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# ESTIMATING THE PARAMETER « OF THE EXPONENTIAL DISTRIBUTION WITH KNOWN COEFFICIENT OF VARIATION BY ORDER STATISTICS

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**ABSTRACT**

In this paper, we have obtained the best linear unbiased estimator $(BLUE)$ of µ of the exponential distribution $E(«,c^2«^2)$ with known coefficient of variation c by order statistics. Also we have obtained the compact form of the estimator derived. Efficiency comparisons are also made on the proposed estimators with some of the usual estimators of « .

## 1. INTRODUCTION

In some of the biological and physical science problems, situations where the scale parameter is proportional to the location parameter are seen reported in the literature. (see, for example, Gleser and Healy, (1976). If the scale parameter is proportional to the location parameter, the constant of proportion is denoted by $c$, $c$ is the known coefficient of variation. If the parent distribution is a normal with mean µ and standard deviation cµ and if it is denoted by $N(«,c^2«^2)$, $c$ is the known coefficient of variation, then the problem of estimating « has been extensively discussed in the available literature, for example see, Searls (1964), Khan (1968), Gleser and Healy (1976), Arnholt and Hebert (1995), Kunte (2000), and Guo and Pal (2003). The Best Linear Unbiased Estimator $(BLUE)$ of « for $N(«,c^2«^2)$ distribution for different values of $c$ using order statistics are discussed in Thomas and Sajeevkumar (2003). Estimating the mean of logistic distribution with known coefficient of variation are discussed by Sajeevkumar and Thomas (2005). Estimating the location parameter of an exponential distribution with known coefficient of variation are discussed in Ghosh and Razmpour (1982) and Samanta (1984).

In this paper, we describe the technique of estimating the location parameter « of the exponential distribution by order statistics, when the coefficient of variation is known.

## 2. ESTIMATING THE LOCATION PARAMETER « OF A DISTRIBUTION WHEN THE SCALE PARAMETER IS $d$« FOR KNOWN $d$

In this section we consider the family $G$ of all absolutely continuous distributions which depend on a location parameter « and a scale parameter $\pm = d$« where d is known. Then any distribution belongs to $G$ has a $p.d.f.$ of the form

$$f(x:«,d«)=\frac{1}{d«}f_0\left(\frac{x-«}{d«}\right), «>0, d>0. \quad (1)$$

Let $\underline{X}=\left(X_{1:n},X_{2:n},...,X_{n:n}\right)'$ be the vector of order statistics of a random sample of size $n$ drawn from (1). Let $\underline{\sim}=\left(\sim_{1:n},\sim_{2:n},...,\sim_{n:n}\right)'$ and $V=\left(\left(v_{r,s:n}\right)\right)$ be the vector of means and dispersion matrix of the vector of order statistics of a random sample of size n arising from $f(x:0,1)$. Then by considering « as the location parameter of (1), a linear unbiased estimator of « based on order statistics is given by (see, Balakrishnan and Rao (1998), p.13)

$$\hat{«}=-\frac{\underline{\sim}'V^{-1}\left(\underline{1}\underline{\sim}'-\underline{\sim}\underline{1}'\right)V^{-1}\underline{X}}{\left(\underline{\sim}'V^{-1}\underline{\sim}\right)\left(\underline{1}'V^{-1}\underline{1}\right)-\left(\underline{\sim}'V^{-1}\underline{1}\right)^2} \quad (2)$$

and

$$V(\hat{«})=\frac{\left(\underline{\sim}'V^{-1}\underline{\sim}\right)d^2«^2}{\left(\underline{\sim}'V^{-1}\underline{\sim}\right)\left(\underline{1}'V^{-1}\underline{1}\right)-\left(\underline{\sim}'V^{-1}\underline{1}\right)^2}, \quad (3)$$

where $\underline{1}$ is a column vector of n ones. Also by considering $d$« as the scale parameter of the $p.d.f.$ defined in (1). A linear unbiased estimator of $d$« is given by (see, Balakrishnan and Rao (1998), p.13)

