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# **APPLICATION OF CONCOMITANTS OF ORDER STATISTICS OF INDEPENDENT NON-IDENTICALLY DISTRIBUTED BIVARIATE NORMAL RANDOM VARIABLES IN ESTIMATION**

T. G. Veena and P. Yageen Thomas

#### **ABSTRACT**

In this paper, we obtain the means, variances and covariances of order statistics arising from independent non-identically distributed bivariate normal random variables. A method of estimation of common parameters involved in several bivariate normal distributions using concomitants of order statistics is also discussed.

## **1. INTRODUCTION**

It is well known that order statistics are very useful in the estimation of location and scale parameters of a distribution. For a survey of literature on the applications of order statistics of *iid* random variables in estimating the location and scale parameters of distributions, see David and Nagaraja (2003) and Balakrishnan and Cohen (1991). Vaughan and Venables (1972) have first discussed about the distribution theory of order statistics of *inid* random variables. For some further results on the order statistics of *inid* random variables, see Beg (1991) and Samuel and Thomas (1998). Sajeevkumar and Thomas (2005) and Thomas and Sajeevkumar (2005) have illustrated some applications of order statistics of independent non-identically distributed random variables in the estimation of common location and scale parameters of several distributions.

In a bivariate setup, study of concomitants of order statistics of *iid* bivariate random variables has gained momentum in a theoretical perspective as well as in terms of its applications. For details, see Beg and Ahsanullah (2007), Chacko (2007), David and Nagaraja (1998) and Nagaraja and David (1994). However as in the case of order statistics of *inid* random variables, not much works have been initiated on the theory and applications of concomitants of order statistics of *inid* random variables. Eryilmaz (2005) introduced the general expression for the *cdf* cdf of concomitants of order statistics of *inid* bivariate random variables. Veena and Thomas (2011) have obtained the general expression for the *pdf* of concomitants of order statistics of *inid* random variables and the means, variances and covariances of order statistics arising from independent non-identically distributed bivariate Pareto distributions. They have also described a method of estimation of common parameters involved in several *Veena T. G. and P. Yageen Thomas*<br>variables. Veena and Thomas (2011) have obtained the general expression for<br>the *pdf* of concomitants of order statistics of *inid* random variables and the<br>means, variances and covarian  $(X_1, Y_1), (X_2, Y_2), ..., (X_n, Y_n)$  are *n* independent bivariate random variables with  $(X_i, Y_i)$  having an absolutely continuous bivariate distribution with *pdf*  $f_i(X_i, Y_i)$ ,  $i = 1, 2, \dots, n$ . If we order  $X_1, X_2, \dots, X_n$  involved in the above bivariate collection of random variables as  $X_{1:n}$ ,  $X_{2:n}$ ,  $X_{n:n}$ , then the accompanying *Y* value of  $X_{r,n}$  in the ordered pair from which  $X_{r,n}$  is taken is called the concomitant of the  $r -$ th order statistic and is denoted by  $Y_{[r:n]}$ . If we write  $F_{X_i}(x)$  to denote the marginal distribution function of  $X_i$  of the bivariate distribution function of the random variable  $(X_i, Y_i)$ ,  $i = 1, 2, ..., n$ then from Veena and Thomas (2011), we can write the *pdf*  $f_{Y_{[r,n]}}(y)$  of  $Y_{[r,n]}$  as  $f_r(X_r, Y_r)$ ,  $i = 1, 2, ..., n$ . If we order  $X_1, X_2, ..., X_n$  involved in the above<br>bivariate collection of random variables as  $X_{1n}, X_{2n}, ..., X_{nn}$ , then the<br>accompanying *Y* value of  $X_{rn}$  in the ordered pair from which  $X_{rn}$  is

$$
f_{Y_{[r:n]}}(y) = \frac{1}{(r-1)!(n-r)!} \int_{-\infty}^{\infty} Per \begin{bmatrix} F_{X_1}(x) & 1 - F_{X_1}(x) & f_1(x, y) \\ \vdots & \vdots & \vdots \\ F_{X_n}(x) & \underbrace{1 - F_{X_n}(x)}_{n-r} & \underbrace{f_n(x, y)}_{n} \end{bmatrix} dx \quad (1)
$$

where per *A* is meant to denote the permanent of a square matrix *A* which is just like the determinant of *A* except that in per *A* all terms in its expansion are taken with positive sign and if a symbol *k* is marked below a column vector *a* in per *A* then it means that *A* includes *k* copies of *a* .

and *Y*[*s*:*n*] has been developed by Veena and Thomas (2011) and is given by,

$$
f_{Y_{[r,s:n]}}(y,z) = \frac{1}{(r-1)!(s-r-1)!(n-s)!}I
$$

where

$$
I = \iint_{u\n(2)
$$

In this work our main interest lies in establishing an application of the above theory of distribution of concomitants of order statistics of *inid* random variables in the estimation of common parameters involved in several bivariate normal distributions.

In section 2, we have considered the problem of estimation of the common correlation coefficient ... (ie., when  $\ldots$  = ...,  $i = 1, 2, \ldots, n$ ) involved in several bivariate normal distributions with different *<sup>i</sup>* 's using concomitants of *inid* normal random variables. In section 3, we consider concomitants of order statistics arising from several bivariate normal distributions with Application of concomitants of order statistics ....... estimation 85<br>
In this work our main interest lies in establishing an application of the above<br>
heory of distribution of concomitants of order statistics of *inid* r values of the correlation coefficient. Further we illustrate an application of concomitants of order statistics of *inid* normal random variables in estimating the parameters  $\sim_2$  and  $\uparrow_2$ . Application of concomitants of order statistics ....... estimation<br>
In this work our main interest lies in establishing an application of the above<br>
verivatives of inidd random<br>
variables in the estimation of common param (*n*) the set of distribution of the extendinates  $x_i$  in the set of distribution of concomitants of order statistics variables in the estimation of concomitants of order statistics variables in the estimation of common p In section 2, we have considered the problem of<br>correlation coefficient ... (ie., when  $\frac{1}{m_i} = \frac{1}{m_i}i = 1,2$ <br>bivariate normal distributions with different  $\uparrow_i$ 's u<br>normal random variables. In section 3, we consist<br>s of estimation of the common<br>1,2,..., *n*) involved in several<br>'s using concomitants of *inid*<br>nonsider concomitants of order<br>normal distributions with<br>..,*n*, but with different known<br>we illustrate an application of<br>rando bivariate normal distributions with different  $\uparrow$ ,'s using concomitants of *inid*<br>normal random variables. In section 3, we consider concomitants of order<br>statistics arising from several bivariate normal distributions w we consider concomitants of order<br>
variate normal distributions with<br>  $i = 1, 2, ..., n$ , but with different known<br>
urther we illustrate an application of<br>
normal random variables in estimating<br> **ION OF** ...<br>
In bivariate rando

### **2. ESTIMATION OF**

$$
\frac{(2f\uparrow_{1}\uparrow_{2i})^{-1}}{\sqrt{1-\dots^{2}}}\exp\left\{\frac{-2^{-1}}{1-\dots^{2}}\left[\frac{(x-\sim_{1})^{2}}{\uparrow_{1}^{2}}-2\dots\frac{(x-\sim_{1})(y-\sim_{2i})}{\uparrow_{1}\uparrow_{2i}}+\frac{(y-\sim_{2i})^{2}}{\uparrow_{2i}^{2}}\right]\right\}
$$
(3)

Clearly the marginal distributions of  $X_i$  and  $Y_i$  are  $N(\sim_1, \uparrow_1)$  and  $N(\sim_2, \uparrow_2)$ function respectively of each of the  $X_i$  's and  $h_i(y|x)$  denote the conditional *pdf* of  $Y_i$  given  $X_i = x$ . Then we have,

Thus, 
$$
u = r_1, \frac{1}{2} \ln \left( \frac{1}{2} \right) = \frac{1}{2} \ln \left( \frac{1}{2} \ln \left( \frac{1}{2} \right) = \frac{1}{2} \ln \left( \frac{1}{2} \ln \left( \frac{1}{2} \right) = \frac{1}{2} \ln \left( \frac{1}{2} \ln \left( \frac{1}{2} \right) = \frac{1}{2} \ln \left( \frac{1}{2} \ln \left( \frac{1}{2} \right) = \frac{1}{2} \ln \left( \frac{1}{2} \ln \left( \frac{1}{2} \right) = \frac{1}{2} \ln \left( \frac{1}{2} \ln \left( \frac{1}{2} \ln \left( \frac{1}{2} \right) \right) = \frac{1}{2} \ln \left( \frac{1}{2} \ln \left( \frac{1}{2} \ln \left( \frac{1}{2} \ln \left( \frac{1}{2} \right) \right) \right) = \frac{1}{2} \ln \left( \frac{1}{2} \right) \right) \right) \right) = \frac{1}{2} \ln \left( \frac{1}{
$$

We know that

$$
\int_{-\infty}^{\infty} yh_i(y \mid x) dy = -\frac{1}{2i} + \dots + \frac{1}{2i} \left( \frac{x - z_1}{\frac{1}{z_1}} \right).
$$

Hence,

*Veena T. G. and P. Yageen Thomas*  
\n
$$
\int_{-\infty}^{\infty} yh_i(y \mid x) dy = -2i + ... + 2i \left( \frac{x - x_1}{1} \right).
$$
\n  
\nce,  
\n
$$
E[Y_{[r:n]}] = \frac{1}{n} \sum_{i} \left( -2i + ... + 2i \right) \left( -2i + ... + 2i \right) \left( -2i + ... \right)
$$
\n  
\nre we write  $\Gamma_{r:n}$  to denote the expected value of the  $r^{th}$  order statistic  $U_{rn}$   
\nng from a random sample of size *n* from  $N(0,1)$ .  
\n
$$
E[Y_{[r:n]}^2] = \int_{0}^{\infty} y^2 f_{r-1}(y) dy
$$

where we write  $\Gamma_{r:n}$  to denote the expected value of the  $r^{th}$  order statistic  $U_{r:n}$ 

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\n
$$
\text{Mean } P, \text{ Yageen Thomas}
$$
\n
$$
\int_{-\infty}^{\infty} yh_y(y|x)dy = -2x + ... + 2\left[\frac{x - z_1}{1 + \frac{1}{2}}\right]
$$
\nHence,  
\n
$$
E[Y_{(pq)}] = \frac{1}{n} \sum_{i=1}^{n} (-2x + ... + 2x)^{n-1} = 0
$$
\n
$$
E[Y_{(pq)}] = \frac{1}{n} \sum_{i=1}^{n} (-2x + ... + 2x)^{n-1} = 0
$$
\n
$$
E[Y_{(pq)}] = \int_{-\infty}^{\infty} y^{2} f_{(pq)}(y)dy
$$
\n
$$
= \frac{1}{(r-1)!(n-r)!} \int_{-\infty}^{\infty} F(x) \left[1 - F(x) - f(x)\right]_{-\infty}^{\infty} y^{2}h_y(y|x)dy
$$
\n
$$
= \frac{1}{(r-1)!(n-r)!} \int_{-\infty}^{\infty} F(x) \left[1 - F(x) - f(x)\right]_{-\infty}^{\infty} y^{2}h_y(y|x)dy
$$
\nWe know that  
\n
$$
\int_{-\infty}^{\infty} y^{2}h_y(y|x)dy = t \frac{1}{2}(1 - x^{2}) + \left[-2x + ... + 2x\right] \left[\frac{x - z_1}{1 + x}\right]_{-\infty}^{\infty} y^{2}h_y(y|x)dy
$$
\n
$$
= \left[x^{2} + x^{2} + x^{2} + \frac{1}{2}(1 - x^{2}) + \frac{1}{2}(1 - x^{2}) + 2x - x^{2} + \frac{1}{2}(1 - x^{2}) + \frac{1}{2}(1 -
$$

We know that

$$
\int_{-\infty}^{\infty} y^2 h_i(y \mid x) dy = \int_{2i}^{2} (1 - \dots^2) + \left[ \left( \frac{x - x_1}{1} \right)^2 \right]^2
$$

$$
E\left[Y_{[r:n]}^{2}\right] = \frac{1}{n}\sum_{i} z_{2i}^{2} + \frac{1}{n}\sum_{i} \frac{z_{2i}}{z_{2i}}(1 - \dots^{2}) + 2\ldots r_{rn} \frac{1}{n}\sum_{i} z_{2i} \frac{z_{2i}}{z_{2i}} + r_{r, rn} \dots^{2} \frac{1}{n}\sum_{i} \frac{z_{2i}}{z_{2i}} \tag{6}
$$

$$
S_{r,r:n} = Var(U_{r:n}) = \Gamma_{r,r:n} - \Gamma_{r:n}^2.
$$

$$
\left[ \frac{F(x)}{r-1} - \frac{1-F(x)}{r-1} - \frac{f(x)}{x} \right] \frac{y^2 h_n(y \mid x) dy}{(1-x)^2}
$$
\nWe know that\n
$$
\int_{-\infty}^{\infty} y^2 h_i(y \mid x) dy = \frac{1}{2} (1 - x^2) + \left[ -2(1 - x^2) + 2x \right] \left[ \frac{x - x_1}{1} \right]^2
$$
\nHence,\n
$$
E\left[ Y_{[r,n]}^2 \right] = \frac{1}{n} \sum_i x^2 + \frac{1}{n} \sum_i \left( \frac{1}{2} (1 - x^2) + 2x \right) \left[ \frac{1}{n} \sum_i x_i \right] + \left[ \frac{1}{n} \sum_i x_i \right] \left( \frac{1}{n} \sum_i x_i \right] + \left[ \frac{1}{n} \sum_i x_i \right] \left( \frac{1}{n} \sum_i x_i \right] \left( \frac{1}{n} \sum_i x_i \right) \left( \frac{1}{n}
$$

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\n
$$
E\Big[Y_{[rn]}Y_{[sn]}\Big] = \frac{1}{(r-1)!(s-r-1)!(n-s)!} \int_{-\infty}^{\infty} \int_{x_1}^{\infty} PerIdx_2 dx_1,
$$
\nHere the matrix *I* is given by

\n
$$
\begin{bmatrix}\n\Gamma & \Gamma\n\end{bmatrix}
$$

where the matrix  $I$  is given by

Application of concomitants of order statistics ....... estimation  
\n
$$
E[Y_{[r,n]}Y_{[s,n]}] = \frac{1}{(r-1)!(s-r-1)!(n-s)!} \int_{-\infty}^{\infty} \int_{x_1}^{\infty} PerIdx_2 dx_1,
$$
\nwhere the matrix *I* is given by  
\n
$$
\begin{bmatrix}\nF(x_1) & F(x_2) - F(x_1) & 1 - F(x_2) & f(x_1) \int_{-\infty}^{\infty} y_1 f_1(y_1 | x_1) dy_1 & f(x_2) \int_{-\infty}^{\infty} y_2 f_1(y_2 | x_2) dy_2 \\
\vdots & \vdots & \vdots & \vdots \\
F(x_1) & F(x_2) - F(x_1) & \frac{1 - F(x_2)}{s} & \frac{f(x_1) \int_{-\infty}^{\infty} y_1 f_1(y_1 | x_1) dy_1}{s} & \frac{f(x_2) \int_{-\infty}^{\infty} y_2 f_1(y_2 | x_2) dy_2}{s}\n\end{bmatrix}
$$
\nNow if we write  $\Gamma_{r,s,n} = E(U_{r,n}U_{s,n})$  then,  
\n
$$
E[Y_{[r,n]}Y_{[s,n]}] = \frac{1}{n(n-1)} \sum_{i \neq j} (\gamma_{2i} \gamma_{2j} + ... \gamma_{2i} \gamma_{2i} \Gamma_{r,n} + ... \gamma_{2j} \gamma_{2i} \Gamma_{s,n} + ...^2 \gamma_{2i} \gamma_{2i} \Gamma_{r,s,n})
$$
\n
$$
Cov[Y_{[r,n]}Y_{[s,n]}] = E[Y_{[r,n]}Y_{[s,n]}] - E[Y_{[r,n]}]E[Y_{[s,n]}]
$$
\n
$$
= \frac{1}{n(n-1)} ...^2 S_{r,s,n} \sum_{i \neq j} \gamma_{2i} \Gamma_{2i} - \frac{1}{n^2(n-1)} \sum_{i \neq j} [(-\gamma_{2i} - \gamma_{2j}) + ... (-\gamma_{2i} - \gamma_{2j}) \Gamma_{r,n}]
$$

\n 
$$
\mathcal{E}\left[Y_{(rn)}Y_{(sn)}\right] = \frac{1}{(r-1)!(s-r-1)!(n-s)!} \int_{-\infty}^{\infty} \int_{x}^{\infty} \text{PerIdx}_2 dx_1,
$$
\n

\n\n The matrix *I* is given by\n

\n\n $F(x_1) F(x_2) - F(x_1) 1 - F(x_2) f(x_1) \int_{-\infty}^{\infty} y_1 f_1(y_1|x_1) dy_1 f(x_2) \int_{-\infty}^{\infty} y_2 f_1(y_2|x_2) dy_2$ \n

