ESTIMATING THE PARAMETER OF THE EXPONENTIAL DISTRIBUTION WITH KNOWN COEFFICIENT OF VARIATION BY ORDER STATISTICS

N. K. Sajeevkumar and Irshad M. R.

ABSTRACT

In the Journal of Statistics

23 (2013), 33-42
 ESTIMATING THE PARAMETER - OF THE EXPONENTIAL
 STRIBUTION WITH KNOWN COEFFICIENT OF VARIATION BY
 ORDER STATISTICS

N. K. Sajeevkumar and Irshad M. R.
 ABSTRACT

In t parh Journal of Statistics

33 (2013), 33-42
 ESTIMATING THE PARAMETER - OF THE EXPONENTIAL
 STRIBUTION WITH KNOWN COEFFICIENT OF VARIATION BY
 ORDER STATISTICS

N. K. Sajeevkumar and Irshad M. R.

In this paper, we by order statistics. Also we have obtained the compact form of the estimator derived. Efficiency comparisons are also made on the proposed estimators with some of the usual estimators of \sim .

1. INTRODUCTION

In some of the biological and physical science problems, situations where the scale parameter is proportional to the location parameter are seen reported in the literature. (see, for example, Gleser and Healy, (1976). If the scale parameter is proportional to the location parameter, the constant of proportion is denoted by *c* , *c* is the known coefficient of variation. If the parent distribution is a normal **EXECTM: EXECTM: EXECTM: ANDENTIFY** IN: **ANDENTACT** In this paper, we have obtained the best linear unbiased estimator (*BLUE*) of μ of the exponential distribution $E(-, c^2 - 2)$ with known coefficient of variatio the known coefficient of variation, then the problem of estimating \sim has been extensively discussed in the available literature, for example see, Searls (1964), Khan (1968), Gleser and Healy (1976), Arnholt and Hebert (1995), Kunte **EXERACT MASTRACT**

and the best linear unbiased estimator (*BLUE*) of μ or the exponential distribution $E(-, e^2 - 2)$ with known coefficient of variation c

by order statistics. Also we have obtained the compact form o **ABSTRACT**

In this paper, we have obtained the best linear unbiased estimator (*BLUE*) of μ of the exponential distribution *E*($\tau_c^2 - 2^2$) with known coefficient of variation c by order statistics. Also we have obt are discussed in Thomas and Sajeevkumar (2003). Estimating the mean of logistic distribution with known coefficient of variation are discussed by Sajeevkumar and Thomas (2005). Estimating the location parameter of an exponential distribution with known coefficient of variation are discussed in Ghosh and Razmpour (1982) and Samanta (1984).

In this paper, we describe the technique of estimating the location parameter \sim of the exponential distribution by order statistics, when the coefficient of variation is known.

2. ESTIMATING THE LOCATION PARAMETER OF A DISTRIBUTION WHEN THE SCALE PARAMETER IS *d* **FOR KNOWN** *d*

In this section we consider the family *G* of all absolutely continuous distributions which depend on a location parameter \sim and a scale parameter *N. K. Sajeevkumar and Irshad M. R.*
 2. ESTIMATING THE LOCATION PARAMETER - OF A
 EXECUTE DUSTRIBUTION WHEN THE SCALE PARAMETER IS $d \sim \textbf{FOR}$

In this section we consider the family *G* of all absolutely continuous
 the form *N. K. Sajeevkun*

2. **ESTIMATING THE LOCATION PARAMETE**
 DISTRIBUTION WHEN THE SCALE PARAMETE
 KNOWN *d*

s section we consider the family *G* of all abso

utions which depend on a location parameter ~ and

~ where d *N. K. Sajeevk*
 **ESTIMATING THE LOCATION PARAMET

TRIBUTION WHEN THE SCALE PARAMET**
 ENOWN *d*

aection we consider the family *G* of all at

ons which depend on a location parameter ~ ar

where d is known. Then any d *N. K. Sajeevkumar and Irshad*
 TING THE LOCATION PARAMETER - OF A
 ON WHEN THE SCALE PARAMETER IS d **- F(

