
\n
$$
= \frac{1}{x} \cdot \sum_{r=0}^{\infty} \lceil_{r+1} |(x^*) \frac{t^n}{n!} + m_{r+2} \sum_{r=0}^{\infty} \sum_{r=0}^{m-1} {m-1 \choose k} \lceil_{r} |(x^*) \frac{t^{r+k}}{r!} \rceil
$$
\n(3.12)
\nOn equating coefficients of $(r!)^{-1} t^r$ on both sides of (3.12) we get (3.9).
\n**Result 3.5** For $r \ge 0$, a recurrence relation for the raw moments $\sim_r(x^*)$ of the *EAHPD* is
\n
$$
\sim_{r+1}(x^*) = \frac{1}{x} \sum_{j=0}^r {r \choose j} {r+1} \cdot m^{j+1} {r+2} \cdot \sim_{r-j}(x^*) + 1
$$
\n(3.13)
\n**Proof** The characteristic function $w_{U}(t)$ of the *EAHPD* with p,q,f . (3.1) has the following series representation. For $t \in R$,
\n
$$
w_{U}(t) = H(e^{it})
$$
\n
$$
= w [1; x; \frac{1}{x} (e^{it} - 1) + x (e^{mt} - 1)]
$$
\n
$$
= \sum_{r=0}^{\infty} \frac{1}{r} (x^*) \frac{(it)^r}{r!}
$$
\n(3.14)
\nOn differentiating (3.14) with respect to t , we obtain

On equating coefficients of $(r!)^{-1} t^r$ on both sides of (3.12) we get (3.9).

Result 3.5 For $r \ge 0$, a recurrence relation for the raw moments $\sim_r(x^*)$ of the *EAHPD* is

$$
\gamma_{r+1}(\mathbf{x}^*) = \frac{1}{\mathbf{x}} \sum_{j=0}^r \binom{r}{j} \binom{r}{r-1} + m^{j+1} \binom{r}{r-1} \gamma_{r-j}(\mathbf{x}^* + 1) \tag{3.13}
$$

$$
I_{11} = \frac{1}{x} \sum_{r=0}^{\infty} \frac{1}{r} \sum_{r=0}^{\infty} \frac{1}{r} \left[(x^{*} + 1) \frac{t^{n}}{n!} + m \right]_{r=2}^{\infty} \sum_{r=0}^{\infty} \frac{1}{n!} \left[(m-1) \frac{1}{r} \right] \left[(x^{*} + 1) \frac{t^{r+k}}{r!} \right]
$$
\n(3.12)
\n
$$
I_{22} = \frac{1}{x} \sum_{r=0}^{\infty} \frac{1}{r} \left[(x^{*})^{-1} t^{r} \text{ on both sides of (3.12) we get (3.9).}
$$
\n(3.13)
\n
$$
I_{33} = \sum_{r=0}^{\infty} \sum_{r=0}^{\infty} \left(\frac{t}{r} \right) (x_{1} + m^{t+1} \cdot x_{2}) \cdot x_{r-1} (x^{*} + 1) \right]
$$
\n(3.14)
\n
$$
I_{34} = \sum_{r=0}^{\infty} \sum_{r=0}^{\infty} \left(\frac{t}{r} \right) (x_{1} + m^{t+1} \cdot x_{2}) \cdot x_{r-1} (x^{*} + 1) \right]
$$
\n(3.15)
\n
$$
I_{33} = \sum_{r=0}^{\infty} \sum_{r=0}^{\infty} \left(\frac{t^{n}}{r} \right)
$$
\n(3.17)
\n
$$
= \sum_{r=0}^{\infty} \sum_{r} (x^{n} \cdot \frac{1}{r!} \right]
$$
\n(3.18)
\n
$$
I_{33} = \sum_{r=0}^{\infty} \sum_{r} (x^{n} \cdot \frac{1}{r!} \frac{1}{r!} \right]
$$
\n(3.19)
\n
$$
= \sum_{r=0}^{\infty} \sum_{r} (x^{n} \cdot \frac{1}{r!} \frac{1}{r!} \left[(x^{n} - 1) + x_{2} (e^{mt} - 1) \right] = \sum_{r=0}^{\infty} \sum_{r} (x^{n} \cdot \frac{1}{r-1}) \cdot (x^{n} - 1) \cdot (
$$