\n\n ∴  $\frac{F(x_1)}{r-1} \left[\frac{F(x_2) - F(x_1)}{s-r-1} \cdot \frac{1-F(x_2)}{s-s} \cdot \frac{f(x_1) \int_{-\infty}^{\infty} y_1 f_1(y_1|x_1) dy_1 f(x_2) \int_{-\infty}^{\infty} y_2 f_1(y_2|x_2) dy_2 \right]$ \n

\n\n ∴  $i$  if we write\n  $\Gamma_{r, x; n} = E(U_{rn}U_{kn})$ \n

\n\n The matrix  $F_{r, x; n} = E(U_{rn}U_{kn})$  then,\n

\n\n The matrix  $\Gamma_{r, x; n} = E(U_{rn}U_{kn})$  then\n

\n\n The matrix  $\Gamma_{r, x; n} = E(U_{rn}U_{kn})$  then\n

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\n\n The matrix  $\Gamma_{r, x; n} = E(U_{rn}U_{kn})$ .\n

\n\n The matrix  $\Gamma_{r, x; n} = E(U_{rn}U_{kn})$ .\n

\n\n The matrix  $\Gamma_{r, x; n} = E(U_{rn}U_{kn}) - E[X_{rn}U_{kn}] = E[X_{rn}U_{kn}] - E[X_{rn}U_{kn}]$ .\n

\n\n The matrix  $\Gamma_{r, x; n} = \frac{1}{n(n-1)} \sum_{i \neq j} \left[ x_{$ 

Consider the units of bivariate sample in which measurement of the *X* variate can be done easily where as a measurement of *Y* is not so easy or economic. In this case we order the *X* observations and make measurements only on the Extraction of the state of the state of  $Y_{1s}$  *x*  $F(x_1) - F(x_2)$   $f(x_1)$   $f(x_2)$   $f(x_1)$   $f(x_2)$   $f(x_2)$   $f(x_2)$   $f(x_2)$   $f(x_2)$ <br>
Now if we write  $\Gamma_{r,sm} = E(U_{rn}U_{snr})$  then,<br>  $E[Y_{1rn}Y_{1sn1}] = \frac{1}{n(n-1)} \sum_{i \neq j} (-x^2z_1 + ... -2x^$  $\begin{aligned}\n\left[ \begin{array}{ll}\n\vdots & \vdots & \vdots \\
\frac{F(x_i)}{r^{-1}} & \frac{F(x_2) - F(x_i)}{r^{-1}} & \frac{1 - F(x_2)}{r^{-1}} & \frac{f(x_i)}{r} \int_{-\infty}^{\infty} f(x_i) \left[ x_i \lambda \delta_i \right] & f(x_2) \int_{-\infty}^{\infty} y_i f_{i}(y_1 | x_2) \delta_i \right] \\
\text{Now if we write } & \frac{F}{r_{i,c}} = E(U_{i,\alpha} V_{i,ca}) \text{ then,} \\
E\left[Y_{i,c|Y_{i,c|1}}\right] = \frac{1}{n$  $z_{2i} = z_2, \forall i = 1, 2, ..., n$  and consider the transformation  $Y_i^* = Y_i - z_2$ . Then the  $\begin{aligned}\n&\sum_{i=1}^{n} y_{2} f_{n} (y_{2} | x_{2}) dy_{2} \\
&\sum_{i=1}^{n} y_{2} f_{n} (y_{2} | x_{2}) \Gamma_{r,n} \\
&= \Gamma_{r,n} \Gamma_{r$  $=\frac{1}{n(n-1)}$   $\int_{0}^{\infty} S_{r,s,n} \sum_{i \neq j} \int_{2i}^{i} \frac{1}{2i} \int_{2j}^{i} - \frac{1}{n^2(n-1)} \sum_{i \leq j} [(\frac{1}{2i} - \frac{1}{2j}) + \dots (\frac{1}{2i} - \frac{1}{2j}) \Gamma_{r,n}]$ <br>  $\times [(\frac{1}{2i} - \frac{1}{2j}) + \dots (\frac{1}{2i} - \frac{1}{2j}) \Gamma_{s,n}]$ <br>
Consider the units of bivariate sampl expectation and variance-covariance matrix which can be expressed in the form if we write  $\Gamma_{r,\text{ion}} = E(U_{rs}U_{sn})$  then,<br>  $\left[\frac{1}{Y_{r,\text{off}}Y_{r,\text{off}}}\right] = \frac{1}{n(n-1)}\sum_{r,r}(-\frac{1}{2}x_{2r}) + \cdots - \frac{1}{2}x_{2r}T_{rs} + \cdots - \frac{1}{2}x_{2r}T_{2r} + \cdots - \frac{1}{2}x_{2r}T_{2r}T_{rsn})$ <br>  $Cov\left[Y_{r,\text{on}}Y_{r,\text{off}}\right] = E\left[Y_{r,\text{on}}Y_{r,\text{off}}\right] - E\$ *n* we write  $\Gamma_{r,xx} = E(U_{rx}U_{rx})$  then,<br>  $Y_{(rs)}Y_{(rs)} = \frac{1}{n(n-1)}\sum_{i\neq j}(-x_i-z_j+\ldots-z_1)^{\frac{1}{2}}\sum_{r,n}(-x_i-z_j+\ldots-z_1)^{\frac{1}{2}}\sum_{r,n}(-x_i+\ldots+z_1)^{\frac{1}{2}}\sum_{r,n}(-x_i-x_i)^{\frac{1}{2}}\sum_{r,n}(-x_i-x_i)^{\frac{1}{2}}\sum_{r,n}(-x_i-x_i)^{\frac{1}{2}}\sum_{r,n}(-x_i-x_i)^{\frac{1}{2}}\sum_{r,n}($ this case we order the *X* observations and m:<br>
concomitants  $Y_{[c+1:n]},..., Y_{[n-c:n]}$ . Now based on this<br>
estimates based on the available concomitant<br>  $z_{2i} = z_2, \forall i = 1, 2,...,n$  and consider the transfor<br>
corresponding vector of  $E_{r,n}[Y_{s,n}]$  =  $E[Y_{t_{r,n}}|Y_{s,n}]$  =  $E[Y_{t_{r,n}}]E[Y_{t_{r,n}}]$ <br>  $\frac{1}{\mu(n-1)}...^{2}S_{r,s,n}\sum_{i\neq j} \frac{1}{i_{2i}}\frac{1}{i_{2j}} - \frac{1}{n^{2}(n-1)}\sum_{i\leq j}[(\sim_{2i}-\sim_{2i})+(\sim_{2i}-\sim_{2i})+(\sim_{2i}-\sim_{2i})+\sim_{2i}]\times [(0,1)-\sim_{2i}]\times [(0,1)-\sim_{2i}]\times [(0,1)-\sim_{2i}]\times [(0,1)-\sim_{$ *i*  $\mathcal{L}_{\text{free}}[f_{\text{real}}] - \frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{i=2}^{n} \sum_{j}^{n} \sum_{j}$  $z_{2i} = z_2, \forall i = 1, 2, ..., n$  and consider the transformation  $Y_i$ <br>corresponding vector of transformed concomitants of rec<br>expectation and variance-covariance matrix which can be ex<br> $E[Y_{[n,c]}^*] = ...$ s,<br>where<br> $S = \frac{1}{n} \sum_{i=1}^n \up$ inding vector of transformed concomitants<br>
on and variance-covariance matrix which ca<br>  $\left[\sum_{n, c_1}^{*} \right] = ...$ S,<br>  $\frac{1}{n} \sum_{i=1}^{n} \uparrow_{2i} (\Gamma_{c+1:n}, \Gamma_{c+2:n}, ..., \Gamma_{n-c:n})$ <br>  $\left[\sum_{n, c_1}^{*} \right] = (1 - ...^2) \frac{1}{n} \sum_{i=1}^{n} \uparrow_{2i}^{2} I + ...^$  $(z_2 - z_1) + ... (t_{2i} - t_{2j})r_{sn}$ <br>its of bivariate sample in which measurem<br>iily where as a measurement of Y is not so<br>der the X observations and make measure<br>of the X observations and make measure<br>1 on the available concomita anding vector of transformed concon and variance-covariance matrix<br>  $\begin{aligned}\n\int_{[n,c]}^{\infty} \left[ \int_{[n,c]}^{n} \mathbf{1}_{2i}(\mathbf{r}_{c+1:n}, \mathbf{r}_{c+2:n}, ..., \mathbf{r}_{n-c:n}) \right] \\
&= (1 - ...^2) \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}_{2i}^2 I + ...^2 H, \n\end{aligned}$  $\frac{1}{2} \left[ (c_{2j} - a_{2j}) + ... (t_{2j} - t_{2j}) \right]$ <br>
sider the units of bivariate sample in which measurement of the *X* variate<br>
be done easily where as a measurement of *Y* is not so easy or economic. In<br>
case we order the *X* ob <sup>2</sup>  $S_{r,s,n}$   $\sum_{i \in J} T_{2i} T_{2j} - \frac{1}{n^2(n-1)} \sum_{i \in J} [(\frac{r_{2i} - r_{2j}}{n} + ...(\frac{r_{2i} - r_{2j}}{n} + \frac{r_{2i} - r_{2j}}{n}) + ...(\frac{r_{2i} - r_{2j}}{n}) + \frac{1}{n^2(n-1)}]$ <br>
of bivariate sample in which measurement of the *X* varia<br>
where as a measure (8)

$$
E\left[Y_{[n,c]}^*\right] = \dots \mathbf{S},\tag{7}
$$

$$
S = \frac{1}{n} \sum_{i=1}^{n} \Upsilon_{2i} (r_{c+1:n}, r_{c+2:n}, ..., r_{n-c:n})'
$$

$$
D\left[Y_{\left(n,c\right)}^{*}\right] = \left(1 - \dots^{2}\right) \frac{1}{n} \sum_{i=1}^{n} \uparrow \frac{2}{2i} I + \dots^{2} H,
$$
\n<sup>(8)</sup>

*Mean T. G. and P. Yageen Thomas*  
\nwhere *I* is the identity matrix of order 
$$
(n-c) \times (n-c)
$$
 and  $H = ||h_{rs}||$ ,  
\n
$$
h_{rr} = \frac{1}{n} \sum_{i} \frac{1}{2} \sum_{r, r, n} \left( \frac{1}{2} \sum_{i < j} (1 - \frac{1}{2})^2 \right) r_{rn}^2
$$
\nand for  $r \neq s$ ,  
\n
$$
h_{rr} = \frac{1}{n} \sum_{i} \left( \frac{1}{2} \sum_{i < j} (1 - \frac{1}{2})^2 \right) r_{rn}^2
$$

*Veena T. G. and P. Yageen Thomas*  
\n**Here** 
$$
I
$$
 is the identity matrix of order  $(n-c) \times (n-c)$  and  $H = ||h_{rs}||$ ,  
\n
$$
h_{rr} = \frac{1}{n} \sum_{i} \frac{1}{2i} S_{r, rn} + \frac{1}{n^2} \sum_{i < j} (\frac{1}{2i} - \frac{1}{2j})^2 \Gamma_{rn}^2
$$
\n**1** for  $r \neq s$ ,  
\n
$$
h_{rs} = \frac{1}{n(n-1)} \sum_{i \neq j} \frac{1}{2i} \sum_{j} S_{r, sn} + \frac{1}{n^2 (n-1)} \sum_{i < j} (\frac{1}{2i} - \frac{1}{2j})^2 \Gamma_{rn} \Gamma_{sn},
$$
\n**here**  $i$  and  $j$  vary from 1 to  $n$  and  $r$  and  $s$  are such that  $c + 1 \leq r < s \leq n - c$ .

88<br> **Solution**<br>
Where *I* is the identity matrix of order  $(n-c) \times (n-c)$  and *H* =  $||h_{rs}||$ ,<br>  $h_{rr} = \frac{1}{n} \sum_{i} \frac{1}{2i} S_{r, rn} + \frac{1}{n^2} \sum_{i \leq j} (\frac{1}{2i} - \frac{1}{2j})^2 \Gamma_{rn}^2$ <br>
and for  $r \neq s$ ,<br>  $h_{rs} = \frac{1}{n(n-1)} \sum_{i \neq j} \frac{1}{2i$ Veena *T*. G. and *P*. Yageen Thomas<br>
veena *T*. G. and *P*. Yageen Thomas<br>  $\int_{\pi}^{\pi} = \frac{1}{n} \sum_{i} \left( \frac{1}{2} S_{r, en} + \frac{1}{n^2} \sum_{i \leq j} (\frac{1}{2} a_i - \frac{1}{2} a_j)^2 \right) \cdot \int_{\pi}^{\pi}$ <br>
for  $r \neq s$ ,<br>  $\int_{\pi}^{\pi} = \frac{1}{n(n-1)} \sum_{i \neq j} \frac$ *Veena T. (*<br>
dentity matrix of order  $(n-c) \times (n-c)$  :<br>  $S_{r,r;n} + \frac{1}{n^2} \sum_{i < j} (\dagger_{2i} - \dagger_{2j})^2 \Gamma_{rn}^2$ <br>  $\sum_{i \neq j} \dagger_{2i} \dagger_{2j} S_{r,s;n} + \frac{1}{n^2 (n-1)} \sum_{i < j} (\dagger_{2i} - \dagger_{2j})^2$ <br>
ary from 1 to *n* and *r* and *s* are such<br>
8) *Veena T. G. and P. Yageen Tha*<br>
is the identity matrix of order  $(n-c) \times (n-c)$  and  $H = ||h_{rs}||$ ,<br>  $\frac{1}{n} \sum_{i} \frac{1}{2s} S_{r, rn} + \frac{1}{n^2} \sum_{i < j} (\frac{1}{2i} - \frac{1}{2j})^2 \Gamma_{rn}^2$ <br>  $\neq s$ ,<br>  $\frac{1}{n(n-1)} \sum_{i \neq j} \frac{1}{2s} \int_{2s}^{1} S_{r, sn} + \$ *Veena T. G. and P. Yageen Thomas*<br> *n*tity matrix of order  $(n-c) \times (n-c)$  and  $H = ||h_{rs}||$ ,<br>  $\frac{1}{n^2 + \frac{1}{n^2} \sum_{i \leq j} (\tau_{2i} - \tau_{2j})^2 \tau_{rsn}^2}$ <br>  $\tau_{2i} \tau_{2j} S_{r,sn} + \frac{1}{n^2 (n-1) \sum_{i \leq j} (\tau_{2i} - \tau_{2j})^2 \tau_{rsn} \tau_{sn},$ <br> *y* fr Veena T. G. and P. Yageen Thomas<br>
I is the identity matrix of order  $(n-c) \times (n-c)$  and  $H = ||h_{rs}||$ ,<br>  $= \frac{1}{n} \sum_{i} \frac{1}{i} \sum_{i}^{2i} S_{r, rn} + \frac{1}{n^2} \sum_{i \leq j} (\frac{1}{2i} - \frac{1}{2j})^2 \Gamma_{rn^2}^2$ <br>  $= \frac{1}{n(n-1)} \sum_{i \neq j} \frac{1}{i} \sum_{i} S_{r, xn$ Veena T. G. and P. Yageen Thomas<br>
the identity matrix of order  $(n-c) \times (n-c)$  and  $H = ||h_n||$ .<br>  $\int_{-1}^{1} \frac{1}{2s} S_{r, cm} + \frac{1}{n^2} \sum_{i \le j} (\uparrow_{2i} - \uparrow_{2j})^2 \Gamma_{rn}^2$ <br>  $\int_{-1}^{1} \sum_{i \ne j} [\uparrow_{2i} S_{r, \text{c.m}} + \frac{1}{n^2 (n-1)} \sum_{i \le j} (\uparrow_{2$ S8<br>
Weena *T*. *G.* and *P.* Yageen Thomas<br>
where *I* is the identity matrix of order  $(n-c) \times (n-c)$  and  $H = ||h_{ij}||$ ,<br>  $h_{rr} = \frac{1}{n} \sum_{i} \sum_{i} \sum_{i} (\sum_{i} (-1)^{i} \sum_{i} (1^{i} - 1^{i})^{2} \sum_{i} (1^{i} - 1^{i})^{2} \sum_{i} (1^{i} - 1^{i})^{2} \sum_{i} (1^{i}$ the BLUE of .... Hence we may obtain two linear unbiased estimators of ... by minimizing the variance in a restricted sense as done in Chacko and Thomas (2008). **Theorem 2.1** Let *R* be a column vector of scalars of order n and  $R^r$ <sub>*F*(*n*  $\pi$ ) **n**  $R^r = \frac{1}{n} \sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{i=1}^{n} (x_i - x_{i})^2 r_{ra}^2$ <br>
and for  $r \neq s$ ,<br>  $h_n = \frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{i=1}^{n} (x_i - x_{i})^2 r_{ra}^2$ </sub> State of  $V$  and  $P$ . Yageen Thomas<br>
where I is the identity matrix of order  $(n-c) \times (n-c)$  and  $H = ||h_{\alpha}||$ ,<br>  $h_{\alpha} = \frac{1}{n} \sum_{i} \frac{1}{i} \frac{2}{i} S_{r,\alpha n} + \frac{1}{n^2} \sum_{i \in J} (\frac{1}{2i} - \frac{1}{2j})^2 r_{\alpha n}^2$ <br>
and for  $r \neq s$ ,<br>  $h_{\alpha} = \$  $\left(-\frac{1}{2}\right) \sum_{i \neq j} \sum_{i} \sum_{j} S_{r,s;n} + \frac{1}{n^2 (n-1)} \sum_{i < j} (\sum_{i} (-\sum_{j} j)^2 \Gamma_{rn} \Gamma_{sn},$ <br> *j* vary from 1 to *n* and *r* and *s* are such that  $c + 1$ <br>
and (8) do not provide a general Gauss-Markov set up<br>
.... Hence we may (a)  $\frac{1}{i+j}$   $\frac{1}{i+j}$   $\frac{1}{i+j}$   $\frac{n^2(n-1)\frac{1}{i<br>
(b)  $i = 1, 2, 3, \ldots, n^2$  of  $n \neq 0$  and  $r$  and  $s$  are such to  $\ln d$  (8) do not provide a general Gauss-Mark  $f(x)$ . Hence we may obtain two linear$ matrix of order  $(n-c) \times (n-c)$ <br>  $\frac{1}{n^2} \sum_{i \le j} (\frac{1}{2i} - \frac{1}{2j})^2 r_{rn}^2$ <br>  $\frac{1}{2i} \frac{1}{2j} S_{r,s;n} + \frac{1}{n^2 (n-1)} \sum_{i \le j} (\frac{1}{2i} - \frac{1}{2j})^2$ <br>
y from 1 to *n* and *r* and *s* are sum do not provide a general Gauss-Nence we *n*<sup>2</sup>(*n*-1)  $\frac{1}{i \neq j}$ <br>*j* vary from 1 to *n* and *r* and *s*<br>*n*<sup>2</sup>(*n*-1)  $\frac{1}{i \leq j}$ <br>*n*<sup>2</sup>(*n*-1)  $\frac{1}{i \leq j}$ <br>*n*<sup>2</sup>(*n*-1)  $\frac{1}{i \leq j}$ <br>*n*<sup>2</sup>(*n*) and *s*<br>**n** is a metric is and  $f(x)$ <br>**n** of  $Y_{[c+1:n]}, Y_{[c$ re *I* is the identity matrix of order  $(n-c) \times (n-c)$  and  $H = ||h_{n}||$ ,<br>  $i_{n} = \frac{1}{n} \sum_{i} \int_{1}^{2} \sum_{2j \leq r, n} \frac{1}{n} \sum_{i \leq j} (f_{2i} - f_{2j})^2 r_{n}^2$ <br>
for  $r \neq s$ ,<br>  $i_{n} = \frac{1}{n(n-1)} \sum_{i \neq j} \int_{2i}^{1} \sum_{2j} (s_{r, kn} + \frac{1}{n^2 (n-1)} \$ *Veena T. G. and P. Yageen Thomas*<br>
matrix of order  $(n-c) \times (n-c)$  and  $H = ||h_{rs}||$ ,<br>  $\frac{1}{n^2} \sum_{i \in J} (\dot{t}_{2i} - \dot{t}_{2j})^2 r_{rn}^2$ <br>  $\int_{2j} S_{r,sn} + \frac{1}{n^2 (n-1)} \sum_{i \in J} (\dot{t}_{2i} - \dot{t}_{2j})^2 r_{rn} r_{sn}$ ,<br>
mm 1 to *n* and *r* and *s* Veena T. G. and P. Yageen Thomas<br>
s the identity matrix of order  $(n-c) \times (n-c)$  and  $H = ||h_n||$ ,<br>  $\sum_{i=1}^{\infty} \frac{1}{2} S_{r,m} + \frac{1}{n^2} \sum_{i \in J} (1 + 1 + 2)^2 r_{rn}^2$ <br>  $s$ ,<br>  $\sum_{j=1}^{\infty} \frac{1}{2} \int_{2}^{1} \frac{1}{2} \int_{2}^{2} S_{r,m} + \frac{1}{n^2 (n-1$ 