$$T=\frac{\underline{1}'V^{-1}\left(\underline{1}\underline{\sim}'-\underline{\sim}\underline{1}'\right)V^{-1}\underline{X}}{\left(\underline{\sim}'V^{-1}\underline{\sim}\right)\left(\underline{1}'V^{-1}\underline{1}\right)-\left(\underline{\sim}'V^{-1}\underline{1}\right)^2} \quad (4)$$

and

$$V(T)=\frac{\left(\underline{1}'V^{-1}\underline{1}\right)d^2«^2}{\left(\underline{\sim}'V^{-1}\underline{\sim}\right)\left(\underline{1}'V^{-1}\underline{1}\right)-\left(\underline{\sim}'V^{-1}\underline{1}\right)^2}. \quad (5)$$

From (4) we can obtain another linear unbiased estimator $«^*$ of « based on order statistics is given by

$$« ^* = \frac{1}{d}\left( \frac{\underline{1}'V^{-1}\left(\underline{1}\underline{\sim}' - \underline{\sim}\underline{1}'\right)V^{-1}\underline{X}}{\left(\underline{\sim}'V^{-1}\underline{\sim}\right)\left(\underline{1}'V^{-1}\underline{1}\right) - \left(\underline{\sim}'V^{-1}\underline{1}\right)^2} \right) \tag{6}$$

and

$$V(« ^*) = \frac{\left(\underline{1}'V^{-1}\underline{1}\right)« ^2}{\left(\underline{\sim}'V^{-1}\underline{\sim}\right)\left(\underline{1}'V^{-1}\underline{1}\right) - \left(\underline{\sim}'V^{-1}\underline{1}\right)^2} . \tag{7}$$

Now we derive the *BLUE* $\tilde{«}$ of $«$ based on order statistics is given in the following theorem.

**Theorem 2.1:** Let $\underline{X} = \left(X_{1:n}, X_{2:n}, ..., X_{n:n}\right)'$ be the vector of order statistics of a random sample of size $n$ drawn from a distribution with $p.d.f.$ defined in (1). Let $\underline{\sim} = \left(\sim_{1:n}, \sim_{2:n}, ..., \sim_{n:n}\right)'$ and $V = \left(\left(v_{r,s:n}\right)\right)$ be the vector of means and dispersion matrix respectively of the vector of order statistics of a random sample of size n drawn from $f(x:0,1)$. Then the *BLUE* $\tilde{«}$ of the parameter $«$ is given by

$$\tilde{«} = \frac{\left(d\underline{\sim}'V^{-1}\underline{X} + \underline{1}'V^{-1}\underline{X}\right)}{\left(d^2\underline{\sim}'V^{-1}\underline{\sim} + 2d\underline{\sim}'V^{-1}\underline{1} + \underline{1}'V^{-1}\underline{1}\right)} \tag{8}$$

and its variance

$$Var\left(\tilde{«}\right) = \frac{d^2« ^2}{\left(d^2\underline{\sim}'V^{-1}\underline{\sim} + 2d\underline{\sim}'V^{-1}\underline{1} + \underline{1}'V^{-1}\underline{1}\right)} , \tag{9}$$

where $\underline{1}$ is a column vector of $n$ ones.

**Proof**: Given $\underline{X} = \left(X_{1:n}, X_{2:n}, ...., X_{n:n}\right)'$ and let $Y_{1:n}, Y_{2:n}, ..., Y_{n:n}$ are the order statistics of a random sample of size $n$ drawn from $f(x:0,1)$. Let $E(Y_{r:n}) = \sim_{r:n}$, $r = 1, 2, ..., n$, and $Cov\left(Y_{r:n}, Y_{s:n}\right) = v_{r,s:n}$ for $1 \le r < s \le n$. Then we have

$$\frac{X_{r:n} - «}{d«} \overset{d}{=} Y_{r:n}, r = 1, 2, ..., n$$

and

$$E\left(X_{r:n}\right) = \left(d\sim_{r:n} + 1\right)«, r = 1, 2, ..., n \tag{10}$$

$$V(X_{r:n}) = d^2« ^2 v_{r,r:n} , \; r = 1, 2, ..., n \tag{11}$$

and

$$Cov\left(X_{r:n}, X_{s:n}\right) = d^2\mu^2 v_{r,s:n}\,,\ 1 \le r < s \le n. \tag{12}$$