On differentiating (3.14) with respect to t , we obtain

$$
\frac{1}{x} \left(\int_{r_1} e^{it} + m_{n_2} e^{mit} \right) \mathbf{W} \left[2; x + 1; \int_{r_1} (e^{it} - 1) + \int_{r_2} (e^{mit} - 1) \right] = \sum_{r=1}^{\infty} \sim_r (x^*) \frac{(it)^{r-1}}{(r-1)!}
$$
\n(3.15)

=
$$
W [1; X;_{n_1}(e^{it} - 1) +_{n_2}(e^{mit} - 1)]
$$

\n= $\sum_{r=0}^{\infty} \sim_r (X^*) \frac{(it)^r}{r!}$ (3.14)
\nOn differentiating (3.14) with respect to *t*, we obtain
\n
$$
\frac{1}{X} ({}_{n_1}e^{it} + m_{n_2}e^{mit}) W [2; X + 1;_{n_1}(e^{it} - 1) +_{n_2}(e^{mit} - 1)] = \sum_{r=1}^{\infty} \sim_r (X^*) \frac{(it)^{r-1}}{(r-1)!}
$$
(3.15)
\nBy using (3.14) with x replaced by x + 1, we get the following from (3.15).
\n
$$
\sum_{r=0}^{\infty} \sim_{r+1} (X^*) \frac{(it)^r}{r!} = \frac{1}{X} ({}_{n_1}e^{it} + m_{n_2}e^{mit}) \sum_{r=0}^{\infty} \sim_r (X^* + 1) \frac{(it)^r}{r!}
$$

\n= $\frac{1}{X} \sum_{r=0}^{\infty} \left[{}_{n_1} \sum_{j=0}^{\infty} \frac{(it)^j}{j!} + m_{n_2} \sum_{j=0}^{\infty} \frac{(mit)^j}{j!} \right] \sim_r (X^* + 1) \frac{(it)^r}{r!}$ (3.16)
\nOn equating the coefficients of $(r!)^{-1}(it)^r$ on both sides we get (3.13).

On equating the coefficients of $(r!)^{-1}(it)^r$ on both sides we get (3.13).

4. MAXIMUM LIKELIHOOD ESTIMATION

In this section we consider the estimation of the parameters of the *EAHPD* by the method of maximum likelihood. Here we assume that m is a fixed known positive integer. Let a(x) be the observed frequency of *x* events and let *y* be the highest value of *n* observed. Then the likelihood function of the sample is **. MAXIMUM LIKE**

e consider the estima

aximum likelihood. I

Let a(x) be the observ

n observed. Then the

^{*})]^{$a(x)$}. **4. MAXIMUM LIKELIH**

ion we consider the estimation

of maximum likelihood. Here

eger. Let a(x) be the observed 1

ue of *n* observed. Then the like
 $[h_x(x^*)]^{a(x)}$.

arithm on both sides of (4.1) we
 $L = \sum_{x=0}^{k} a(x) \log[h_x$ *C. Sa*
 4. MAXIMUM LIKELIHO

ction we consider the estimation of

od of maximum likelihood. Here

nteger. Let a(x) be the observed fr

alue of *n* observed. Then the likel
 $\prod_{y=0}^{y} [h_x(x^*)]^{a(x)}$.

garithm on both sid *C. Satheesh Kt*
 4. MAXIMUM LIKELIHOOD ES

is section we consider the estimation of the pa

method of maximum likelihood. Here we assure

ive integer. Let a(x) be the observed frequency

est value of *n* observed. Then C. Satheesh Kumar and B. Umikrishnan Nair
 4. MAXIMUM LIKELIHOOD ESTIMATION

section we consider the estimation of the parameters of the *EAHPD* by

ethod of maximum likelihood. Here we assume that m is a fixed known

w

$$
L = \prod_{x=0}^{y} [h_x(\mathbf{x}^*)]^{a(x)}.
$$
 (4.1)

Taking logarithm on both sides of (4.1) we get

$$
l = \log L = \sum_{x=0}^{k} a(x) \log [h_x(\mathbf{x}^*)].
$$
 (4.2)