**Theorem 2.1** Let R be a column vector of scalars of order n and  $R'Y_{[n,c]}^*$  be a

$$
Var\left(R'Y_{[n,c]}^*\right) = (1 - ...^2) \frac{1}{n} \sum_{i=1}^n \frac{1}{2i} R'R + ...^2 R'HR
$$
\n(9)

Then an estimator  $\hat{u}_1$  obtained by minimizing  $R'R$  involved in (9) subject to the obtained by minimizing *R'R* involved in (9) subject to the<br>involved by minimizing *R'RR* involved in (9) subject to the<br>involved by minimizing *R'RR* involved in (9) subject to the<br>involved by minimizing *R'RR* involved  $h_{rr} = \frac{1}{n} \sum_{i} \frac{1}{i} \sum_{i} S_{r, rn} + \frac{1}{n^2} \sum_{i < j} (\frac{1}{2i} - \frac{1}{2i})^2 f$ <br>and for  $r \neq s$ ,<br>and for  $r \neq s$ ,<br> $h_{rs} = \frac{1}{n(n-1)} \sum_{i \neq j} \frac{1}{2i} \sum_{j} S_{r, sn} + \frac{1}{n^2(n-1)}$ <br>where *i* and *j* vary from 1 to *n* and *r* and<br>Clea  ${}^{t}Y_{[n,c]}^{*}$  is unbiased for  $\dots$  is given by  $\widehat{...}_1 = \frac{S'}{S'S} Y_{[n,c]}^{*}$  and an d estimators of ... by<br>
Chacko and Thomas<br>
ir n and  $R'Y_{[n,c]}^*$  be a<br>
in by<br>
(9)<br>
d in (9) subject to the<br>  $Y_1 = \frac{S'}{S'}Y_{[n,c]}^*$  and an<br>
in (9) subject to the<br>  $\frac{H^{-1}}{S}Y_{[n,c]}^*$ *n*,  $\cdot$ <br>+1≤ *r* < *s* ≤ *n* − *c*.<br>up so as to derive<br>timators of ... by<br>acko and Thomas<br>and  $R'Y_{[n,c]}^*$  be a<br>y<br>y<br>(9)<br>(9)<br>(9)<br>subject to the<br> $\frac{S'}{S}Y_{[n,c]}^*$  and an<br>9) subject to the<br> $\frac{S'}{S}Y_{[n,c]}^*$ . estimator  $\hat{u}_2$  obtained by minimizing *R'HR* involved in (9) subject to the  $\int \sum_{i=1}^{\infty} \int_{2i}^{1} z_i \, \frac{1}{2i} \, S_{r,gen} + \frac{1}{n^2 (n-1)} \sum_{i \leq j} (1 \, z_i - 1 \, z_j)^2 \, \Gamma_{r,n} \Gamma_{sn},$ <br>
vary from 1 to *n* and *r* and *s* are such that  $c + 1 \leq r < s \leq n - c$ .<br>
(8) do not provide a general Gauss-Markov set up so  $h_{rs} = \frac{1}{n(n-1)} \sum_{i \neq j} \frac{1}{1} \sum_{2i} \frac{1}{5} \sum_{r,s;n} + \frac{1}{n^2(n-1)}$ <br>where *i* and *j* vary from 1 to *n* and *r i*<br>Clearly (7) and (8) do not provide a gene<br>the BLUE of .... Hence we may obtain t<br>minimizing the variance  $Y_{[n,c]}^*$  is unbiased for ... is given by  $\widehat{N}_{i,j} = \frac{S'H^{-1}}{(r-1)} Y_{[n,c]}^*$ . of order n and  $R'Y_{[n,c]}$  be a<br>
ce given by<br>
(9)<br>
involved in (9) subject to the<br>
n by  $\widehat{H}_{-1} = \frac{S'}{S'} Y_{[n,c]}^*$  and an<br>
olved in (9) subject to the<br>  $\widehat{H}_{2} = \frac{S'H^{-1}}{S'H^{-1}S} Y_{[n,c]}^*$ .  $H^{-1}S$   $^{[n,c]}$  $\widehat{...}_{2} = \frac{S'H^{-1}}{1.5 \cdot 1.5 \cdot 1} Y_{[n,c]}^{*}.$ *i*  $r_{r,n}r_{s,n}$ ,<br>
that  $c+1 \le r < s \le n-c$ .<br>
(sover the solution of and the sased estimators of and the same of the same of the same of the all iven by<br>
only and the same of the  $-1$  and  $-1$  and  $-1$  and  $-1$  and  $-1$  $\widehat{C}_2 = \frac{S'H^{-1}}{S'H^{-1}S}Y_{[n,c]}^*.$ Var $(R'Y_{[n,c]}^*) = (1 - ...^2) \frac{1}{n} \sum_{i=1}^n \frac{1}{2i} R'R + ...^2 R'HR$ <br>
Then an estimator  $\frac{1}{n}$  obtained by minimizing R'R involved in (9) so<br>
condition that  $R'Y_{[n,c]}^*$  is unbiased for ... is given by  $\frac{1}{n} = \frac{S'}{S'S'}$ <br>
estimat (7) and (8) do not provide a general Gauss-Markov set up so as<br>
UE of .... Hence we may obtain two linear unbiased estimators<br>
zing the variance in a restricted sense as done in Chacko and<br> **m 2.1** Let R be a column vecto bela Ualus-Malkov set up so as<br>
two linear unbiased estimator<br>
d sense as done in Chacko and<br>
or of scalars of order n and R'I<br>  $\frac{1}{1}$  with variance given by<br>  $+\frac{2}{1}$  with variance given by<br>  $+\frac{2}{1}$  with variance g and *j* vary from 1 to *n* and *r* and *s* are such that  $c+1 \le r < s \le n-c$ .<br>
(7) and (8) do not provide a general Gauss-Markov set up so as to derive<br> *IE* of .... Hence we may obtain two linear unbiased estimators of ... by of provide a general Gauss-Markov set up so as to derive<br>we may obtain two linear unbiased estimators of ... by<br> *i* in a restricted sense as done in Chacko and Thomas<br>
a column vector of scalars of order n and  $R'Y_{[n,c]}^{$ and *j* vary from 1 to *n* and *r* and *s* are such that  $c + 1 \le r < s \le n - c$ .<br>
7) and (8) do not provide a general Gauss-Markov set up so as to derive<br>
E of .... Hence we may obtain two linear unbiased estimators of ... by<br> that  $R'Y_{[n,c]}$  is unbiased for ... is given by  $\frac{1}{\log 2} = \frac{1}{S \cdot S} Y_{[n,c]}$  and<br>  $\frac{1}{\log 2}$  obtained by minimizing  $R'HR$  involved in (9) subject to<br>
that  $R'Y_{[n,c]}^*$  is unbiased for ... is given by  $\frac{1}{\log 2} = \frac{S'H^{$ **n 2.1** Let *R* be a column vector of scalars of order n and  $R'Y_{[n,c]}^*$  be a<br>
notion of  $Y_{[c+1a]}^*$ ,  $Y_{[c+2a]}^*$ ,  $Y_{[n-ca]}^*$  with variance given by<br>  $(R'Y_{[n,c]}^*) = (1 - \dots^2) \frac{1}{n} \sum_{i=1}^{n} \frac{2}{3} R' R + \dots^2 R' H R$  (9)<br>
esti mn vector of scalars of order n and<br>
...,  $Y_{[n-cn]}$  with variance given by<br>  $\sum t_{2i}^2 R'R + ...^2 R'HR$ <br>
by minimizing  $R'R$  involved in (9)<br>
iased for ... is given by  $\frac{S}{m_1} = \frac{S'}{S'S}$ <br>
nimizing  $R'HR$  involved in (9)<br>
ad for be a column vector of scalars of order n and  $RY'_{\{u,c\}}$  be a<br>  $n^2 Y_{\{u,c\}} \cdots Y_{\{u-cn\}}$  with variance given by<br>  $\therefore \frac{1}{n} \sum_{i=1}^{n} \frac{1}{1} \frac{2}{2i} R' R + \dots^2 R' H R$  (9)<br>
bbtained by minimizing R'R involved in (9) subject to Ing the variance in a restricted sense as done in Chacko and Thomas<br> **2.1** Let *R* be a column vector of scalars of order n and  $R'Y_{[n,c]}^*$  be a<br>
lection of  $Y_{[c+i:n]}, Y_{[c+2:n]}, ..., Y_{[n-c:n]}$  with variance given by<br>  $RY_{[n,c]}^*$  = nn vector of scalars of order n and  $R'Y_{[n,c]}^*$  be a<br>
...,  $Y_{[n,-cn]}$  with variance given by<br>  $\pm \frac{2}{3}R'R + ...^2R'HR$  (9)<br>
by minimizing  $R'R$  involved in (9) subject to the<br>
seed for ... is given by  $\frac{2}{n_1} = \frac{S'}{S}Y_{[n,c]}$ for ... is given by  $m_1 = \frac{1}{S^2} Y_{[n,c]}$  and an<br>  $\lim_{s \to c} R' H R$  involved in (9) subject to the<br>  $\lim_{s \to c} \frac{1}{S^2} R'' R^{2s} = \frac{1}{S^2 H^{-1} S} Y_{[n,c]}^*$ .<br>  $\lim_{s \to c} \frac{1}{S^2} S^{2s} = \frac{1}{S^2 H^{-1} S}$  (10) 2.1 Let R be a column vector of scalars of order n and  $R'Y_{\text{[n,c]}}^*$  be a<br>
cition of  $Y_{\{\text{[n,c]}}}, Y_{\{\text{[n,c]}}}, \dots, Y_{\{\text{[n-c]}}\}$  with variance given by<br>  $R'Y_{\text{[n,c]}}^*$  =  $(1 - \frac{3}{n})\frac{1}{n^2} \sum_{i=1}^{n} \frac{1}{2i}RR + \frac{3}{n^$  $Var(RY_{[n,c]}^{K}) = (1 - ...^{2}) \frac{1}{n} \sum_{i=1}^{n} \frac{1}{2} \lambda^{2} R^{2} + ...^{2} R^{2} H R$  (9)<br>
Then an estimator  $\frac{1}{n_{c1}}$  obtained by minimizing *R'R* involved in (9) subject to the<br>
condition that  $R'Y_{[n,c]}^{K}$  is unbiased for ... is given

$$
Var(\widehat{\ldots}_1) = (1 - \ldots^2) \frac{1}{n} \sum_{i=1}^n \frac{1}{2i} \frac{1}{S^2} + \ldots^2 \frac{S^2}{(S^2)^2}
$$
(10)

and

$$
Var(\hat{...}_2) = (1 - ...^2) \frac{1}{n} \sum_{i=1}^n \frac{1}{2i} \frac{1}{(S'H^{-1}S)^2} + ...^2 \frac{1}{S'H^{-1}S}
$$
(11)

**Proof**

Using (7), we have

$$
E(R'Y_{[n,c]}^*) = R'S...
$$

Hence,  $R'Y_{n,c}^*$  will be an unbiased estimator for  $\ldots$  if

*Application of concomitants of order statistics ……. estimation 89*

$$
R'S = 1.
$$
 (12)

To minimize  $R'R$  subject to the condition that  $R'Y_{[n,c]}^*$  is unbiased for ..., we

$$
E_1 = R'R - 2 \, \frac{1}{1} (R'S - 1), \tag{13}
$$

Application of concomitants of order statistics ....... estimation<br>  $R'S = 1$ .<br>
To minimize  $R'R$  subject to the condition that  $R'Y_{[n,c]}^*$  is unbiased for ...,<br>
have to minimize<br>  $\mathbb{E}_1 = R'R - 2\}_1 (R'S - 1)$ ,<br>
Where  $\}_1$  is Application of concomitants of order statistics ....... estimation<br>  $R' = 1$ . (12)<br>
To minimize  $R'R$  subject to the condition that  $R'Y_{[n,c]}^*$  is unbiased for ..., we<br>
have to minimize<br>  $\mathbb{E}_1 = R'R - 2\mathbb{1}_1(R'S - 1)$ , (13)<br> Where  $\}$ <sub>1</sub> is the Lagrangian multiplier. Differentiating (13) with respect to Application of concomitants of order statistics ....... estimation<br>  $R'$   $R'$   $S = 1$ .<br> **Comminize**<br>  $R'$   $R'$  lication of concomitants of order statistics ....... estim<br>  $R'S = 1$ .<br>
minimize  $R'R$  subject to the condition that  $R'Y_{\lfloor n}^*$ <br>
e to minimize<br>  $E_1 = R'R - 2\}_1 (R'S - 1)$ ,<br>
ere  $\Big|_1$  is the Lagrangian multiplier. Different<br> *m* of concomitants of order statistics ....... estimational<br>  $= 1$ .<br>  $\text{size } R'R \text{ subject to the condition that } R'Y^*_{[n,c]}$ <br>  $\text{minimize}$ <br>  $R'R - 2\}_1 (R'S - 1),$ <br>  $\frac{1}{1}$  is the Lagrangian multiplier. Differentiat<br>  $\text{using to zero, we get } 2R - 2\}_1 S = 0.$ <br>  $\text{S}$ .<br>