From (10) to (12) one can also write,

$$E\left(\underline{X}\right) = \left(d\underline{\alpha} + \underline{1}\right)\mu \tag{13}$$

and

$$D\left(\underline{X}\right) = V d^2 \mu^2\,, \tag{14}$$

where $\underline{1}$ is a column vector of n ones, $\underline{\alpha} = \left(\alpha_{1:n}, \alpha_{2:n}, \ldots, \alpha_{n:n}\right)'$ and $V = \left(\left(v_{r,s:n}\right)\right)$. Then by generalized Gauss-Markoff theorem, the $BLUE$ $\tilde{\mu}$ of $\mu$ is given by,

$$\tilde{\mu} = \frac{\left(d\underline{\alpha} + \underline{1}\right)' V^{-1} \underline{X}}{\left(d\underline{\alpha} + \underline{1}\right)' V^{-1} \left(d\underline{\alpha} + \underline{1}\right)},$$

That is

$$\tilde{\mu} = \frac{\left(d\underline{\alpha}' V^{-1} \underline{X} + \underline{1}' V^{-1} \underline{X}\right)}{d^2 \underline{\alpha}' V^{-1} \underline{\alpha} + 2d\underline{\alpha}' V^{-1}\underline{1} + \underline{1}' V^{-1} \underline{1}}$$

and

$$V\left(\tilde{\mu}\right) = \frac{d^2\mu^2}{\left(d\underline{\alpha} + \underline{1}\right)' V^{-1} \left(d\underline{\alpha} + \underline{1}\right)}$$

$$= \frac{\mathrm{d}^2\mu^2}{\mathrm{d}^2 \underline{\alpha}' \mathrm{V}^{-1} \underline{\alpha} + 2\mathrm{d}\underline{\alpha}' \mathrm{V}^{-1}\underline{1} + \underline{1}' \mathrm{V}^{-1} \underline{1}}.$$

This proves the theorem.

## 3. ESTIMATING THE PARAMETER $\mu$ OF THE EXPONENTIAL DISTRIBUTION BY ORDER STATISTICS WHEN THE COEFFICIENT OF VARIATION C IS KNOWN

In this section we consider an exponential distribution $E\left(\mu, d^2\mu^2\right)$ with $p.d.f.$

$$f\left(x : \mu, d\mu\right) = \frac{1}{d\mu} e^{-\frac{(x-\mu)}{d\mu}}\,, \mu, d > 0, x \ge \mu\ . \tag{15}$$

Clearly the mean and variance of the $p.d.f.$ defined in (15) are respectively $\mu + d\mu$ and $d^2\mu^2$. Therefore the coefficient of variation of the model defined in (15) is

$$c = \frac{s\tan darddeviation}{arithmeticmean} = \frac{d}{1+d}.$$

Let $\underline{X}^E = \left(X^E_{1:n}, X^E_{2:n}, ..., X^E_{n:n}\right)'$ be the vector of order statistics of a random sample of size $n$ drawn from (15). Let $\underline{\sim}^E = \left(\sim^E_{1:n}, \sim^E_{2:n}, ..., \sim^E_{n:n}\right)'$ and $V^E = \left(\left(v^E_{r,s:n}\right)\right)$ be the mean vector and dispersion matrix of the vector of order statistics of a random sample of size n drawn from the standard exponential distribution $f(x:0,1)$. Then by considering $\text{«}$ as the location parameter of (15), a linear unbiased estimator of $\text{«}$ is obtained by putting $V = V^E$ in (2) and is given by