C. Satheesh Kun

4. **MAXIMUM LIKELIHOOD EST**

section we consider the estimation of the parallel

hod of maximum likelihood. Here we assum

integer. Let a(x) be the observed frequency c

value of *n* observed. Then the li *C. Satheesh Kumar and B. Un*
 4. MAXIMUM LIKELIHOOD ESTIMATION

is section we consider the estimation of the parameters of the

method of maximum likelihood. Here we assume that m is a

tive integer. Let $a(x)$ be the o Let \hat{i}_1 , \hat{i}_2 and \hat{x} denote the maximum likelihood estimators of i_1 , i_2 and x respectively. Now \hat{i}_1 , \hat{i}_2 and \hat{x} are computed by solving the following equations, obtained from (4.2) on differentiation with respect to $_{n_1}$, $_{n_2}$ and x respectively and equating to zero. (4.1) we get

(4.2)

imum likelihood estimators of $_{n_1}$, $_{n_2}$ and x
 \hat{x} are computed by solving the following

differentiation with respect to $_{n_1}$, $_{n_2}$ and x
 $_{2}W[1+u; x + u; -({}_{n_1} + {}_{n_2})] - \langle x \rangle = 0,$ (4. logarithm on both sides of (4.1) v

log $L = \sum_{x=0}^{k} a(x) \log[h_x(x^*)]$.

, \int_{R} and \hat{x} denote the maximum

ively. Now $\int_{n_1}^{\infty}$, $\int_{n_2}^{\infty}$ and \hat{x} are

nons, obtained from (4.2) on differe

ively and equati **4. MAXIMUM LIKELIHOOD ESTIMATION**
we consider the estimation of the parameters of the *EAHPD* by
maximum likelihood. Here we assume that m is a fixed known
for n Let a(x) be the observed frequency of *x* events and let **4. MAXIMUM LIKELIHOOD ESTIMATION**
section we consider the estimation of the parameters of the *EAHPD* by
ethod of maximum likelihood. Here we assume that m is a fixed known
ve integer. Let a(x) be the observed frequency **4. MAXIMUM LIKELIHOOD ESTIMATION**

is section we consider the estimation of the parameters of the *EAHPD* by

the the dof maximum likelihood. Here we assume that m is a fixed known

ive integer. Let a(x) be the observed C. Satheresh Kumar and B. Umikrishnan Nair
 4. MAXIMUM LIKELHIGOD ESTIMATION

is section we consider the estimation of the parameters of the *FAHPD* by

its section of maximum likelihood. Here we assume that m is a fixe (4.2)

naximum likelihood estimators of $_{n_1}$, $_{n_2}$ and $_{n_3}$

nd \hat{x} are computed by solving the followin

on differentiation with respect to $_{n_1}$, $_{n_2}$ and $_{n_3}$
 $_{n_2}$ w[1+u;x +u;-($_{n_1}$ +, $_{n_2}$)] and \hat{x} denote the maximum likel

y. Now \hat{n}_1 , \hat{n}_2 and \hat{x} are co

obtained from (4.2) on differentiat

y and equating to zero.
 $\sum_{z=0}^{y} \left[\frac{x}{m}\right] \Delta(x;k)\{(x-mk)_{n-2}w[1+u;x-1]\}$
 $\sum_{z=0}^{y} \sum_{k=0}^{\left[\frac{x}{m}\right]} \$ t a(x) be the observed frequency of x events and let y be the
observed. Then the likelihood function of the sample is
(4.1)
 $\varphi^{(x)}$. (4.1)
n both sides of (4.1) we get
(x)log[$h_x(x^*)$]. (4.2)
denote the maximum likelihoo *l* **d** of maximum likelihood. Here we assume that m is a fixed known

theger. Let a(x) be the observed frequency of *x* events and let *y* be the
 lue of *n* observed. Then the likelihood function of the sample is
 \int old of maximum likelihood. Here we assume that m is a fixed known

integr. Let a(x) be the observed frequency of x events and let y be the

alue of n observed. Then the likelihood function of the sample is
 $\sum_{z=0}^{\infty} [$ C. Satheesh Kumar and B. Umakrishnan Nair
 4. MAXIMUM LIKELIHOOD ESTIMATION

ection we consider the estimation of the parameters of the *FAHPD* by

ed of maximum likelihood. Here we assume that m is a fixed known

entin

$$
\frac{\partial l}{\partial_{n_1}} = \sum_{x=0}^{y} \sum_{k=0}^{\left[\frac{x}{m}\right]} \Delta(x;k) \{ (x-mk)_{n_2} \mathbf{W}[1+\mathbf{u}; \mathbf{x}+\mathbf{u}; -(n_1+n_2)] - \langle \mathbf{X} \rangle \} = 0, \qquad (4.3)
$$