That is,

Substituting the value of  $R$  in (12), we get

$$
\bigg\}^{\,1} = \frac{1}{\mathsf{S}^{\,\prime}\mathsf{S}}
$$

Therefore,

$$
R = \frac{S}{S'S}
$$

*ion of concomitants of order statistics* ....... *estima*<br>  $= 1$ .<br>
mize *R'R* subject to the condition that  $R'Y_{[n,c]}^*$ <br>
minimize<br>  $R'R - 2Y_1(R'S - 1)$ ,<br>  $Y_1$  is the Lagrangian multiplier. Differential<br>
quating to zero, we Thus the required unbiased estimator  $\hat{u}_1$  of  $\hat{u}_2$  is given by  $\hat{u}_1 = \frac{S'}{S'S} Y_{[n,c]}^*$ .  $\hat{Y}_1 = \frac{S'}{S'S} Y_{[n,c]}^*$ . (12)<br>
..., we<br>
(13)<br>
respect to<br>  $\frac{S'}{S} Y_{[n,c]}^*$ <br>  $\frac{S'}{S} Y_{[n,c]}^*$ Then by using (8), the variance of  $\hat{u}_1$  is given by  $\frac{1}{s}$ <br>  $\frac{s}{s}$ <br>
required unbiased estimator  $\frac{1}{s}$  of  $\ldots$  is given by  $\frac{1}{s}$ <br>
ising (8), the variance of  $\frac{1}{s}$  is given by<br>  $\frac{1}{s}$ ) = (1 – ...<sup>2</sup>)  $\frac{1}{n} \sum_{i=1}^{n} \frac{1}{s} \cdot \frac{1}{s} + \ldots^2 \frac{s'HS}{(s's)^2}$ <br> 1 1 1 ( ) (1 ) er. Differentiating (13) with <br>  ${}_{1}S = 0$ .<br>
get<br>
get<br>  $\widehat{...}_{1}$  of  $...$  is given by  $\widehat{...}_{1} = \frac{1}{2}$ <br>
given by<br>  $S'HS$ <br>  $(S'S)^{2}$ <br>
to the condition that  $R'Y_{[n,c]}^{*}$  is minimize<br>  $R'R - 2\frac{1}{2}(R'S - 1)$ , (13)<br>  $\}$ , is the Lagrangian multiplier. Differentiating (13) with respect to<br>
quating to zero, we get  $2R - 2\frac{1}{2}S = 0$ .<br>  $\}$ , S.<br>
ting the value of R in (12), we get<br>  $\frac{1}{S'S}$ <br>  $\frac{1$ 1),<br>
(13)<br>
ingian multiplier. Differentiating (13) with respect to<br>
we get  $2R-2$ <sub>1</sub>, s = 0.<br> *i* R in (12), we get<br> *i* R in (12), we get<br>
assed estimator  $\hat{u}_1$  of  $\hat{u}_2$  is given by  $\hat{u}_1 = \frac{S'}{S'S'} Y_{[n,c]}^*$ .<br>
ari minimize<br>  $R'R = 2\frac{1}{2}(R'S - 1)$ , (13)<br>  $\frac{1}{2}$ , is the Lagrangian multiplier. Differentiating (13) with respect to<br>  $\frac{1}{2}$ , is the Lagrangian multiplier. Differentiating (13) with respect to<br>  $\frac{1}{2\sqrt{5}}$ <br>  $\frac{5}{5}$ <br> R and equating to zero, we get  $2R - 2$ ,  $\le 9$  .<br>
That is,<br>  $R = \frac{1}{5}$ .<br>
Substituting the value of R in (12), we get<br>  $\int_1 = \frac{1}{55}$ <br>
Therefore,<br>  $R = \frac{5}{55}$ <br>
Thus the required unbiased estimator  $\frac{5}{10}$  of  $\ldots$  is That is,<br>  $R = \frac{1}{2}$ , S.<br>
Substituting the value of R in (12), we get<br>  $\frac{1}{2} = \frac{1}{5}$ <br>
Therefore,<br>  $R = \frac{S}{S}$ <br>
Thus the required unbiased estimator  $\frac{1}{m_1}$  of  $\ldots$  is given by  $\frac{S}{m_1} = \frac{S'}{S}$ <br>  $V_{\text{p},e,1}^$  $J_1 = \frac{1}{s \text{ s}}$ <br>  $R = \frac{s}{s \text{ s}}$ <br>  $R = \frac{s}{s \text{ s}}$ <br>
the required unbiased estimator  $\frac{1}{n_1}$  of  $\ldots$  is given by  $\frac{1}{n_1} = \frac{s'}{s} Y_{\text{in},d}^*$ <br>  $Var(\frac{1}{n_1}) = (1 - \ldots^2) \frac{1}{n} \sum_{i=1}^n \frac{1}{s_i} \frac{1}{s_i} + \ldots \frac{s'}{(s \text{ s})^2}$ <br> refore,<br>  $R = \frac{S}{S'S}$ <br>
as the required unbiased estimator  $\hat{m}_1$  of  $\hat{m}$  is given by  $\hat{m}_1$ <br>
in by using (8), the variance of  $\hat{m}_1$  is given by<br>  $Var(\hat{m}_1) = (1 - \hat{m}^2) \frac{1}{n} \sum_{i=1}^{n} \frac{1}{i} \frac{1}{i} \frac{1}{i} \frac{1}{i}$ the required unbiased estimator  $\hat{m}_1$  of  $\hat{m}_2$ <br>by using (8), the variance of  $\hat{m}_1$  is given by<br> $r(\hat{m}_1) = (1 - \hat{m}^2) \frac{1}{n} \sum_{i=1}^{n} \hat{m}^2 \frac{1}{i} \frac{1}{s} + \hat{m}^2 \frac{S'HS}{(S^2)^2}$ <br>rly, to minimize *R'HR* subject to

$$
Var(\widehat{...}_{1}) = (1 - ...^{2}) \frac{1}{n} \sum_{i=1}^{n} \frac{1}{2^{i}} \frac{1}{S^{2}} + ...^{2} \frac{S^{'}HS}{(S^{'}S)^{2}}
$$

Similarly, to minimize R'HR subject to the condition that  $R'Y_{[n,c]}^*$  is unbiased

$$
\mathbb{E}_2 = R' H R - 2 \frac{1}{2} (R' S - 1),\tag{14}
$$

where  $\}$ <sub>2</sub> is the Lagrangian multiplier. Differerentiating (14) with respect to *R* and equating to zero, we get

That is,

$$
R=\frac{1}{2},H^{-1}\mathsf{S}\,.
$$

Substituting the value of  $R$  in (12), we get

$$
\big\}_2 = \frac{1}{\mathsf{S}'H^{-1}\mathsf{S}}.
$$

Therefore,

$$
R=\frac{H^{-1}\mathsf{S}}{\mathsf{S}'H^{-1}\mathsf{S}}.
$$

*Veena T. G.*<br> $\frac{H^{-1}S}{S'H^{-1}S}$ .<br>required unbiased estimator  $\widehat{a_2}$  of  $\ldots$  is given by  $\widehat{a_2}$ . Thus the required unbiased estimator  $\hat{f}_{2}$  of  $\hat{f}_{2}$  is given by  $\hat{f}_{2} = \frac{S'H^{-1}}{S'H^{-1}S}Y_{n}^{*}Y_{n}$ . and P. Yageen Thomas<br>  $\sum_{i=2}^{\infty} \frac{1}{1 - s} Y_{[n,c]}^*$  $H^{-1}S$   $^{[n,c]}$  $\hat{w}_{2} = \frac{S'H^{-1}}{I(z-1)}Y_{[n,c]}^{*}.$ *P. Yageen Thomas*<br> $\frac{S'H^{-1}}{S'H^{-1}S}Y_{[n,c]}^*$ .  $-1$  and  $-1$  and  $-1$  and  $-1$  and  $-1$  $\widehat{C}_2 = \frac{S'H^{-1}}{S'H^{-1}S}Y_{[n,c]}^*$ . Then by using (8), the variance of  $\hat{w}_2$  is given by Veena T. G. and P. You<br>
mbiased estimator  $\hat{m}_2$  of  $\hat{m}$  is given by  $\hat{m}_2 = \frac{S'H}{S'H}$ <br>
the variance of  $\hat{m}_2$  is given by<br>  $\hat{m}_2 = \frac{S'H}{S'H}$ <br>  $\hat{m}_1 = \frac{S'H}{I} + \frac{1}{2I} \frac{S'H^{-2}S}{(S'H^{-1}S)^2} + \frac{1}{2I} \frac{S'H^{-1}S}{S'H^{-1}$ 90<br>  $\text{Mean } T. G. \text{ and } P. \text{ Yageen } T.$ <br>  $R = \frac{H^{-1}S}{S'H^{-1}S}.$ <br>
Thus the required unbiased estimator  $\frac{1}{2}$  of ... is given by  $\frac{1}{2} = \frac{S'H^{-1}}{S'H^{-1}S}Y_{[n,c]}^{*}$ <br>
Then by using (8), the variance of  $\frac{1}{2}$  is given by<br>  $Var(\frac{$ *Veena T. G. and P. Yageen Thomas*<br>  $\frac{H^{-1}S}{S'H^{-1}S}$ .<br>  $\frac{1}{S'H^{-1}S}$ .<br>  $\therefore$  required unbiased estimator  $\frac{1}{\cdots 2}$  of  $\cdots$  is given by  $\frac{1}{\cdots 2} = \frac{S'H^{-1}}{S'H^{-1}S}Y_{[n,c]}^*$ .<br>  $\therefore$  using (8), the variance of  $\frac{$ Veena T. G. and P. Yageen Thomas<br>
or  $\hat{H}$  of  $\hat{H}$  is given by  $\hat{H}$  =  $\frac{S'H^{-1}}{S'H^{-1}S}Y_{[n,c]}^*$ .<br>  $\hat{H}$  is given by<br>  $\hat{H}$  is given by<br>  $\hat{H}$  =  $\frac{S'H^{-1}}{S'H^{-1}S}$ <br>  $H$  =  $\hat{H}$  =  $\frac{S'H^{-1}}{S'H^{-1}S}Y_{[n,s]}^*$ 

$$
Var(\widehat{...}_{2}) = (1 - ...^{2}) \frac{1}{n} \sum_{i=1}^{n} \frac{1}{2^{i}} \frac{1}{(s' H^{-1} s)^{2}} + ...^{2} \frac{1}{s' H^{-1} s}
$$