$$\hat{\text{«}}_E = -\frac{\left(\underline{\sim}^E\right)'\left(V^E\right)^{-1}\left(\underline{1}\left(\underline{\sim}^E\right)' - \left(\underline{\sim}^E\right)\underline{1}'\right)\left(V^E\right)^{-1}\underline{X}^E}{\left(\left(\underline{\sim}^E\right)'\left(V^E\right)^{-1}\left(\underline{\sim}^E\right)\right)\left(\underline{1}'\left(V^E\right)^{-1}\underline{1}\right) - \left(\left(\underline{\sim}^E\right)'\left(V^E\right)^{-1}\underline{1}\right)^2} \tag{16}$$

and

$$V\left(\hat{\text{«}}_E\right) = \frac{\left(\underline{\sim}^E\right)'\left(V^E\right)^{-1}\left(\underline{\sim}^E\right)d^2\text{«}^2}{\left(\left(\underline{\sim}^E\right)'\left(V^E\right)^{-1}\left(\underline{\sim}^E\right)\right)\left(\underline{1}'\left(V^E\right)^{-1}\underline{1}\right) - \left(\left(\underline{\sim}^E\right)'\left(V^E\right)^{-1}\underline{1}\right)^2}, \tag{17}$$

where $\underline{1}$ is a column vector of n ones.

Using the results of Sarhan (1954), p.322, then (16) and (17) reduces to,

$$\hat{\text{«}}_E = \frac{1}{n-1}\left[nX^E_{1:n} - \overline{X}^E\right] \tag{18}$$

and

$$V\left(\hat{\text{«}}_E\right) = \frac{d^2\text{«}^2}{n(n-1)} \tag{19}$$

where $\overline{X}^E$ is the sample mean. Also by considering $d\text{«}$ as the scale parameter, another linear unbiased estimator $\text{«}^*_E$ of $\text{«}$ corresponding to (6) is given by.

$$\text{«}^*_E = \frac{\left(\underline{1}\right)'\left(V^E\right)^{-1}\left(\underline{1}\left(\underline{\sim}^E\right)' - \left(\underline{\sim}^E\right)\underline{1}'\right)\left(V^E\right)^{-1}\underline{X}^E}{d\left[\left(\left(\underline{\sim}^E\right)'\left(V^E\right)^{-1}\left(\underline{\sim}^E\right)\right)\left(\underline{1}'\left(V^E\right)^{-1}\underline{1}\right) - \left(\left(\underline{\sim}^E\right)'\left(V^E\right)^{-1}\underline{1}\right)^2\right]} \tag{20}$$

and

$$V\left(\text{«}_E^{*}\right)=\frac{\left(\underline{1}'\left(V^{E}\right)^{-1}\underline{1}\right)\text{«}^{2}}{\left(\left(\underline{\sim}^{E}\right)'\left(V^{E}\right)^{-1}\left(\underline{\sim}^{E}\right)\right)\left(\underline{1}'\left(V^{E}\right)^{-1}\underline{1}\right)-\left(\left(\underline{\sim}^{E}\right)'\left(V^{E}\right)^{-1}\underline{1}\right)^{2}}\,. \tag{21}$$

Using the results of Sarhan (1954), p.322, then (20) and (21) reduces to,

$$\text{«}_E^{*}=\frac{n}{d\left(n-1\right)}\left[\overline{X}^{E}-X_{1:n}^{E}\right] \tag{22}$$

and

$$V(\text{«}_E^{*})=\frac{\text{«}^{2}}{\left(n-1\right)} \tag{23}$$

Using theorem 2.1, the $BLUE$ of « corresponding to (8) for the exponential distribution is given by

$$\tilde{\text{«}}_E=\frac{d\left(\underline{\sim}^{E}\right)'\left(V^{E}\right)^{-1}\underline{X}^{E}+\underline{1}'\left(V^{E}\right)^{-1}\underline{X}^{E}}{d^{2}\left(\underline{\sim}^{E}\right)'\left(V^{E}\right)^{-1}\left(\underline{\sim}^{E}\right)+2d\left(\underline{\sim}^{E}\right)'\left(V^{E}\right)^{-1}\underline{1}+\underline{1}'\left(V^{E}\right)^{-1}\underline{1}} \tag{24}$$

and

$$V\left(\tilde{\text{«}}_E\right)=\frac{d^{2}\text{«}^{2}}{d^{2}\left(\underline{\sim}^{E}\right)'\left(V^{E}\right)^{-1}\left(\underline{\sim}^{E}\right)+2d\left(\underline{\sim}^{E}\right)'\left(V^{E}\right)^{-1}\underline{1}+\underline{1}'\left(V^{E}\right)^{-1}\underline{1}} \tag{25}$$