$$
\frac{\partial l}{\partial_{n_2}} = \sum_{x=0}^{y} \sum_{k=0}^{\left[\frac{x}{m}\right]} \Delta(x;k) \{k_{n_1}W[1+u;x+u;-(x_{n_1}+x_{n_2})] - \langle X \rangle\} = 0, \tag{4.4}
$$

and

124 *C. Sahheesh Kumar and B. Umikrishnan Nair*
\n4. **MAXIMUM LIKELIHOOD ESTIMATION**
\nIn this section we consider the estimation of the parameters of the EAHPD by the method of maximum likelihood. Here we assume that m is a fixed known positive integer. Let a(x) be the observed frequency of x events and let y be the highest value of n observed. Then the likelihood function of the sample is\n
$$
L = \prod_{x=0}^{x} [h_x(x^x)]^{w(x)}.
$$
\n(4.1)
\nTaking logarithm on both sides of (4.1) we get\n
$$
l = \log L = \sum_{x=0}^{x} a(x) \log[h_x(x^x)].
$$
\n(4.2)
\nLet \hat{i}_1, \hat{i}_2 and *X* denote the maximum likelihood estimators of i_{-1}, i_{-2} and *X* respectively. Now \hat{i}_1, \hat{i}_2 and *X* are computed by solving the following equations, obtained from (4.2) on differentiation with respect to i_1, i_2 and *X* respectively and equating to zero.
\n
$$
\frac{\partial l}{\partial \hat{i}_1} = \sum_{x=0}^{x} \sum_{k=0}^{x} \Delta(x;k) \{ (x-mk)_x \cdot y[1+u; x+u;-(x_1+x_2)] - \langle x \rangle \} = 0, \qquad (4.3)
$$
\nand\n
$$
\frac{\partial l}{\partial x} = \sum_{x=0}^{x} \sum_{k=0}^{x} \Delta(x;k) \{ k_x \cdot y[1+u; x+u;-(x_1+x_2)] - \langle x \rangle \} = 0, \qquad (4.4)
$$
\nand\n
$$
\frac{\partial l}{\partial x} = \sum_{x=0}^{x} \sum_{k=0}^{x} \Delta(x;k) \{ w_1[1+u; x+u;-(x_1+x_2)] - \langle x \rangle \} = 0, \qquad (4.5)
$$
\nand\n
$$
\frac{\partial l}{\partial x} = \sum_{x=0}^{x} \sum_{k=0}^{x} \Delta(x;k) \{ w_1[1+u; x+u;-(x_1+x_2)] - \langle x \rangle \} = 0, \qquad (4.6)
$$
\nand\n
$$
\frac{\partial l}{\partial x} = \sum_{x=0}^{x} \sum_{k=0}^{x} \Delta(x;k) \{ w_1[1+u; x+u;-(
$$

$$
\Delta(x;k) = a(x)\frac{1}{h_x(x^*)} \frac{[n-(m-1)k]!_{n_1}^{n-mk-1} {n-1 \choose n_2}}{(n-mk)!k!(x)_{u}},
$$
\n(4.6)

 $1+U_{\text{m}+2}$ (1) $(1+U_{\text{m}})^2$ $\ddot{}$

5. CONCLUDING REMARKS

In section 2, we have shown that the *EAHPD* possess a random sum structure. Such random sum distributions have found extensive applications in several areas of scientific research. For a detailed account of random sum distributions refer chapter 9 of Johnson *et al*. (2005). Here we consider two real life data sets for demonstrating the estimation procedures discussed in section 4 and for illustrating the usefulness of the model. We have obtained maximum likelihood estimates and variances of the parameters of *EAHPD* for $m = 1, 2, 3, 4$ and On extended alternative hyper-poisson distribution 125
5. **CONCLUDING REMARKS**
In section 2, we have shown that the *EAHPD* possess a random sum structure.
Such random sum distributions have found extensive applications i using *MATHCAD* software and there by computed expected frequencies, t^2 values and $p -$ values. The results obtained are given in Table1 and 2. It can be observed from tables that both *HPD* and *AHPD* are not giving good fit to both data sets, where as the *EAHPD* with $m = 4$ gives the best fit in case of first data set and the *EAHPD* with $m=3$ gives the best fit in case of second data set compared to the existing model.

Acknowledgement

The authors are grateful to the Chief Editor and the anonymous referee for their valuable suggestions. The second author is particularly thankful to University Grants Commission, New Delhi, India for the financial support.