Veena T. G. and P. Yageen Thomas<br>  $=\frac{H^{-1}S}{S'H^{-1}S}$ .<br>
Le required unbiased estimator  $\frac{1}{\cdot 2}$  of ... is given by  $\frac{1}{\cdot 2} = \frac{S'H^{-1}}{S'H^{-1}S}Y_{[n,c]}^*$ .<br>  $\left(\frac{1}{\cdot 2}\right) = (1 - \frac{1}{n})\frac{1}{n}\sum_{i=1}^{n} \frac{1}{2i} \frac{S'H^{-2}S}{(S'H$ Veena T. G. and P.<br>
imator  $\frac{1}{m_2}$  of ... is given by  $\frac{1}{m_2} = \frac{S}{S'}$ <br>
e of  $\frac{1}{m_2}$  is given by<br>  $\frac{S'H^{-2}S}{(S'H^{-1}S)^2} + \frac{1}{S'H^{-1}S}$ <br>  $\frac{*}{[S'H^{-1}S]}$ <br>  $\frac{1}{S'H^{-1}S}$ <br>  $\frac{1}{S'H^{-1}S}Y^*_{[n,c]}$  are<br>
es given by (10 *Veena T. G. and P. Yageen Thomas*<br>
iased estimator  $\frac{1}{n_2}$  of ... is given by  $\frac{1}{n_2} = \frac{S'H^{-1}}{S'H^{-1}S}Y_{[n,c]}^*$ .<br>
variance of  $\frac{1}{n_2}$  is given by<br>  $\frac{1}{n} \sum_{i=1}^n \frac{1}{2i} \frac{S'H^{-2}S}{(S'H^{-1}S)^2} + \frac{1}{n_2} \frac{1}{S'H$ *Veena T. G. and P. Yageen Thomas*<br>
mator  $\hat{H}_{2}$  of  $\hat{H}_{2}$  is given by  $\hat{H}_{2} = \frac{S'H^{-1}}{S'H^{-1}S}Y_{[n,c]}^{*}$ <br>
of  $\hat{H}_{2}$  is given by<br>  $\frac{S'H^{-2}S}{S'H^{-1}S} + \frac{1}{S'H^{-1}S}$ <br>  $\frac{1}{S'H^{-1}S}[H^{-1}S]^{*}$  are two unbiased<br>
s giv Veena T. G. and P. Yageen Thomas<br>  $\frac{H^{-1}S}{S'H^{-1}S}$ .<br>
required unbiased estimator  $\frac{1}{m_2}$  of ... is given by  $\frac{1}{m_2} = \frac{S'H^{-1}}{S'H^{-1}S}Y_{[n,n]}^*$ .<br>
using (8), the variance of  $\frac{1}{m_2}$  is given by<br>  $\frac{1}{m_2} = (1 - \frac$ biased estimator  $\hat{H}_2$  of  $\hat{H}_1$  is given by  $\hat{H}_2 = \frac{S'H^{-1}}{S'H^{-1}S}Y_1$ <br>
e variance of  $\hat{H}_2$  is given by<br>  $\frac{1}{n} \sum_{i=1}^n \frac{1}{n} \sum_{i=1}^n \frac{1}{2i} \frac{1}{(S'H^{-1}S)^2} + \frac{1}{2i} \frac{1}{S'H^{-1}S}$ <br>  $\hat{H}_1 = \frac{S'}{S'S}Y_{1n,c}^*$  $\hat{m}_1 = \frac{S'}{S'S} Y_{[n,c]}^*$  *and*  $\hat{m}_2 = \frac{S'H^{-1}}{S'H^{-1}S} Y_{[n,c]}^*$  are two unbiased *Veena T. G. and P. Yage*<br>
ed estimator  $\frac{1}{n_2}$  of ... is given by  $\frac{1}{n_2} = \frac{S'H^{-1}}{S'H^{-1}}S$ <br>
ariance of  $\frac{1}{n_2}$  is given by<br>  $\frac{n}{\sum_{i=1}^{n_2}} + \frac{1}{2i} \frac{S'H^{-2}S}{(S'H^{-1}S)^2} + \dots^2 \frac{1}{S'H^{-1}S}$ <br>  $\frac{S'}{S'S'} Y_{[n,c]}^*$ of ... is given by  $\hat{H}_{2} = \frac{S'H^{-1}}{S'H^{-1}S}Y_{[n,c]}^{*}$ .<br>
iven by<br>  $H = \frac{2}{\pi} \frac{1}{S'H^{-1}S}$ <br>  $H = \frac{2}{\pi} \frac{S'H^{-1}}{S'H^{-1}S}Y_{[n,c]}^{*}$  are two unbiased<br>
(10) and (11) respectively, then  $\hat{H}_{1}$  is<br>  $H = \frac{1}{\pi} \frac{S'H^{-1}S}{S+1}$  $H^{-1}$ S  $[0, c]$  $\hat{m}_2 = \frac{S'H^{-1}}{I(z)}Y_{[n,c]}^*$  are two unbiased *Veena T. G. and P. Yageen Thomas*<br>
is given by  $\hat{H}_{2} = \frac{S'H^{-1}}{S'H^{-1}S}Y_{[n,c]}^{*}$ ,<br>
by<br>  $\frac{1}{S'H^{-1}S}$ <br>  $\frac{S'H^{-1}}{S'H^{-1}S}Y_{[n,c]}^{*}$  are two unbiased<br>
and (11) respectively, then  $\hat{H}_{n,c}$  is<br>
where  $-1$  $\widehat{C}_2 = \frac{S'H^{-1}}{S'H^{-1}S}Y_{[n,c]}^*$  are two unbiased estimators of  $\ldots$  with variances given by (10) and (11) respectively, then  $\ldots$  is more efficient than  $\widehat{u}_2$  if  $||...|| \leq \sqrt{\frac{K_2}{K_1 + K_2}}$  where 1 2 Veena T. G. and P. Yageen Thomas<br>
estimator  $\frac{1}{m_2}$  of ... is given by  $\frac{1}{m_2} = \frac{S'H^{-1}}{S'H^{-1}S}Y_{[n,e]}^*$ .<br>
ance of  $\frac{m_2}{m_2}$  is given by<br>  $\int_{-2i}^{2} \frac{S'H^{-2}S}{(S'H^{-1}S)^2} + \frac{1}{m^2} \frac{1}{S'H^{-1}S}$ <br>  $\int_{-2i}^{i} \frac{1}{$ *Veena T. G. and P. Yageen Thomas*<br> *Kor*  $\frac{1}{m_2}$  of ... is given by  $\frac{1}{m_2} = \frac{S'H^{-1}}{S'H^{-1}S}Y_{[n,c]}^*$ .<br>  $\frac{1}{m_2}$  is given by<br>  $\frac{H^{-2}S}{S'H^{-1}S} + \frac{1}{S'H^{-1}S}Y_{[n,c]}^*$  are two unbiased<br> *Koren by* (10) and (11) r  $+ K_2$ where **with**  $ar(\hat{x}_2) = (1 - \hat{x}^2) \frac{1}{n} \sum_{i=1}^n \frac{1}{2i} \frac{1}{(S'H^{-1}S)^2} + \dots^2 \frac{1}{S'H^{-1}S}$ <br> **rem 2.2** If  $\hat{x}_1 = \frac{S'}{S'S} Y_{[n,c]}^*$  and  $\hat{x}_2 = \frac{S'H^{-1}}{S'H^{-1}S} Y_{[n,c]}^*$ <br>
ators of  $\hat{x}_1$  with variances given by (10) and (11) respectivel  $\frac{H^{-1}S}{S'H^{-1}S}$ .<br>
required unbiased estimator  $\frac{1}{m^2}$ <br>
using (8), the variance of  $\frac{1}{m^2}$  is<br>  $\frac{1}{m^2}$  =  $(1-\dots^2)\frac{1}{n}\sum_{i=1}^n + \frac{1}{2i}\frac{S'H^{-2}S}{(S'H^{-1}S)}$ <br>
n 2.2 If  $\frac{1}{m} = \frac{S'}{S'S}Y_{[n,c]}^*$  and<br>
rs of ... *H*<sup>-1</sup>S<br> *s'H*<sup>-1</sup>S<br>
required unbiased estimator  $\frac{1}{m_2}$  of  $\ldots$  is given by<br>
using (8), the variance of  $\frac{1}{m_2}$  is given by<br>  $\sum_2$ ) =  $(1-\ldots^2)\frac{1}{n}\sum_{i=1}^n 1\frac{1}{2i} \frac{S'H^{-2}S}{(S'H^{-1}S)^2} + \ldots^2 \frac{1}{S'H^{-1}S}$ <br> *H*<sup>-1</sup>S<br> *s*<sup>-1</sup>*S*<br> *s* (*8*), the variance of  $\frac{1}{n^2}$  is given b<br>  $\frac{1}{n^2}$ <br>  $\left(1 - \frac{1}{n^2}\right) \frac{1}{n} \sum_{i=1}^{n} \frac{1}{2i} \frac{1}{(s'H^{-1}s)^2} + \frac{1}{n^2} \frac{1}{s^2}$ <br> **2.2** *If*  $\frac{1}{n^2} = \frac{s'}{s's} Y_{[n,c]}^*$  *and*  $\frac{1}{n^2} =$ Veena T. G. and P. Yageen Thomas<br>  $= \frac{H^{-1}S}{S'H^{-1}S}$ .<br>
the required unbiased estimator  $\frac{5}{12}$  of ... is given by  $\frac{5}{12} = \frac{S'H^{-1}}{S'H^{-1}S}Y_{[n,c]}^{*}$ .<br>
y using (8), the variance of  $\frac{5}{12}$  is given by<br>  $(\frac{5}{12}) = ($ Veena T. G. and P. Yageen Thomas<br>  $V = \frac{1}{15}$ <br>
quired unbiased estimator  $\frac{1}{\alpha_2}$  of ... is given by  $\frac{1}{\alpha_2} = \frac{S'H^{-1}}{S'H^{-1}S}Y_{(n,c)}^*$ <br>  $V_{(n,c)}^*$ <br>  $V_{(n,c)}^*$  is given by<br>  $V_{(n,c)}^* = (1 - \frac{1}{n})\frac{1}{n} \sum_{i=1}^{n} \frac{$ rem 2.2 If  $\pi_1 = \frac{1}{S^2} Y_{[n,c]}$  and  $\pi_2 = \frac{1}{S^2 H^{-1} S} Y_{[n,c]}$ <br>ators of  $\pi_2$  with variances given by (10) and (11) re<br>efficient than  $\pi_2$  if  $|\pi_1| \le \sqrt{\frac{K_2}{K_1 + K_2}}$  where<br> $\pi_1 = \frac{S^2 H S}{(S^2 S)^2} - \frac{1}{S^2 H^{-1} S}$ <br> ed estimator  $\frac{1}{m_2}$  of ... is given by  $\frac{1}{m_2} = \frac{S'H^{-1}}{S'H^{-1}}$ <br>
riance of  $\frac{1}{m_2}$  is given by<br>  $\frac{n}{s} + \frac{1}{2i} \frac{S'H^{-2}S}{(S'H^{-1}S)^2} + \frac{1}{s'H^{-1}S}$ <br>  $\frac{S'}{S'S'} Y_{[n,c]}^*$  and  $\frac{m_2}{m_2} = \frac{S'H^{-1}}{S'H^{-1}S} Y_{[n,c]}^*$ equired unbiased estimator  $\hat{u}_2$  of ...<br>
sing (8), the variance of  $\hat{u}_2$  is given<br>  $y_1 = (1 - \hat{u}^2) \frac{1}{n} \sum_{i=1}^n \frac{1}{2i} \frac{s'H^{-2}s}{(s'H^{-1}s)^2} + \dots$ <br>
2.2 If  $\hat{u}_1 = \frac{s'}{s's} Y_{[n,e]}^*$  and  $\hat{u}_2 =$ <br>
of ... with variances fired unbiased estimator  $\frac{1}{m_2}$  of ... is given by  $\frac{1}{m_2} = \frac{S'H'}{S'H'}$ <br>
(8), the variance of  $\frac{1}{m_2}$  is given by<br>  $(1-\frac{1}{m^2})\frac{1}{n}\sum_{i=1}^{n}1\frac{1}{2i}\frac{S'H'^2S}{(S'H'^2S)^2} + \frac{1}{m^2}\frac{1}{S'H'^2S}$ <br>
2 If  $\frac{1}{m_1} = \frac$ <sup>11</sup>S.<br> *H*<sup>-1</sup>S.<br> *A*<sub>*I*</sub> (8), the variance of  $\frac{1}{n^2}$  is given by  $\frac{1}{n^2} = \frac{S'H^{-1}}{S'H^{-1}}Y_{[n,c]}^*$ ,<br> *mg* (8), the variance of  $\frac{1}{n^2}$  is given by<br>  $= (1 - \frac{1}{n^2})\frac{1}{n}\sum_{i=1}^n \frac{1}{i^2} \frac{S'H^{-2}S}{(S'H^{-1}S)^2} + \$ *H* <sup>1</sup>S<br>
quired unbiased estimator  $\frac{1}{m_2}$  of ... is given by  $\frac{1}{m_2} = \frac{S'H^{-1}}{S'H^{-1}}$ <br>
ing (8), the variance of  $\frac{m_2}{m_2}$  is given by<br>  $\vec{B} = (1 - \frac{m^2}{m_2}) \frac{1}{n_2} \frac{m_1}{m_2} + \frac{2}{n_2} \frac{S'H^{-2}S}{(S'H^{-1}S)^2} + \frac$ *R* =  $\frac{H \cdot S}{S'H^{-1}S}$ .<br>
Thus the required unbiased estimator  $\frac{2}{12}$  of ... is given by  $\frac{2}{12} = \frac{S'H^{-1}}{S'H^{-1}S}Y_{[0,2]}^+$ .<br>
Then by using (8), the variance of  $\frac{2}{12}$  is given by<br>  $Var(\frac{2}{12}) = (1 - \frac{2}{12})\frac{1}{12}$ y using (8), the variance of  $\frac{1}{n^2}$  is given by<br>  $(\frac{1}{n^2}) = (1 - \frac{1}{n^2}) \frac{1}{n^2} \int_{1}^{n^2} \frac{1}{12i} \frac{1}{(5'H^{-1}S)^2} + \frac{1}{n^2} \frac{1}{5'H^{-1}S}$ <br>  $\frac{1}{5TH^{-1}S}$ <br>  $\frac{1}{12}$ <br>  $\frac{1}{12}$ <br>  $\frac{1}{12}$ <br>  $\frac{1}{12}$ <br>  $\frac{1}{12}$ <br> *Var*( $\frac{1}{\alpha_2}$ ) = (1 - ...<sup>2</sup>)  $\frac{1}{n} \sum_{i=1}^{\infty} \frac{1}{i} \frac{s^2 H^{-2} s}{(s^2 H^{-1} s)^2} + ...^2 \frac{1}{s^2 H^{-1} s}$ <br> **i corem 2.2** *If*  $\frac{1}{\alpha_1} = \frac{s^2}{s^2} Y_{(s,c)}^*$  *and*  $\frac{1}{\alpha_2} = \frac{s^2 H^{-1}}{s^2 H^{-1} s} Y_{(s,c)}^*$  are two unbiase  $\frac{1}{n}\sum_{i=1}^{n} \frac{1}{2i} \frac{1}{(s'H^{-1}s)^2} + \frac{1}{s'H^{-1}s}$ <br>  $= \frac{s'}{s's} Y_{[n,c]}^* \text{ and } \frac{1}{n^2 s} = \frac{s'H^{-1}}{s'H^{-1}s} Y_{[n,c]}^* \text{ are two unbiased  
variances given by (10) and (11) respectively, then  $\frac{1}{n_1}$  is  
at  $2 \text{ if } |m| \le \sqrt{\frac{K_2}{K_1 + K_2}}$  where  

$$
\frac{1}{s's} \frac{1}{n} \sum_{i=1}^{n} \frac{1}{
$$$ using (8), the variance of  $\frac{2}{n_2}$  is given by<br>  $\frac{2}{n_2}$  =  $(1-\frac{2}{n_2})\frac{1}{n_2}\frac{n_2}{n_3} + \frac{3}{2n_3}\frac{5'H^{-3}S}{(S'H^{-1}S)^2} + \frac{3}{2} \frac{1}{S'H^{-1}S}$ <br> **n** 2.2 If  $\frac{n_1}{n_1} = \frac{5'}{8} Y_{[n,c]}^s$  and  $\frac{n_2}{n_2} = \frac{5'H^{-1}}{8'H^{-1}S$ e of  $\frac{S'H^{-2}S}{(S'H^{-1}S)^2} + \dots^2 \frac{1}{S'H^{-1}S}$ <br>  $\frac{s'H^{-2}S}{(S'H^{-1}S)^2} + \dots^2 \frac{1}{S'H^{-1}S}Y_{[n,c]}^*$  are two<br>
es given by (10) and (11) respectively,<br>  $K\sqrt{\frac{K_2}{K_1+K_2}}$  where<br>  $\frac{1}{S'S} + \dots^2 \frac{1}{S'H^{-1}S} + K_1 \dots^2$ ,<br>  $\frac{1}{S'S} + \dots$ using (8), the variance of  $\frac{5}{a_2}$  is given by<br>  $\binom{5}{a_2} = (1 - \frac{3}{a_1}) \frac{1}{n} \sum_{i=1}^{n} \frac{1}{t_i^2} \frac{1}{(s'H^{-1}s)^2} + \frac{1}{s'(H^{-1}s)}$ <br> **n** 2.2 If  $\frac{5}{a_1} = \frac{s'}{s} Y_{\lfloor n, s \rfloor}$  and  $\frac{5}{a_2} = \frac{s'H^{-1}}{s'(H^{-1}s)} Y_{\lfloor n, s \rfloor}$  a (angle in the space of  $\frac{1}{n} \sum_{i=1}^{n} Y_i^*$  (s'  $H^{-1}S$ )<sup>2</sup> if  $S'$   $H^{-1}S$ <br>
(m 2.2 If  $\therefore \frac{1}{n} = \frac{S'}{S} Y_{[n,c]}^*$  and  $\therefore \frac{1}{2} = \frac{S'H^{-1}}{S'H^{-1}} Y_{[n,c]}^*$  are two unbiased<br>
ors of ... with variances given by (10) and **Example 2.2** If  $\hat{L}_1 = \frac{5'}{5'} \sum_{k=1}^{K} Y_{k,k}^2$  and  $\hat{L}_2 = \frac{5'H^{-1}}{5'} Y_{k,k}^*$  are two unbiased<br>
mators of ... with variances given by (10) and (11) respectively, then  $\hat{L}_1$  is<br>
re efficient than  $\hat{L}_2$  if  $|...|<$  $=\frac{S'}{S'S'}I_{(n,c)}^*$  and  $\hat{=}$   $=\frac{S'H^{-1}}{S'H^{-1}S}I_{(n,c)}^*$  are two unbiased<br>variances given by (10) and (11) respectively, then  $\hat{=}$  is<br>if  $|...|< \sqrt{\frac{K_2}{K_1+K_2}}$  where<br> $-\frac{1}{S'S}\left|\frac{1}{n}\sum_{i=1}^{n}1\frac{2}{2i}\right|$ .<br> $-\frac{1}{S'S}\left|\$  $\frac{5}{2}$  =  $(1 - \frac{3}{2}) \frac{1}{R} \sum_{1}^{R} \frac{1}{3} \frac{3}{(S'H^{-1}S)^2} + \frac{3}{S'H^{-1}S}$ <br> **n** 2.2 If  $\frac{5}{2} \sum_{1}^{R} \sum_{n=1}^{R} \frac{1}{2} \frac{1}{(S'H^{-1}S)^2} + \frac{3}{S'H^{-1}S} \sum_{1}^{R} \sum_{1}^{R} \sum_{1}^{R} \sum_{1}^{R} \sum_{1}^{R} \sum_{1}^{R} \sum_{1}^{R} \sum_{1}^{R} \sum_{1}^{R} \$  $(s'H^{-1}s)^2$   $s'H^{-1}s$ <br>
\*<br>
\*<br>
\*<br>
\*<br>
\*<br>
\*<br>
\*<br>
\*<br>  $s'(H^{-1}s)^2 = s'H^{-1}s$ <br>  $s'(H^{-1}s)^*$ <br>  $s'(H^{-1}s)^*$ <br>  $s'(H^{-1}s)^*$ <br>  $s'(H^{-1}s)^*$ <br>  $s'(H^{-1}s)^*$ <br>  $s'(H^{-1}s)^* + K_1 \dots^2$ <br>  $s'(H^{-1}s)^* + K_1 \dots^2$ <br>  $s'(H^{-1}s)^* + K_2(1 - \dots^2)$ <br>  $s'(H^{-1}s)^* + K_2(1 - \dots^2)$ <br>  $s'(H^{-1}s)^* + K_$  $E_2$ ) = (1 – *x*<sup>2</sup>)  $\frac{1}{n} \sum_{i=1}^{n} \frac{1}{i} \frac{2i}{3} \frac{(3i-3i-3i)}{(5i-1i-5i)^2} + \cdots^2 \frac{5}{5} \frac{4i-3i}{3} \frac{5i}{5}$ <br> **n** 2.2 *H*  $\frac{1}{n_1} = \frac{5^2}{55} Y_{\text{final}}^4$  *and*  $\frac{1}{n_2} = \frac{5^2 H^{-1}}{5^2 H^{-1} 5} Y_{\text{final}}^4$  are two unbi estimators of ... with variances given by (10) as<br>
more efficient than  $\frac{1}{n^2}$  if  $|\ln |\le \sqrt{\frac{K_2}{K_1 + K_2}}$  whe<br>  $K_1 = \frac{S'HS}{(S^2S)^2} - \frac{1}{S'H^{-1}S}$ <br>
and<br>  $K_2 = \left[ \frac{S'H^{-2}S}{(S'H^{-1}S)^2} - \frac{1}{S^2S} \right] \frac{1}{n} \sum_{i=1}^n \frac{1}{2i$ So d:... with variances given by (10) and (11) res<br>
cient than  $\frac{1}{n^2}$  if  $|\ln |\le \sqrt{\frac{K_2}{K_1 + K_2}}$  where<br>  $\frac{S'HS}{(S'S)^2} - \frac{1}{S'H^{-1}S}$ <br>  $\left[\frac{S'H^{-2}S}{(S'H^{-1}S)^2} - \frac{1}{S'S}\right] \frac{1}{n} \sum_{i=1}^n \frac{1}{2i}$ .<br>  $\left[\frac{S'H^{-2}S}{(S'H^{-1}S$ estimators of ... with variances given by (10) and (11) respectively, then ..., is<br>
more efficient than ...<sub>2</sub> if  $|-|< \sqrt{\frac{K_2}{K_1 + K_2}}|$  where<br>  $K_1 = \frac{S'HS}{(S+S)^2} = \frac{1}{S'H^{-1}S}$ <br>
and<br>  $K_2 = \left[ \frac{S'H^{-2}S}{(S'H^{-1}S)^2} - \frac{1}{S'S}$ 

$$
K_1 = \frac{S'HS}{(S'S)^2} - \frac{1}{S'H^{-1}S}
$$

and

$$
K_2 = \left[ \frac{S'H^{-2}S}{(S'H^{-1}S)^2} - \frac{1}{S'S}\right] \frac{1}{n} \sum_{i=1}^n \frac{1}{2i}.
$$