By using the results of Sarhan (1954), p.322, we have found out for the exponential distribution given in (15) ,the following results

$\underline{1}'\left(V^{E}\right)^{-1}\underline{1}=n^{2},\ \underline{1}'\left(V^{E}\right)^{-1}\underline{\sim}^{E}=n,\ \left(\underline{\sim}^{E}\right)'\left(V^{E}\right)^{-1}\underline{\sim}^{E}=n,$

$\underline{1}'\left(V^{E}\right)^{-1}=\left(n^{2},0,0,...,0\right)$ , a vector of order $1\times n$ and

$\left(\underline{\sim}^{E}\right)'\left(V^{E}\right)^{-1}=\left(1,1,...,1\right)$, a row vector of $n$ ones.

Using the above results, (24) and (25) reduces to,

$$\tilde{\text{«}}_E=\frac{n}{2d+d^{2}+n}X_{1:n}^{E}+\frac{d}{2d+d^{2}+n}\overline{X}^{E} \tag{26}$$

$$=e_1X_{1:n}^{E}+e_2\overline{X}^{E}, \tag{27}$$

where $e_1=\dfrac{n}{2d+d^{2}+n},e_2=\dfrac{d}{2d+d^{2}+n}$ and $\overline{X}^{E}$ is the sample mean of a random sample of size $n$ taken from (15), and

$$V\left(\tilde{«}_E\right) = \frac{d^2«^2}{n\left(2d + d^2 + n\right)}, \qquad (28)$$

where $d = \frac{c}{1-c}$, $c$ is the known coefficient of variation. The main advantage of the results given in (27) and (28) is that, one can obtain the *BLUE* and its variance of the location parameter « of the exponential distribution with known coefficient of variation without knowing the values of means, variances and covariances of the entire order statistics arising from the standard exponential distribution.

## 4. MOMENT ESTIMATOR OF THE PARAMETER « OF THE EXPONENTIAL DISTRIBUTION WHEN d IS KNOWN

In this section we consider an exponential distribution $E\left(«, d^2«^2\right)$ with *p.d.f.* given in (15). Let $X_1, X_2, ..., X_n$ are random sample of size n drawn from the *p.d.f.* given in (15). Let « be the first population raw moment of the exponential distribution given in (15) and let $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ be the first raw moment of the sample. Equating the first population raw moment and first raw moment of the sample, we get the moment estimator of «, namely

$$«'' = \frac{\overline{X}}{1+d}$$

and

$$V\left(«''\right) = \frac{d^2«^2}{n\left(1+d\right)^2}$$

## 5. NUMERICAL ILLUSTRATION

Now we have evaluated the coefficients of $X^E_{1:n}$ and $\overline{X}^E$ of the *BLUE* $\tilde{«}_E$ given in (27) for $n = 2(1)20$ and for $c = 0.15$ and $0.2$, where $c$ is the coefficient of variation and are given in table 5.1. Also we have evaluated $V\left(\hat{«}_E\right), V\left(«^*_E\right), V\left(«''_E\right), V\left(\tilde{«}_E\right)$, the relative efficiency $RE_1 = RE\left(\tilde{«}_E / \hat{«}_E\right)$ of $\tilde{«}_E$ relative to $\hat{«}_E$, the relative efficiency $RE_2 = RE\left(\tilde{«}_E / «^*_E\right)$ of $\tilde{«}_E$ relative to $«^*_E$, the relative efficiency $RE_3 = RE\left(\tilde{«}_E / «''_E\right)$ of $\tilde{«}_E$ relative to $«''_E$, for $n = 2(1)20$ and $c = 0.15$ and $0.2$, and are presented in table 5.2 . It may be noted that in all the cases our estimator $\tilde{«}_E$ is much better than that of $\hat{«}_E, «_E^*$, and $«_E^{''}$.