Table 1: Observed distribution of *Ribes* [Fracker and Brischle 1944] and the expected frequencies computed using *HPD* , *AHPD* and the *EAHPD* for $m = 2, 3, 4$ and 5.

Estimated value of parameters		$\hat{r}_1 = 4.39 \hat{r}_1 = 3.65$	$\hat{x} = 1.104$	$\hat{r}_1 = 0.436$ $\hat{r}_1 = 0.736$ $\hat{x} = 6.748 \hat{x} = 3.167 \hat{z} = 0.339 \hat{z} = 0.216$ $\hat{x} = 1.374$	\hat{i} = 1.671	$\hat{r}_1 = 33041$ $\hat{r}_1 = 42.938$ \hat{i} = 1.102 $\hat{x} = 36117$ $\hat{x} = 43.413$
$+^2$	8.13	49.694	9.012	207.13	1.869	2.742
P-value	0.004	0.014	0.011	0.097	0.172	0.098

Table 2: Observed distribution of the counts of Red mites on Apple leaves [P.Garman,1951] and the expected frequencies computed using *HPD* and the *AHPD* as well as the *EAHPD* for $m = 2$ and $m = 3$ by method of maximum likelihood.

REFERENCES

Ahmad, M. (2007): A short note on Conway-Maxwell-hyper Poisson distribution. *Pakistan J. Statist.*, **23**, 135-137.

Bardwell, G. E. and Crow, E. L. (1964): A two parameter family of hyper- Poisson distributions. *J. Amer. Statist. Assoc.*. **59**, 133-141.

Beall, G., and Rescia, R. R. (1953): A generalization of Neyman's contagious distributions. *Biometrics*, **9**, 354-386.

Crow, E.L. and Bardwell, G.E. (1965): *Estimation of the parameters of the hyper-Poisson distributions*. *Classical and Contagious Discrete Distributions*. G. P. Patil (editor), 127-140, Pergamon Press, Oxford.

Fracker, S. B. and Brischle, H. A. (1944): Measuring the local distribution of Ribes. *Ecology*, **25**, 283-303.

Gupta, R. P. and Jain, G. C. (1974): A generalized Hermite distribution and its properties. *SIAM J. Appl. Math.*, **27**, 359-363.

Hand, D. J. , Daly, F., Lunn, A.D., Mc. Conway, K. J. and Ostrowski, E. (1994): *A hand book of small data sets*, Chapman and Hall. London.

Kemp, C. D. (2002): q-analogues of the hyper-Poisson distribution. *J. Statist. Plann. Inference*. **101**, 179-183.

Kemp, C.D. and Kemp, A. W. (1965): Some properties of Hermite distribution, *Biometrika*. **52**, 381-394.

Kumar, C. S. and Nair, B. U. (2011): A modified version of hyper-Poisson distribution and its applications. *Journal of Statistics and Applications*. **6**, 23- 34.

Kumar, C. S and Nair, B.U. (2012a): An alternative hyper-Poisson distribution, *Statistica,* **72**(**3**), 357-369.

Kumar, C. S. and Nair, B.U. (2012b): An extended version of hyper Poisson distribution and some of its applications, *Journal of Applied Statistical Sciences* **19**(**1**), 81-88.

Mathai, A. M. and Haubold, H.J. (2008): *Special Functions of Applied Sciences*, Springer, NewYork.

Nisida, T. (1962): On the multiple exponential channel queuing system with hyper-Poisson arrivals. *J. Oper. Res. Soc. Japan*, **5**, 57-66.

Ong, S. H. and Lee, P. A. (1979): The noncentral negative binomial distribution. *Biometrics*, **21**, 611-627.

Riordan, J. (1968): *Combinatorial Identities*, Wiley, New York.

Roohi, A. and Ahmad, M. (2003a): Estimation of the parameter of hyper- Poisson distribution using negative moments. *Pakistan J. Statist.*, **19**, 99-105.

Roohi A. and Ahmad, M. (2003b): Inverse ascending factorial moments of the hyper-Poisson probability distribution. *Pakistan J. Statist.*, **19**, 273-280.

Slater, L. J. (1960): *Confluent Hypergeometric Functions*. Cambridge: Cambridge University Press.

Staff, P. J. (1964): The displaced Poisson distribution. *Australian Journal of Statistics*, **6**, 2-20.

Trivandrum-695 581, India E-mail: drcsatheeshkumar@gmail.com E-mail: ukswathy@gmail.com