Since either that 
$$
m_2 = 1 + \frac{1}{1} + \frac{1}{1}
$$

$$
Var(\dots_1) = (1 - \dots) \frac{1}{n} \sum_{i=1}^{n} \frac{1}{2i} \frac{1}{S} + \dots \frac{1}{S'H^{-1}S} + K_1 \dots,
$$
\n
$$
Var(\dots_2) = (1 - \dots^2) \frac{1}{n} \sum_{i=1}^{n} \frac{1}{S} + \frac{2}{2i} \frac{1}{S'S} + \dots^2 \frac{1}{S'H^{-1}S} + K_2(1 - \dots^2)
$$
\n
$$
Var(\dots_2) = (1 - \dots^2) \frac{1}{n} \sum_{i=1}^{n} \frac{1}{S'S} + \dots^2 \frac{1}{S'H^{-1}S} + K_2(1 - \dots^2)
$$
\n
$$
Var(\dots_1) < Var(\dots_2) \text{ if } K_1 \dots^2 < K_2(1 - \dots^2).
$$
\n
$$
Var(\dots_1) < Var(\dots_2) \text{ if } K_1 \dots^2 < K_2(1 - \dots^2).
$$
\n
$$
Var(\dots_1) < Var(\dots_2) \text{ if } K_1 \dots^2 < K_2(1 - \dots^2).
$$
\n
$$
Var(\dots_1) < Var(\dots_2) \text{ if } K_1 \dots^2 < K_2(1 - \dots^2).
$$
\n
$$
Var(\dots_1) < Var(\dots_2) \text{ if } K_1 \dots^2 < K_2(1 - \dots^2).
$$
\n
$$
Var(\dots_1) < Var(\dots_2) \text{ if } K_1 \dots^2 < K_2(1 - \dots^2).
$$
\n
$$
Var(\dots_1) < Var(\dots_2) \text{ if } K_1 \dots^2 < K_2(1 - \dots^2).
$$
\n
$$
Var(\dots_1) < Var(\dots_2) \text{ if } K_1 \dots^2 < K_2(1 - \dots^2).
$$
\n
$$
Var(\dots_1) < Var(\dots_2) \text{ if } K_1 \dots^2 < K_2(1 - \dots^2).
$$
\n
$$
Var(\dots_1) < Var(\dots_2) \text{ if } K_1 \dots^2 < K_2(1 - \dots^2).
$$
\n
$$
Var(\dots_1) <
$$

From  $(15)$  and  $(16)$ , we have

From (15) and (16), we have  
\n
$$
Var(\widehat{...}_1) < Var(\widehat{...}_2)
$$
 if  $K_1 \dots^2 < K_2(1 - \dots^2)$ .  
\nThus  $\widehat{...}_1$  is more efficient than  $\widehat{...}_2$  if  
\n
$$
\frac{K_2}{K_1 + K_2}
$$
.  
\nThat is,  
\n
$$
|\dots| < \sqrt{\frac{K_2}{K_1 + K_2}}
$$
.

is more efficient than  $\hat{u}_2$  if

$$
...^{2} < \frac{K_{2}}{K_{1} + K_{2}}.
$$

That is,

$$
|\ldots| < \sqrt{\frac{K_2}{K_1 + K_2}}.
$$

# **3. ESTIMATION OF COMMON PARAMETERS**  $\sim_2$  AND  $\uparrow_2$

Application of concomitants of order statistics ....... estimation  
\n3. ESTIMATION OF COMMON PARAMETERS ~2 AND 
$$
\uparrow
$$
 2  
\nLet  $(X_i, Y_i)$ ,  $i = 1, 2, ..., n$  be independent bivariate random variables with  
\n $(X_i, Y_i)$  having pdf  $f_i(x, y)$  of the form  
\n
$$
\frac{(2f\uparrow_1\uparrow_2)^{-1}}{\sqrt{1-\frac{y^2}{n}}}\exp\left\{\frac{-2^{-1}}{1-\frac{y^2}{n}}\left[\frac{(x-\frac{y}{n})^2}{\frac{y^2}{n}}-2\frac{(x-\frac{y}{n})(y-\frac{y}{n})}{\frac{y^2}{n}}+\frac{(y-\frac{y}{n})^2}{\frac{y^2}{n}}\right]\right\}
$$
\nfor  $i = 1, 2, ..., n$ . In this section we estimate the common parameters ~2 and  
\n $\uparrow$  under the assumption that  $j = 1, 2, ..., n$  are known using concomitants of

*lication of concomitants of order statistics* ....... *estimation*<br>
3. **ESTIMATION OF COMMON PARAMETERS**  $\sim_2$  **AND**  $\uparrow_2$ <br>  $(X_i, Y_i)$ ,  $i = 1, 2, ..., n$  be independent bivariate random variables with<br>  $(Y_i)$  having  $pdf \, f_i(x, y$ promitants of order statistics ....... estimation<br> **MATION OF COMMON PARAMETERS**  $\sim_2$  **AND**  $\uparrow$ <sub>2</sub><br>
=1,2,...,*n* be independent bivariate random variables wit<br>
pdf  $f_i(x, y)$  of the form<br>  $\exp\left\{\frac{-2^{-1}}{1-\frac{x^2}{n^2}}\left[\frac{(x$ *ation of concomitants of order statistics* ....... *estimation* 91<br> **3. ESTIMATION OF COMMON PARAMETERS**  $\sim_2$  **AND**  $\uparrow_2$ <br>  $(X_i, Y_i), i = 1, 2, ..., n$  be independent bivariate random variables with<br>
() having pdf  $f_i(x, y)$  of *a of concomitants of order statistics* ....... *estimation*<br> **STIMATION OF COMMON PARAMETERS**  $\sim_2$  **AND**  $\uparrow_2$ <br>  $Y_i$ ,  $i = 1, 2, ..., n$  be independent bivariate random variables with<br>
viving pdf  $f_i(x, y)$  of the form<br>  $\frac{$ concomitants of order statistics ....... estimation<br> **IMATION OF COMMON PARAMETERS**  $\sim_2$  **AND**  $\uparrow_2$ <br>  $i = 1, 2, ..., n$  be independent bivariate random variables with<br>  $\log pdf f_i(x, y)$  of the form<br>  $-\exp\left\{\frac{-2^{-1}}{1 - \frac{y^2}{n_1}} \left$ ion of concomitants of order statistics ....... estimation<br> **ESTIMATION OF COMMON PARAMETERS**  $z_2$  AND  $\uparrow z_2$ <br>  $\left(\frac{1}{2}, Y_i\right), i = 1, 2, ..., n$  be independent bivariate random variables with<br>
thaving  $pdf \ f_1(x, y)$  of the form<br> Application of concomitants of order statistics ....... estimation 91<br>
3. **ESTIMATION OF COMMON PARAMETERS**  $\sim_2$  AND  $\uparrow_2$ <br>
Let  $(X_i, Y_i)$ ,  $i = 1, 2, ..., n$  be independent bivariate random variables with  $(X_i, Y_i)$  having pdf for  $i=1,2,...,n$ . In this section we estimate the common parameters  $\sim_2$  and  $\dagger$ , under the assumption that  $\ldots$ ;  $i = 1, 2, \ldots, n$  are known using concomitants of order statistics of *inid* random variables.

lication of concomitants of order statistics ....... estimation 91<br>
3. **ESTIMATION OF COMMON PARAMETERS**  $\sim_2$  **AND**  $\uparrow_2$ <br>  $(X_i, Y_i)$ ,  $i = 1, 2, ..., n$  be independent bivariate random variables with<br>  $\downarrow$ ,  $\uparrow$ ) having pd Clearly the marginal distributions of  $X_i$  and  $Y_i$  are  $N(\sim_1, \cdot)$ FIERS  $\sim_2$  AND  $\uparrow_2$ <br>iate random variables with<br> $\left(\frac{y - \sim_2}{\uparrow_2} + \frac{(y - \sim_2)^2}{\uparrow_2^2}\right)$ <br>common parameters  $\sim_2$  and<br>known using concomitants of<br>are  $N(\sim_1, \uparrow_1)$  and  $N(\sim_2, \uparrow_2)$ <br>denote the marginal pdf and<br> $\$ Application of concomitants of order statistics ....... estimation 91<br> **3. ESTIMATION OF COMMON PARAMETERS**  $-2$  **AND**  $\uparrow$ <sub>2</sub><br>
Let  $(X_i, Y_i)$  having  $pdf \ f_i(x, y)$  of the form<br>  $\frac{(2f\uparrow, \uparrow, )^{-1}}{\sqrt{1-\frac{2}{\sqrt{1-\cdots}}}}exp\left\{\frac{-2^{-1}}{$ distribution function respectively of each of the  $X_i$ 's. Let  $f_i(y|x)$  denote the FIERS  $\sim_2$  AND  $\uparrow_2$ <br>te random variables with<br> $\left[\frac{y - \sim_2}{2} + \frac{(y - \sim_2)^2}{\uparrow_2^2}\right]$ <br>ommon parameters  $\sim_2$  and<br>nown using concomitants of<br>re  $N(\sim_1, \uparrow_1)$  and  $N(\sim_2, \uparrow_2)$ <br>enote the marginal *pdf* and<br>'s. Let  $f$ conditional *pdf* of  $Y_i$  given  $X_i = x$ ,  $i = 1, 2, ..., n$ . Application of concomitants of order statistics ....... estimation<br>
3. **ESTIMATION OF COMMON PARAMETERS**  $\tau_2$  **AND**  $\tau_1$ <br>
Let  $(X_i, Y_i)$  having pdf  $f_i(x, y)$  of the form<br>  $\frac{(2f_1^{\dagger} + 1_2)^{-1}}{\sqrt{1 - x_i^2}} \exp\left\{\frac{-2^{-1}}{1 - x_i$ 

Application of concomitants of order statistics ......, estimation  
\n3. ESTIMATION OF COMMON PARMATTERS 
$$
\sim_2
$$
 AND 1,  
\nLet  $(X_1, Y_1)$ , i=1,2,...,n be independent bivariate random variables with  
\n $(X_1, Y_1)$  having pdf  $f_1(x, y)$  of the form  
\n
$$
\frac{(2f_{1+1}^{-1})^2}{\sqrt{1-\frac{1}{n}}}\exp\left\{\frac{-2r_1}{1-\frac{1}{n}}\left[\frac{(x-x_1)^2}{1-\frac{1}{n}}-2\omega_{1} - \frac{(x-x_1)(y-x_2)}{1+\frac{1}{n}}+\frac{(y-x_2)^2}{1-\frac{2}{n}}\right]\right\}
$$
\nfor  $i=1,2,...,n$ . In this section we estimate the common parameters  $\sim_3$  and  
\n $\uparrow$ , under the assumption that  $\omega_{11}i=1,2,...,n$  are known using concomitants of  
\norder statistics of *ind* random variables.  
\nrespectively for  $i=1,2,...,n$ . Let  $f(x)$  and  $Y_i$  are  $N(\sim_i, \uparrow, \uparrow)$  and  $N(\sim_i, \uparrow, \uparrow)$   
\nrespectively for  $i=1,2,...,n$ . Let  $f(x)$  and  $F(x)$  denote the marginal *pdf* and  
\ndistribution function respectively of each of the  $X_i$ 's. Let  $f_i(y|x)$  denote the  
\nconditional *pdf* of  $Y_i$  given  $X_i = x$ .  $i=1,2,...,n$ .  
\nThen we have,  
\n
$$
E[Y_{i=0}^2] = \frac{1}{n}\sum_{i=1}^{n-1}(-\frac{2}{i}+1\frac{2}{3}(1-\omega_{i}^2)+2\omega_{1}^2+5\omega_{12}+\frac{1}{3}\omega_{11}+\frac{2}{3}\omega_{12}+\frac{2}{3}\omega_{13}+\frac{2}{3}\omega_{14}+\frac{2}{3}\omega_{15}+\frac{2}{3}\omega_{16}+\frac{2}{3}\omega_{17}+\frac{2}{3}\omega_{18}+\frac{2}{3}\omega_{19}+\frac{2}{3}\omega_{10}+\frac{2}{3}\omega_{10}+\frac{2}{3}\omega_{11}+\frac{2}{3}\omega_{12}
$$

$$
Var(Y_{[r:n]}) = \frac{1}{2} + (S_{r,r:n} - 1) \frac{1}{n} \sum_{i} \frac{1}{n^{2}} + \frac{1}{2} \frac{1}{n^{2}} \frac{1}{2} \sum_{i} \sum_{i < j} (\frac{1}{n^{2}} - \frac{1}{n^{2}})^{2}
$$
 (18)

$$
E\left[Y_{[rn]}\right] = \frac{1}{n} \sum_{i} \left[1 - \frac{1}{n} \sum_{i} \left(1 - \frac{1}{n} \sum_{i} \frac{1}{n} \right) + 2 - \frac{1}{n} \sum_{i} \frac{
$$

Hence,

$$
Cov\Big[Y_{[r:n]}Y_{[s:n]}\Big] = E\Big[Y_{[r:n]}Y_{[s:n]}\Big] - E\Big[Y_{[r:n]}\Big]E\Big[Y_{[s:n]}\Big]
$$
  
= 
$$
\frac{1}{n(n-1)}\Big|_{2}^{2}S_{r,s:n}\sum_{i \neq j}\cdots_{i} \cdots_{j} - \frac{1}{n^{2}(n-1)}\Big|_{2}^{2}T_{r,n}\Gamma_{s:n}\sum_{i < j}(\cdots_{i} - \cdots_{j})^{2}.
$$
 (19)

Consider the units of bivariate sample in which measurement of the *X* variate can be done easily where as a measurement of *Y* is not so easy or economic. In this case we order the *X* observations and make measurements only on the *Veena T. G. and P. Yageen Thomas*<br>Consider the units of bivariate sample in which measurement of the *X* variate<br>can be done easily where as a measurement of *Y* is not so easy or economic. In<br>this case we order the *X* estimates based on the available concomitants of order statistics. *92*<br> *Veena T. G. and P. Yageen TR*<br>
Consider the units of bivariate sample in which measurement of the X va<br>
can be done easily where as a measurement of Y is not so easy or econom<br>
this case we order the X observations *Veena T. G. and P. Yageen Thomas*<br>
sisider the units of bivariate sample in which measurement of the X variate<br>
be done easily where as a measurement of Y is not so easy or economic. In<br>
contitants  $Y_{[t-r;1]} \dots Y_{[t-r;2]}$ . *Yeena T. G. and P. Yageen Tho*<br>
Consider the units of bivariate sample in which measurement of the *X* var<br>
can be done easily where as a measurement of *Y* is not so easy or economic<br>
this case we order the *X* observat *Veena T. G. and*<br>
sily where as a measurement of *Y* is not so easy<br>
sily where as a measurement of *Y* is not so easy<br>
order the *X* observations and make measurement<br>  $Y_{[c+1:n]},..., Y_{[n-c:n]}.$  Now based on this restricted si *Veena T. G. and P. Yageen Thomas*<br>
isider the units of bivariate sample in which measurement of the *X* variate<br>
be done easily where as a measurement of *Y* is not so easy or economic. In<br>
case we order the *X* observat *Veena T. G. and P. Yageen Thomas*<br>
sisider the units of bivariate sample in which measurement of the X variate<br>
be done casily where as a measurement of Y is not so casy or economic. In<br>
case we order the X observations an be done easily where as a measurement of *Y* is not so easy or econon<br>
bis case we order the *X* observations and make measurements only<br>
oncominants  $Y_{(c+lm)},..., Y_{(n-cn)}$ . Now based on this restricted sample we n<br>
stimate the variance that is  $Y_{[r+k]}$ ,  $\cdots$ ,  $Y_{[n-cn]}$ . Now based on this restricted sample we may<br>attes based on the available concomitants of order statistics.<br>and  $\pi_{n,c} = (Y_{[r+k]1}, Y_{[r+k]2}, \cdots, Y_{[n-cn]})'$ .<br> $[(Y_{[n,c]}] = -2 + 1 + 2$ .<br>order the *X* observations and make measurements only on the  $Y_{\{i+1:n\},\ldots,Y_{\{n-e:n\}}\}}$ . Now based on this restricted sample we may use<br>ed on the available concomitants of order statistics.<br>  $x+\tan Y_{\{i+2:n\}}\ldots Y_{\{n-e:n\}}'$ .<br> Veena T. G. and P. Yageen Thomas<br>
or the units of bivariate sample in which measurement of the X variate<br>
done easily where as a measurement of Y is not so easy or economic. In<br>
se we order the X observations and make mea Veeno T. G. and P. November 1 Weights are a measurement of the X variates<br>
units of bivariate same<br>
units of which measurement of Y is not so easy or ecconomic. In<br>
order the X observations and make measurements only on t

Let

$$
Y_{[n,c]} = (Y_{[c+1:n]}, Y_{[c+2:n]}, \ldots, Y_{[n-c:n]})'.
$$

$$
E[Y_{[n,c]}] = -21 + \frac{1}{2}r \tag{20}
$$

$$
\Gamma = \frac{1}{n} \sum_{i=1}^{n} ..._{i} (\Gamma_{c+1:n}, \Gamma_{c+2:n}, ..., \Gamma_{n-c:n})'
$$

Let  
\n
$$
Y_{[n,c]} = (Y_{[c+1:n]}, Y_{[c+2:n]}, ..., Y_{[n-c:n]})'.
$$
\nThen  
\n
$$
E[Y_{[n,c]}] = -21 + 12r
$$
\nwhere 1 is a column vector of  $n - 2c$  ones,  
\n
$$
r = \frac{1}{n} \sum_{i=1}^{n} ..._{i} (r_{c+1:n}, r_{c+2:n}, ..., r_{n-c:n})'
$$
\nand the variance covariance matrix of  $Y_{[n,c]}$  can be written in the form  
\n
$$
D[Y_{[n,c]}] = 12c
$$
\nwhere  
\n
$$
(21)
$$

where

$$
\Gamma = \frac{1}{n} \sum_{i=1}^{n} ..._{i} (\Gamma_{c+1:n}, \Gamma_{c+2:n}, ..., \Gamma_{n-c:n})'
$$
  
the variance covariance matrix of  $Y_{[n,c]}$  can be wi  

$$
D[Y_{[n,c]}] = \frac{1}{2}G,
$$
  
ere  

$$
= ||g_{rs}||
$$
 given by  

$$
g_{rr} = 1 + (S_{r,rn} - 1) \frac{1}{n} \sum_{i} ..._{i}^{2} + \frac{1}{n^{2}} \Gamma_{rn}^{2} \sum_{i < j} (..._{i} - ..._{j})^{2}
$$
  
for  $r \neq s$ ,  

$$
g_{rs} = \frac{1}{n^{2}} S_{rs} \sum_{i} ... \sum_{i} \frac{1}{n^{2}} \sum_{j} (..._{j} - ..._{j})^{2}
$$

mates based on the available concomitants of order statistics.