**Table 5.1:** Coefficients of $X_{1:n}^E$ and $\overline{X}^E$ in the $BLUE$, $\tilde{«}_E$, for different values of $n$ and $c$.

| $n$ | $c=0.15$ | | $c=0.2$ | |
|---|---|---|---|---|
| | $e_1$ | $e_2$ | $e_1$ | $e_2$ |
| 2 | 0.83890 | 0.07402 | 0.78049 | 0.09756 |
| 3 | 0.88650 | 0.05215 | 0.84211 | 0.07018 |
| 4 | 0.91239 | 0.04025 | 0.87671 | 0.05479 |
| 5 | 0.92866 | 0.03278 | 0.89888 | 0.04494 |
| 6 | 0.93984 | 0.02764 | 0.91429 | 0.03810 |
| 7 | 0.94799 | 0.02390 | 0.92562 | 0.03306 |
| 8 | 0.95419 | 0.02105 | 0.93431 | 0.02920 |
| 9 | 0.95907 | 0.01881 | 0.94118 | 0.02614 |
| 10 | 0.96301 | 0.01699 | 0.94675 | 0.02367 |
| 11 | 0.96626 | 0.01550 | 0.95135 | 0.02162 |
| 12 | 0.96899 | 0.01425 | 0.95522 | 0.01990 |
| 13 | 0.97130 | 0.01319 | 0.95853 | 0.01843 |
| 14 | 0.97330 | 0.01227 | 0.96137 | 0.01717 |
| 15 | 0.97503 | 0.01147 | 0.96386 | 0.01606 |
| 16 | 0.97656 | 0.01077 | 0.96604 | 0.01509 |
| 17 | 0.97791 | 0.01015 | 0.96797 | 0.01423 |
| 18 | 0.97911 | 0.00960 | 0.96970 | 0.01347 |
| 19 | 0.98019 | 0.00910 | 0.97125 | 0.01278 |
| 20 | 0.98116 | 0.00866 | 0.97264 | 0.01216 |

**Table 5.2:** Variances of the estimators, $\hat{«}_E, «_E^*, «_E^{''}$, $\tilde{«}_E$ and the relative efficiencies $RE_1, RE_2$ and $RE_3$ for $c=0.15$

| $n$ | $V(\hat{«}_E)$ | $V(«_E^*)$ | $V(«_E^{''})$ | $V(\tilde{«}_E)$ | $RE_1$ | $RE_2$ | $RE_3$ |
|---|---|---|---|---|---|---|---|
| 2 | 0.01557 | 1.00000 | 0.01125 | 0.00653 | 2.38438 | 153.13936 | 1.72282 |
| 3 | 0.00519 | 0.50000 | 0.00750 | 0.00307 | 1.69055 | 162.86645 | 2.44300 |
| 4 | 0.00260 | 0.33333 | 0.00563 | 0.00178 | 1.46067 | 187.26404 | 3.16292 |
| 5 | 0.00156 | 0.25000 | 0.00450 | 0.00116 | 1.34483 | 215.51724 | 3.87931 |
| 6 | 0.00104 | 0.20000 | 0.00375 | 0.00081 | 1.28395 | 246.91358 | 4.62963 |
| 7 | 0.00074 | 0.16667 | 0.00321 | 0.00060 | 1.23333 | 277.78333 | 5.35000 |
| 8 | 0.00056 | 0.14286 | 0.00281 | 0.00046 | 1.21739 | 310.56522 | 6.10870 |
| 9 | 0.00043 | 0.12500 | 0.00250 | 0.00037 | 1.16216 | 337.83784 | 6.75676 |
| 0 | 0.00035 | 0.11111 | 0.00225 | 0.00030 | 1.16667 | 370.36667 | 7.50000 |
| 11 | 0.00028 | 0.10000 | 0.00205 | 0.00025 | 1.12000 | 400.00000 | 8.20000 |
| 12 | 0.00024 | 0.09091 | 0.00188 | 0.00021 | 1.14286 | 432.90476 | 8.95238 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 13 | 0.00020 | 0.08333 | 0.00173 | 0.00018 | 1.11111 | 462.94444 | 9.61111 |
| 14 | 0.00017 | 0.07692 | 0.00161 | 0.00015 | 1.13333 | 512.80000 | 10.73333 |
| 15 | 0.00015 | 0.07143 | 0.00150 | 0.00013 | 1.15385 | 549.46154 | 11.53846 |
| 16 | 0.00013 | 0.06667 | 0.00141 | 0.00012 | 1.08333 | 555.58333 | 11.75000 |
| 17 | 0.00012 | 0.06250 | 0.00132 | 0.00011 | 1.09090 | 568.18182 | 12.00000 |
| 18 | 0.00010 | 0.05882 | 0.00125 | 0.00009 | 1.11111 | 653.55556 | 13.88889 |
| 19 | 0.00009 | 0.05556 | 0.00118 | 0.00008 | 1.12500 | 694.50000 | 14.75000 |
| 20 | 0.00008 | 0.05263 | 0.00113 | 0.00008 | 1.01250 | 657.87500 | 14.12500 |