KNOWN** d

consider the family G of all absolutely continued

depend on a location parameter - and a scale pa *N. K. Sajeevkumar and Irshad M. R.*
 **G THE LOCATION PARAMETER - OF A

WHEN THE SCALE PARAMETER IS** d **- FOR

KNOWN** d **

sajder the family** G **of all absolutely continuous

and on a location parameter - and a scale parame** *N. K. Sajeevkumar and Irshad M. R.*
 MATING THE LOCATION PARAMETER - OF A
 EXECUTE PARAMETER IS d - **FOR**
 EXECUTE PARAMETER IS d - **FOR**
 EXECUTE ARAMETER IS d - **FOR**

we consider the family *G* of all absol *N. K. Sajeevkumar and Irshad M. R.*
 LOCATION PARAMETER - OF A
 ETHE SCALE PARAMETER IS d - **FOR**
 KNOWN d

ne family G of all absolutely continuous

location parameter - and a scale parameter

any distributio

$$
f(x:-,d-) = \frac{1}{d-}f_0\left(\frac{x--}{d-}\right), \quad - > 0, d > 0. \tag{1}
$$

N. K. Sajeevkumar and Irshad
 **2. ESTIMATING THE LOCATION PARAMETER - OF A

DISTRIBUTION WHEN THE SCALE PARAMETER IS** d **- FO

KNOWN** *d*

is section we consider the family *G* of all absolutely contin

butions which d Let $\underline{X} = (X_{1:n}, X_{2:n}, ..., X_{n:n})$ be the vector of order statistics of *N. K. Sajeevkumar and Irshad M. R.*
 **2. ESTIMATING THE LOCATION PARAMETER - OF A

EXECUTION WHEN THE SCALE PARAMETER IS** $d \sim \text{FOR}$
 EXECUTION WHEN THE SCALE PARAMETER IS $d \sim \text{FOR}$

this section we consider the fami of size *n* drawn from (1). Let $\underline{r} = (r_{1n}, r_{2n},...,r_{nn})$ and $V = ((v_{r,sn})$ be the *N. K. Sajeevkumar and Irshad M. R.*
 1: THE SCALE PARAMETER IS d **- FOR**
 1: THE SCALE PARAMETER IS d **- FOR**
 1: KNOWN d

the family G of all absolutely continuous

a location parameter - and a scale parameter vector of means and dispersion matrix of the vector of order statistics of a *N. K. Sajeevkumar and Irshad M. R.*

2. **ESTIMATING THE LOCATION PARAMETER - OF A**
 INSTRIBUTION WHEN THE SCALE PARAMETER IS d - **FOR**

In this section we consider the family G of all absolutely continuous

distribu location parameter of (1) , a linear unbiased estimator of \sim based on order statistics is given by (see, Balakrishnan and Rao (1998), p.13) $y = \frac{1}{d} f_0 \left(\frac{x - z}{d - z} \right),$ $\sim > 0, d > 0.$
 $X_{2m},..., X_{n:n}$ be the vector of order statistics of

in from (1). Let $\underline{r} = (\Gamma_{1n}, \Gamma_{2n}, ..., \Gamma_{n:n})$ and $V =$

is and dispersion matrix of the vector of ord

of size n arising fr $l \rightarrow = \frac{1}{d} f_0 \left(\frac{x - x}{d - x} \right), \sim > 0, d > 0.$
 $l, X_{2n}, ..., X_{n:n}$ be the vector of order statistics

awn from (1). Let $\Gamma = (\Gamma_{1n}, \Gamma_{2n}, ..., \Gamma_{n:n})$ and

cans and dispersion matrix of the vector of of

ole of size n arising from f **TING THE LOCATION PARAMETER** ~
 CON WHEN THE SCALE PARAMETER IS
 KNOWN d