\n
$$
Y_{[n,c]} = (Y_{[c+1:n]}, Y_{[c+2:n]}, \dots, Y_{[n-c:n]})'.
$$
\nin

\n
$$
E[Y_{[n,c]}] = -21 + 12r
$$
\nwhere 1 is a column vector of  $n - 2c$  ones,

\n
$$
r = \frac{1}{n} \sum_{i=1}^{n} \dots \left( r_{c+1:n}, r_{c+2:n}, \dots, r_{n-c:n} \right)'
$$
\nthe variance covariance matrix of  $Y_{[n,c]}$  can be written in the form

\n
$$
D[Y_{[n,c]}] = 12r^2 G,
$$
\nwhere

\n
$$
= \|g_{rs}\| \text{ given by}
$$
\n
$$
g_{rr} = 1 + (S_{r,rn} - 1) \frac{1}{n} \sum_{i} \dots \sum_{i=1}^{2} \frac{1}{n^2} r_{rn}^2 \sum_{i < j} \left( \dots - \dots \right)^2
$$
\nfor  $r \neq s$ ,

\n
$$
g_{rs} = \frac{1}{n(n-1)} S_{r, sn} \sum_{i \neq j} \dots \sum_{i} \dots \sum_{i} \frac{1}{n^2 (n-1)} \sum_{i < j} \left( \dots - \dots \right)^2 r_{rn} r_{sn},
$$
\nwhere  $i$  and  $j$  vary from 1 to  $n$  and  $r$  and  $s$  are such that  $c + 1 \leq r < s$ .

Let<br>  $Y_{[n,c]} = (Y_{[c+1:n]}, Y_{[c+2:n]}, ..., Y_{[n-c:n]})'.$ <br>
Then<br>  $E[Y_{[n,c]}] = -\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$ <br>
where 1 is a column vector of  $n-2c$  ones,<br>  $\Gamma = \frac{1}{n} \sum_{i=1}^{n} ..._{i} (\Gamma_{c+1:n}, \Gamma_{c+2:n}, ..., \Gamma_{n-c:n})'$ <br>
and the variance covariance matrix of  $\begin{aligned}\n\mathbb{E}[Y_{[n+1]}, Y_{[n+2n]}, \dots, Y_{[n-cn]})'.\n\end{aligned}$   $\mathbb{E}[Y_{[n,c]}] = -\frac{1}{2} + \frac{1}{2} \int_{-i}^{T} (r_{c+1:n}, r_{c+2:n}, \dots, r_{n-cn})'.\n\end{aligned}$   $\begin{aligned}\n\mathbb{E}[Y_{[n,c]}] &= -\frac{1}{n} \sum_{i=1}^{n} \dots (r_{c+1:n}, r_{c+2:n}, \dots, r_{n-cn})'.\n\end{aligned}$ the variance covari  $\frac{1}{n} \sum_{i} \frac{1}{n^{2}} + \frac{1}{n^{2}} \sum_{i} \frac{1}{n^{2}} (\frac{1}{n^{2}} - \frac{1}{n^{3}})$ <br> $\sum_{i \neq j} \frac{1}{n^{2}} + \frac{1}{n^{2}} \sum_{i \neq j} (\frac{1}{n^{2}} - \frac{1}{n^{3}}) \sum_{i \neq j} (\frac{1}{n^{2}} - \frac{1}{n^{3}})$ <br>rom 1 to *n* and *r* and *s* are such<br>21) together defines a gen *n*<sub>1</sub>] =  $\frac{1}{2}$  1 +  $\frac{1}{2}$  *r*<br>
is a column vector of  $n - 2c$  ones,<br>  $\sum_{i=1}^{n} \cdots (r_{c+1:n}, r_{c+2:n}, \dots, r_{n-c:n})'$ <br>
ariance covariance matrix of  $Y_{[n,c]}$  can be written in the form<br>  $\cdots$ <br>  $\cdots$ ] =  $\frac{1}{2}$   $\cdots$ <br>  $\vdots$  $Y_{\{x+2n\}},...,Y_{\{n-cn\}})'$ .<br>  $+ \uparrow \frac{1}{2}r$  (20)<br>
mm vector of  $n-2c$  ones,<br>  $\sum_{e+kn} \Gamma_{e+2n},..., \Gamma_{n-cn}y'$ <br>
covariance matrix of  $Y_{\{n,c\}}$  can be written in the form<br> *G*,<br>
(21)<br>
by<br>
by<br>  $y_y$ <br>  $y_y$ <br>  $y_y$ <br>  $y_y$ <br>  $y_y$ <br>  $y_y$ <br>  $y$  $I_{\text{[n,c]}} = (Y_{\text{[c+la]}}, Y_{\text{[c+2a]}}, ..., Y_{\text{[n-ca]}})'$ .<br>
(20)<br>
i is a column vector of  $n-2c$  ones,<br>  $\frac{1}{n} \sum_{i=1}^{n} ... \left(\mathbf{r}_{\text{[c+la]}} \mathbf{r}_{\text{[c+2a]}}, \mathbf{r}_{\text{[c+2a]}}, ..., \mathbf{r}_{\text{[c+2a]}}\right)'$ <br>
variance covariance matrix of  $Y_{$ Then<br>  $E[Y_{(n,r)}] = -\frac{1}{n} + \frac{r}{n}$  (20)<br>
where 1 is a column vector of  $n - 2c$  ones,<br>  $\Gamma = \frac{1}{n} \sum_{i=1}^{n} \cdots (r_{c,1:n}, r_{c,2:n}, \cdots, r_{n,cn})'$ <br>
and the variance covariance matrix of  $Y_{[n,c]}$  can be written in the form<br>  $D\left[Y_{[n$ when the  $\frac{1}{s}$ 's are known and then the *BLUEs*  $\frac{1}{s}$  and  $\frac{1}{s}$  are given by  $S_{r,r;n} - 1) \frac{1}{n} \sum_{i} \frac{1}{n^{2}} + \frac{1}{n^{2}} \Gamma_{rn}^{2} \sum_{i < j} (\frac{1}{n^{2}} - \frac{1}{n^{3}})^{2}$ <br>  $\left(-1\right)^{\frac{1}{n} \sum_{i \neq j} \frac{1}{n^{2}} \cdots \frac{1}{n^{2}} - \frac{1}{n^{2}(n-1)} \sum_{i < j} (\frac{1}{n^{2}} - \frac{1}{n^{3}})^{2} \Gamma_{rn} \Gamma_{sn},$ <br> *j* vary from 1 to *n* and *r* and  $\sum_{r}$  = 1 + (S<sub>r,rn</sub> - 1)  $\frac{1}{n} \sum_{i} ..._{i}^{2} + \frac{1}{n^{2}} \Gamma_{rn}^{2} \sum_{i < j} (..., -1)$ <br>
for  $r \neq s$ ,<br>  $\sum_{rs} = \frac{1}{n(n-1)} S_{r,s:n} \sum_{i \neq j} ..._{i} ..._{j} - \frac{1}{n^{2}(n-1)} \sum_{i < j} (..., -1)$ <br>
e *i* and *j* vary from 1 to *n* and *r* and *s* is<br>
tion ( $r_{c+1:n}, r_{c+2:n},...,r_{n-c:n}$ )'<br>
ce covariance matrix of  $Y_{[n,c]}$  can be<br>  $\frac{1}{2}G$ ,<br>
en by<br>  $r_{r,n} - 1$ ) $\frac{1}{n} \sum_{i} ..._{i}^{2} + \frac{1}{n^{2}} r_{rn}^{2} \sum_{i < j} (..._{i} - ..._{j})$ <br>  $\frac{1}{n^{3}} S_{r,s:n} \sum_{i \neq j} ..._{i} ..._{j} - \frac{1}{n^{2}(n-1)} \sum_{i < j} (..._{i} - ..._{j})$ *n*<sub>*n<sub>c</sub>*]  $\int_{t_{c-1}}^{u} ..._{t} (\Gamma_{c+\ln t}, \Gamma_{c+2m}, ..., \Gamma_{n-cm})'$ <br>
variance covariance matrix of  $Y_{\lfloor n,c \rfloor}$  can be written in the form<br>  $n_c$ ]  $= \Gamma^2_2 G$ , (21)<br>  $\parallel$  given by<br>  $1 + (S_{r,ren} - 1) \frac{1}{n} \sum_{i} ..._{i}^{2} + \frac{1}{n^{2}} \Gamma^2_{rn} \sum_{$   $\int_{\frac{1}{2}}^{n} \int_{-\frac{1}{2}}^{n} (r_{c+1m}, \Gamma_{c+2m}, \ldots, \Gamma_{n-cm})'$ <br>
riance covariance matrix of  $Y_{\{n,c\}}$  can be written in the form<br>  $\int_{0}^{1} = 1 \frac{1}{s} G,$  (21)<br>
given by<br>  $+(S_{r,cn} - 1) \frac{1}{n} \sum_{i} \ldots^{2}_{i} + \frac{1}{n^{2}} \Gamma_{rs}^{2} \sum_{i \leq$  $g_{rs} = \frac{1}{n(n-1)} S_{r,s;n} \sum_{i \neq j} ..._{i} ..._{j} - \frac{1}{n^{2}(n-1)} \sum_{i < j} (..._{i} - \frac{1}{n(n-1)})$ <br>where *i* and *j* vary from 1 to *n* and *r* and *s* are<br>Equations (20) and (21) together defines a gen<br>when the  $..._{i}$ 's are known and then th  $I_{rs} = \frac{1}{n(n-1)} S_{r,s:n} \sum_{i \neq j} ..._{i} ..._{j} - \frac{1}{n^{2}(n-1)} \sum_{i < j} (...,$ <br>
e *i* and *j* vary from 1 to *n* and *r* and *s*<br>
tions (20) and (21) together defines a g<br>
the  $...$ 's are known and then the *BLUEs*<br>  $I_{2} = \frac{\Gamma'G^{-1}(\Gamma'1'$ *n,c*]  $\left[\int_0^{\pi} 1 + (S_{r,r,n} - 1) \frac{1}{n} \sum_{i} \frac{1}{n} + \frac{1}{n^2} \int_0^2 \sum_{i \in J} (m_i - m_j)t^2 dt\right]$ <br>  $\pi \neq s$ ,<br>  $\frac{1}{n(n-1)} S_{r,s,n} \sum_{i \neq j} \frac{1}{n(n-1)} \sum_{i \in J} (m_i - m_i)t^2 dt$ <br>
and *j* vary from 1 to *n* and *r* and *s* are<br>
is (20) and ( , *n c G G Y*  s,<br>  $\frac{1}{(n-1)}$ S<sub>r,sn</sub>  $\sum_{i \neq j}$ <sub>*"i'i"*j</sub>  $-\frac{1}{n^2(n-1)} \sum_{i \leq j}$  (*...<sub>i</sub>*  $-\frac{1}{n^3}$ )<sup>2</sup>r<sub>rn</sub>r<sub>sn</sub>,<br>
1 *j* vary from 1 to *n* and *r* and *s* are such that  $c + 1 \leq r < s$ <br>
20) and (21) together defines a generalized mante extrainmental on  $\sin A = \frac{1}{\ln a}$  and a strain in the lotting<br>  $\sin A = \frac{1}{a} \int_{a}^{a} \cos A = \frac{1}{b} \int_{a}^{a} \frac{1}{a^{2}} \int_{a}^{a} \frac{1}{a^{2}} \int_{a}^{a} \frac{1}{a^{2}} \int_{a}^{a} (m_{1} - m_{1})^{2}$ <br>  $\neq s$ ,<br>  $\frac{1}{n(n-1)} S_{r,m} \sum_{m} m_{1} = \frac{1}{n^{2}(n-1$  $G = ||g_{tt}||$  given by<br>  $g_{tt} = 1 + (S_{t,rm} - 1) \frac{1}{n} \sum_{i} -\frac{2}{i} + \frac{1}{n^2} \Gamma_{em}^2 \sum_{i \in I} (-1 - \frac{1}{n})^2$ <br>
and for  $r \neq s$ ,<br>  $g_{xt} = \frac{1}{n(n-1)} S_{t,xx} \sum_{i \neq j} (-1 - \frac{1}{n^2} \sum_{i \neq j} (-1 - \frac{1}{n})^2 \Gamma_{tx} \Gamma_{ext},$ <br>
where *i* and *j* vary fr

$$
E_2 = \frac{\Gamma' G^{-1} (\Gamma' 1' - 1 \Gamma') G^{-1}}{\Delta} Y_{[n,c]}
$$

$$
T_2 = \frac{1'G^{-1}(1r' - r1')G^{-1}}{\Delta} Y_{[n,c]},
$$

$$
\Delta = (\Gamma' G^{-1} \Gamma)(1' G^{-1} 1) - (\Gamma' G^{-1} 1)^2.
$$

The variances of the above estimators are given by

lication of concomitants of order statistics ...... estimation

\nvariances of the above estimators are given by

\n
$$
Var(\epsilon_2) = \frac{\Gamma' G^{-1} \Gamma}{\Delta} \frac{1}{2}
$$
\n
$$
Var(\tau_2) = \frac{\Gamma' G^{-1} \Gamma}{\Delta} \frac{1}{2}.
$$

and

\n *ication of concomitants of order statistics ... estimation*  
\n   
\n variances of the above estimators are given by\n

\n\n
$$
Var(\hat{\zeta}_2) = \frac{\Gamma' G^{-1} \Gamma}{\Delta} \hat{\zeta}_2^2
$$
\n

\n\n
$$
Var(\hat{\zeta}_2) = \frac{\Gamma' G^{-1} \Gamma}{\Delta} \hat{\zeta}_2^2.
$$
\n

ion of concomitants of order statistics ....... estimation<br>
iances of the above estimators are given by<br>  $(\frac{c}{2}) = \frac{\Gamma' G^{-1} \Gamma}{\Delta} \frac{1}{2}$ <br>  $(\Gamma_2) = \frac{\Gamma' G^{-1} \Gamma}{\Delta} \frac{1}{2}$ .<br>  $\Gamma$  that  $\frac{c}{2}$  and  $\Gamma_2$  are linear functio *Variances* of the above estimators are given by<br>  $Var(\hat{\zeta}_2) = \frac{\Gamma' G^{-1} \Gamma}{\Delta} \hat{\zeta}_2^2$ <br>  $Var(\hat{\zeta}_2) = \frac{\Gamma' G^{-1} \Gamma}{\Delta} \hat{\zeta}_2^2$ <br>  $Var(\hat{\zeta}_2) = \frac{\Gamma G^{-1} \Gamma}{\Delta} \hat{\zeta}_2^2$ <br>  $Var(\hat{\zeta}_2) = \frac{\Gamma G^{-1} \Gamma}{\Delta} \hat{\zeta}_2^2$ <br>  $Var(\hat{\zeta}_2) = \frac{\Gamma G^{-1} \Gamma$ It is clear that  $\hat{z}_2$  and  $\hat{T}_2$  are linear functions of the concomitants and hence we can write variances of the above estimate<br>  $Var(\hat{\tau}_2) = \frac{\Gamma' G^{-1} \Gamma}{\Delta} \hat{\tau}_2^2$ <br>  $Var(\hat{\tau}_2) = \frac{\Gamma' G^{-1} \Gamma}{\Delta} \hat{\tau}_2^2$ .<br>
Elear that  $\hat{\tau}_2$  and  $\hat{\tau}_2$  are linear<br>
vite<br>  $\tau_2 = \sum_{r=c+1}^{n-c} a_r Y_{[r:n]}$ all integrals of the above estint<br>  $\hat{f}_2$ ) =  $\frac{\Gamma' G^{-1} \Gamma}{\Delta} \frac{d}{dt}$ <br>  $\Gamma_2$ ) =  $\frac{d^2 G^{-1} \Gamma}{d} \frac{d}{dt} \frac{d}{dt}$ <br>  $\Gamma_2$  and  $\Gamma_2$  are li<br>  $\sum_{r=0}^{n-c} a_r Y_{r,n}$ of the above estimators and<br>  $r = \frac{r'G^{-1}r}{\Delta} + \frac{2}{2}$ <br>  $\frac{1'}{\Delta} = \frac{1'}{2}$ <br>  $\frac{1'}{2}$  and  $\frac{1}{2}$  are linear functions.<br>  $r = rT_{[r,n]}$ *lication of concomitants of order statistics* ...... *estimation*<br>
variances of the above estimators are given by<br>  $Var(\tilde{\tau}_2) = \frac{\Gamma'G^{-1}r}{\Delta} \tilde{\tau}_2^2$ <br>  $Var(\tilde{\tau}_2) = \frac{\Gamma'G^{-1}1}{\Delta} \tilde{\tau}_2^2$ .<br>
clear that  $\tilde{\tau}_2$  and  $\tilde{\$ Transies of the above estimators are given by<br>  $tr(\hat{\tau}_2) = \frac{r'G^{-1}r}{\Delta} \hat{\tau}_2^2$ <br>  $tr(\hat{\tau}_2) = \frac{1'G^{-1}1}{\Delta} \hat{\tau}_2^2$ .<br>
ear that  $\hat{\tau}_2$  and  $\hat{\tau}_2$  are linear functions of the concomitants and hence we<br>
tite<br>  $= \sum_{r=c+1}^{$  $Var(\overline{\Gamma}_2) = \frac{1'G^{-1}1}{\Delta} \overline{\Gamma}_2^2.$ <br>
Elear that  $\frac{1}{r_2}$  and  $\overline{\Gamma}_2$  are linea<br>
vrite<br>  $2 = \sum_{r=c+1}^{n-c} a_r Y_{[r:n]}$ <br>  $2 = \sum_{r=c+1}^{n-c} b_r Y_{[r:n]}$ ,<br>  $2 = a_r b_r$ ,  $r = c + 1, c + 2, ..., n - c$  $\mathcal{F}_2$ ) =  $\frac{1'G^{-1}1}{\Delta} \mathcal{F}_2^2$ .<br>
that  $\mathcal{F}_2$  and  $\mathcal{F}_2$  are li<br>  $\sum_{n=c+1}^{n-c} a_r Y_{[r:n]}$ <br>  $\sum_{n=c+1}^{n-c} b_r Y_{[r:n]}$ ,<br>  $b_r$ ,  $r = c + 1, c + 2, ..., n - 1$  $r = \frac{1'G^{-1}1}{\Delta} \tau_2^2$ .<br>  $\epsilon_2$  and  $\tau_2$  are linear functions  $r_r Y_{r,n}$ <br>  $r_r Y_{r,n}$ ,<br>  $r = c + 1, c + 2, ..., n - c$  are considered in the set of different velocies  $\begin{aligned} \n\text{tr}(\mathbf{f}_2) &= \frac{\mathbf{1}'G^{-1}\mathbf{1}}{\Delta}\mathbf{1}\frac{2}{2}. \n\end{aligned}$ <br>
ear that  $\mathcal{F}_2$  and  $\mathbf{f}_2$  are linear functions of the concomitants and hence we<br>  $\text{tr} = \sum_{r=c+1}^{n-c} a_r Y_{[r,n]}$ <br>  $= \sum_{r=c+1}^{n-c} b_r Y_{[r,n]}$ ,<br>  $a_r b_r, r = c +$ 