$c = 0.2$ ,

| $n$ | $V(\hat{\alpha}_E)$ | $V(\alpha^*_E)$ | $V(\alpha''_E)$ | $V(\tilde{\alpha}_E)$ | $RE_1$ | $RE_2$ | $RE_3$ |
|---|---|---|---|---|---|---|---|
| 2 | 0.03125 | 1.00000 | 0.02000 | 0.01220 | 2.56148 | 81.96721 | 1.63934 |
| 3 | 0.01042 | 0.50000 | 0.01333 | 0.00585 | 1.78120 | 85.47009 | 2.27863 |
| 4 | 0.00521 | 0.33333 | 0.01000 | 0.00342 | 1.52339 | 97.46491 | 2.92398 |
| 5 | 0.00313 | 0.25000 | 0.00800 | 0.00225 | 1.39111 | 111.11111 | 3.55556 |
| 6 | 0.00208 | 0.20000 | 0.00667 | 0.00159 | 1.30818 | 125.78616 | 4.19497 |
| 7 | 0.00149 | 0.16667 | 0.00571 | 0.00118 | 1.26271 | 141.24576 | 4.83898 |
| 8 | 0.00112 | 0.14286 | 0.00500 | 0.00091 | 1.23077 | 156.98901 | 5.49451 |
| 9 | 0.00087 | 0.12500 | 0.00444 | 0.00073 | 1.19178 | 171.23288 | 6.08219 |
| 10 | 0.00069 | 0.11111 | 0.00400 | 0.00059 | 1.16949 | 188.32203 | 6.77966 |
| 11 | 0.00057 | 0.10000 | 0.00364 | 0.00049 | 1.16327 | 204.08163 | 7.42857 |
| 12 | 0.00047 | 0.09091 | 0.00333 | 0.00041 | 1.14634 | 221.73171 | 8.12195 |
| 13 | 0.00040 | 0.08333 | 0.00308 | 0.00035 | 1.14286 | 238.08571 | 8.80000 |
| 14 | 0.00034 | 0.07692 | 0.00286 | 0.00031 | 1.09677 | 248.12903 | 9.22581 |
| 14 | 0.00030 | 0.07143 | 0.00267 | 0.00027 | 1.11111 | 264.55556 | 9.88889 |
| 16 | 0.00026 | 0.0667 | 0.00250 | 0.00024 | 1.08333 | 277.79167 | 10.41667 |
| 17 | 0.00023 | 0.06250 | 0.00235 | 0.00021 | 1.09524 | 297.61905 | 11.19048 |
| 18 | 0.00020 | 0.05882 | 00222 | 0.00019 | 1.05263 | 309.57895 | 11.68421 |
| 19 | 0.00018 | 0.05556 | 0.00211 | 0.00017 | 1.05882 | 326.82353 | 12.41176 |
| 20 | 0.00016 | 0.05263 | 0.00200 | 0.00015 | 1.06667 | 350.86667 | 13.33333 |

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