$$
\widehat{Z}_2 = \sum_{r=c+1}^{n-c} a_r Y_{[rn]}
$$

and

$$
\Gamma_2 = \sum_{r=c+1}^{n-c} b_r Y_{[r:n]},
$$

*riances* of the above estimators are given by<br>  $Var(\tilde{\zeta}_2) = \frac{\Gamma' G^{-1} r}{\Delta} \tilde{\zeta}_2^2$ <br>  $Var(\tilde{\zeta}_2) = \frac{r' G^{-1} r}{\Delta} \tilde{\zeta}_2^2$ <br>  $Var(\tilde{\zeta}_2) = \frac{r' G^{-1} r}{\Delta} \tilde{\zeta}_2^2$ <br>  $Var(\tilde{\zeta}_2) = \frac{r}{\Delta} \tilde{\zeta}_2^2$ <br>  $Var(\tilde{\zeta}_2) = \sum_{r=c+1}^{n-c$ Application of concomitants of order statistics ........<br>
The variances of the above estimators are given<br>  $Var(\tilde{\tau}_2) = \frac{\Gamma'G^{-1}\Gamma}{\Delta} \tilde{\tau}_2^2$ <br>
and<br>  $Var(\tilde{\tau}_2) = \frac{\Gamma'G^{-1}\Gamma}{\Delta} \tilde{\tau}_2^2$ .<br>
It is clear that  $\tilde{\tau}_2$  and  $\til$ *riances* of the above estimators are given by<br>  $r(\frac{c}{2}) = \frac{r'G^{-1}r}{\Delta} + \frac{2}{2}$ <br>  $r(\frac{r}{2}) = \frac{r'G^{-1}r}{\Delta} + \frac{2}{2}$ .<br>  $r(\frac{r}{2}) = \frac{rG^{-1}r}{\Delta} + \frac{2}{2}$ .<br>  $r(\frac{r}{2}) = \sum_{r=1}^{n} a_r Y_{(r,n)}$ <br>  $a_r \sum_{r=1}^{n} a_r Y_{(r,n)}$ .<br>  $a_r \sum_{r=1}^{$ It is clear that  $\frac{1}{r_2}$  and  $\frac{1}{r_2}$  are<br>can write<br> $\frac{1}{r_2} = \sum_{r=c+1}^{n-c} a_r Y_{r,n}$ <br>and<br> $\frac{1}{r_2} = \sum_{r=c+1}^{n-c} b_r Y_{r,n}$ ,<br>where  $a_r b_r$ ,  $r = c + 1$ ,  $c + 2$ ,..., *n*<br>The values of  $\Gamma_{r,n}$  for differe<br>and those of  $S_{r,s$ The values of  $r_{r_n}$  for different values of *r* and *n* are given in Harter (1961) can write<br>  $\epsilon_2 = \sum_{r=e+1}^{n-c} a_r Y_{[r,n]}$ <br>
and<br>  $\Gamma_2 = \sum_{r=e+1}^{n-c} b_r Y_{[r,n]}$ ,<br>
where  $a_r b_r$ ,  $r = c + 1, c + 2, ..., n - c$  are constants which can be determined.<br>
The values of  $r_{r,n}$  for different values of *r* and *n* are given in Sa  $Var(\tilde{\tau}_2) = \frac{16\pi^4}{\Delta} \tilde{\tau}_2^2$ <br>
and<br>
and<br>  $Var(\tilde{\tau}_2) = \frac{16\pi^4}{\Delta} \tilde{\tau}_2^2$ ,<br>
It is clear that  $\tilde{\tau}_2$  and  $\tilde{\tau}_2$  are linear functions of the concomitants and hence we<br>
can write<br>  $\tilde{\tau}_2 = \sum_{r=1}^{n_c} a_r Y_{[rn]}$ <br>
a *r r f f f <i>f f f <i>f f f <i>f* **<b>***<i>f <i>f <i>f <i>f <i>f <i>f <i>f <i>f <i>f <i>f* **<b>***<i>f <i>f <i>f <i>f <i>f <i>f <i><i>f**<i><i>f**<i><i>* easily determined using *MATHCAD* software. For an illustration, let us assume that *k* of the observations are from a bivariate normal population with correlation coefficient  $\mathbb{I}$  and the remaining  $n-k$  observations are from a remaintant in the concomitants and hence we<br> *n*  $h - c$  are constants which can be determined.<br>
From tradues of r and n are given in Harter (1961)<br>
remt values of r, s and n are given in Sarhan and<br>
the constants  $a,b_r, r = c+$ bivariate normal population with correlation coefficient  $\ldots$ . Then we have It is clear that  $\frac{1}{2}$  and  $\frac{1}{2}$  are linear functions of the concomitants and hence we<br>can write<br>  $\frac{1}{2} = \sum_{n=1}^{n} a_n Y_{[rn]}$ ,<br>
where  $a_i b_r, r = c + 1, c + 2, ..., n - c$  are constants which can be determined.<br>
The values of ants which can be determined.<br> *r* and *n* are given in Harter (1961)<br> *r*, *s* and *n* are given in Harter (1961)<br> *r*, *s* and *n* are given in Sarhan and<br>  $a_r b_r$ ,  $r = c + 1$ ,  $c + 2$ , ...,  $n - c$  can be<br>
be bivariate norma variances of the estimators  $\epsilon_2$  and  $\epsilon_1$  for  $\epsilon_3$  (..., ..., ...) = (0.8, 0.7), (0.6, for the concomitants and hence we<br>
tants which can be determined.<br> *r* and *n* are given in Harter (1961)<br> *r*, *s* and *n* are given in Sarhan and<br>  $a, b, r = c + 1, c + 2, ..., n - c$  can be<br>
are. For an illustration, let us assume<br>  $n = 5$ . The values are given in the following tables. From tables numbered 1 to 4, we observe that both  $\sim_2$  and  $\rm \dagger$  are estimated with more precision for larger values of  $\mu_i$  's than for the cases with smaller values of  $\mu_i$ 's. Since  $Var(\hat{\zeta}_2)$  is be determined.<br>
ven in Harter (1961)<br>
given in Sarhan and<br>  $+2,...,n-c$  can be<br>
ration, let us assume<br>
aal population with<br>
rvations are from a<br>  $\frac{1}{2}$ . Then we have<br>  $\frac{2}{2},...,n-c$  and the<br>  $\frac{8}{2},0.7$ , (0.6,0.5) for<br>
able and<br>  $\Gamma_2 = \sum_{r=\epsilon+1}^{n_{\epsilon}} b_r Y_{[r,n]}$ ,<br>
where  $a_r b_r$ ,  $r = c + 1, c + 2, ..., n - c$  are constants which can be determined<br>
The values of  $r_{r,n}$  for different values of  $r$  and  $n$  are given in Harter<br>
and those of  $S_{r,n}$  for differe  $\mathcal{F}_2$ ) for the concomitants of order statistics for a given collection of random variables we infer that concomitants of order statistics of *inid* n normal random variables can be more profitably used for estimating  $\sim$ , than for estimating  $\dagger$ <sub>2</sub>. Very rarely in problems of estimation of common parameters of several distributions, explicit expression for the values of the proposed estimator exists. However in our method it can be explicitly expressed and hence we can determine the quality of our estimator as well.

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								<b>Table 1:</b> Coefficients $a_i$ 's in the <i>BLUE</i> $\hat{z}_2 = \sum_{n=1}^{n-c} a_r Y_{[rn]}$ and $Var(\hat{z}_2) / \hat{z}_2^2$ for
			$m_1 = 0.8, m_2 = 0.7$					
$\boldsymbol{n}$	$\boldsymbol{c}$	$\boldsymbol{k}$	$a_{1}$	$a_{2}$	a <sub>3</sub>	$\mathfrak{a}_4$	a <sub>5</sub>	$Var(\widehat{\cdot}_2)/\!\uparrow^2_2$
5	$\boldsymbol{0}$	$\mathbf{1}$	0.1995	0.2003	0.2004	0.2003	0.1995	0.2000
		$\overline{c}$	0.1991	0.2005	0.2008	0.2005	0.1991	0.2000
		3	0.1991	0.2005	0.2008	0.2005	0.1991	0.2000
		4	0.1994	0.2003	0.2006	0.2003	0.1994	0.2000
	$\mathbf{I}$	$\perp$		0.3370	0.3260	0.3370		0.2779
		$\overline{c}$		0.3374	0.3253	0.3374		0.2744
		3		0.3379	0.3242	0.3379		0.2713
		4		0.3385	0.3230	0.3385		0.2685
								<b>Table 2:</b> Coefficients $b_i$ 's in the <i>BLUE</i> $\mathbf{f}_2 = \sum_{r=a+1}^{n-c} b_r Y_{[r:n]}$ and $Var(\mathbf{f}_2) / \mathbf{f}_2^2$ for
			$m_1 = 0.8, m_2 = 0.7$ .					
$\boldsymbol{n}$	$\boldsymbol{c}$	$\boldsymbol{k}$	b <sub>1</sub>	b <sub>2</sub>	b <sub>3</sub>	b <sub>4</sub>	$b_5$	$Var(\hat{\mathsf{T}}_2)/\hat{\mathsf{T}}_2^2$
5	$\boldsymbol{0}$	1	$-0.5064$	$-0.2132$	0.0000	0.2132	0.5064	0.4243
		$\overline{c}$	$-0.4927$	$-0.2076$	0.0000	0.2076	0.4927	0.3921
					0.0000	0.2018	0.4798	0.3625

**Table 1:** Coefficients *a<sub>i</sub>* 's in the *BLUE*  $\sum_{r=c+1}^{n-c} a_r Y_{[rn]}$  and  $Var(\hat{\xi}_2) / \hat{\tau}_2^2$  for <br>  $\sum_{r=1}^{n} a_r = 0.8, \quad \sum_{r=2}^{n} a_r = 0.7$ *ena T. G. and P. Yageer*<br> $\sum_{n=c+1}^{n-c} a_r Y_{[r:n]}$  and  $Var(\hat{z}_2)$ *r r f*<sub>(*rm*]</sub> and *Var*( $\frac{2}{3}$ )/ $\frac{1}{2}$  for  $\epsilon_2 = \sum_{r=0}^{n-c} a_r Y_{(r,n)}$  and  $Var(\epsilon_2)/\tau_2^2$  for Veena T. G. and P. Yageen Thomas<br>=  $\sum_{r=c+1}^{n-c} a_r Y_{[r:n]}$  and  $Var(\tilde{\cdot}_2) / \tilde{\cdot}_2^2$  for

 $\frac{1}{4}$  $\mathcal{F}_2 = \sum_{r=0}^{n-c} b_r Y_{r-1}$  and  $Var(\mathcal{F}_2) / \mathcal{F}_2^2$  for

			$m_1 = 0.8, m_2 = 0.7$ .					
n	$\mathcal{C}$	$\boldsymbol{k}$	b <sub>1</sub>	b <sub>2</sub>	$b_3$	$b_4$	$b_5$	$Var(\hat{T}_2)/\hat{T}_2^2$
5	$\boldsymbol{0}$	$\mathbf{1}$	$-0.5064$	$-0.2132$	0.0000	0.2132	0.5064	0.4243
		$\overline{2}$	$-0.4927$	$-0.2076$	0.0000	0.2076	0.4927	0.3921
		3	$-0.4798$	$-0.2018$	0.0000	0.2018	0.4798	0.3625
		$\overline{4}$	$-0.4678$	$-0.1959$	0.0000	0.1959	0.4678	0.3350
		1		$-1.4033$	0.0000	1.4033		2.2230
	1	$\overline{2}$		$-1.3644$	0.0000	1.3644		2.0105
		3		$-1.3291$	0.0000	1.3291		1.8175
		4		$-1.2949$	0.0000	1.2949		1.6404

Application of concomitants of order statistics ....... estimation<br> **Table 3:** Coefficients  $a_i$ 's in the BLUE  $\hat{=}$ <sub>2</sub> =  $\sum_{r=c+1}^{n-c} a_r Y_{[r:n]}$  and  $Var(\hat{=}$ <sub>2</sub>)/<br>  $\therefore$ <sub>1</sub> = 0.6,  $\therefore$ <sub>2</sub> = 0.5 *n*-c<br> $\sum_{n=c+1}^{n-c} a_r Y_{[r:n]}$  and  $Var(\hat{\sigma})$ *rtion* 9<br> *r*  $\left( \frac{r}{r} Y_{[r:n]} \right)$  and  $Var(\hat{z}_2) / \hat{z}_2$  for  $\left( \frac{r}{r} \right)$  is  $\left( \frac{r}{r} \right)$  is  $\left( \frac{r}{r} \right)$  $\epsilon_2 = \sum_{r=0}^{n-c} a_r Y_{r,n}$  and  $Var(\epsilon_2)/\tau_2^2$  for .. estimation 95<br>=  $\sum_{r=c+1}^{n-c} a_r Y_{[rn]}$  and  $Var(\hat{\cdot}_2) / \hat{\cdot}_2^2$  for

		Application of concomitants of order statistics  estimation					95
							<b>Table 3:</b> Coefficients $a_i$ 's in the BLUE $\hat{z}_2 = \sum_{n=1}^{n-c} a_n Y_{[rn]}$ and $Var(\hat{z}_2) / \hat{z}_2$ for
		$m_1 = 0.6, m_2 = 0.5$					
$\boldsymbol{c}$ $\boldsymbol{n}$	$\boldsymbol{k}$	$a_{1}$	$a_{2}$	a <sub>3</sub>	$\boldsymbol{a}_4$	a <sub>5</sub>	$Var(\frac{c}{2})/\frac{1}{2}$
	$\mathbf{1}$	0.1995	0.2003	0.2004	0.2003	0.1995	0.2001
5 $\boldsymbol{0}$							
	$\overline{c}$	0.1994	0.2003	0.2006	0.2003	0.1994	0.2001
	3	0.1994	0.2003	0.2006	0.2003	0.1994	0.2001
	4	0.1996	0.2002	0.2004	0.2002	0.1996	0.2001
	$\mathbf{1}$		0.3351	0.3298	0.3351		0.3050
2	$\overline{c}$		0.3350	0.3300	0.3350		0.3010
	3		0.3350	0.3300	0.3350		0.2999

 $\mathcal{F}_2 = \sum_{r=0}^{n-c} b_r Y_{r+n}$  and  $Var(\mathcal{F}_2) / \mathcal{F}_2^2$  for  $n_1 = 0.6, \ldots, = 0.5$ .



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