consider the family G of all absolutely

depend on a location parameter ~ and a scandary

sknown. Then any distribution belongs to G has

thro **DISTRIBUTION WHEN THE SCALE PARAMETER IS** $d \sim \text{FOW}$ **

is section we consider the family** G **of all absolutely contin

butions which depend on a location parameter** \sim **and a scale paranty-

where d is known. Then any dis IMATING THE LOCATION PARAMETER - OF A**
 EUTION WHEN THE SCALE PARAMETER IS d **- FOR**
 KNOWN d

i we consider the family G of all absolutely continuous

inch depend on a location parameter - and a scale parameter

d is *V V V* **TIMATING THE LOCATION PARAMETER - OF A**
 IBUTION WHEN THE SCALE PARAMETER IS d **- FO**
 KNOWN d

on we consider the family G of all absolutely contin

which depend on a location parameter - and a scale parar

re d is k 2. ESTIMATING THE LOCATION PARAMETER - OF A

DISTRIBUTION WHEN THE SCALE PARAMETER IS d - FOR

His section we consider the family G of all absolutely continuous

ibutions which depend on a location parameter - and a sca is known. Then any distribution belongs to *G* has a $p.d.f.$ of
 $=\frac{1}{d} f_0 \left(\frac{x - \epsilon}{d \epsilon} \right), \epsilon > 0, d > 0.$ (1)
 \ldots (1)
 \ldots (1)
 \ldots (1). Let $\Gamma = (\Gamma_{1n}, \Gamma_{2n}, \ldots, \Gamma_{nn})$ and $V = ((v_{r, sn})$ be the

and dispersion matrix $J = \frac{1}{d} f_0 \left(\frac{x - z}{d - z} \right), \sim > 0, d > 0.$ (1)
 $X_{2n},..., X_{n:n}$ be the vector of order statistics of a random sample
 m from (1). Let $\underline{r} = (\underline{r}_{1:n}, \underline{r}_{2:n}, ..., \underline{r}_{n:n})$ and $V = ((\underline{v}_{r,sn})$ be the

ns and dispersion matrix o **2. ESTIMATING THE LOCATION PARAMETER - OF A**
 DISTRIBUTION WHEN THE SCALE PARAMETER IS d - **FOR**
 KNOWN d

s section we consider the family G of all absolutely continuous

utions which depend on a location paramete therefore of order statistic

1). Let $\underline{r} = (r_{1n}, r_{2n},...,r_{nn})$ and

spersion matrix of the vector of

arising from $f(x:0,1)$. Then by

1), a linear unbiased estimator

1, Balakrishnan and Rao (1998), p.
 $-\underline{r_1}^{\prime}V^{-1}\$ on non (1). Let $\underline{L} = (1 \ln y, 1 \ln z, ..., 1 \ln z)$ and v

and dispersion matrix of the vector of or

exist of size n arising from $f(x:0,1)$. Then by con

eter of (1), a linear unbiased estimator of

en by (see, Balakrishnan and ∼ where d is known. Then any distribution belongs to *G* has a *p.d.f*.

Th

(x: -,*d* -) = $\frac{1}{d}$ - f₀ $\left(\frac{x-\epsilon}{d-\epsilon}\right)$, - > 0,*d* > 0.

= (X_{Ln}, X_{2n},..., X_{nn}) be the vector of order statistics of a random sam
 epend on a location parameter \sim and a scal

nown. Then any distribution belongs to *G* has
 $f_0\left(\frac{x-\epsilon}{d\epsilon}\right), \sim > 0, d > 0.$
 $X_{n:n}$ b the vector of order statistics of a ran

1 (1). Let $\underline{r} = (\Gamma_{1:n}, \Gamma_{2:n}, ..., \Gamma_{n:n})$ and $y = \frac{1}{d^2} f_0\left(\frac{x-\epsilon}{d^2}\right), ->0, d>0.$ (1)
 $X_{2m},..., X_{mn}$ be the vector of order statistics of a random sample
 *V Y*_{2*m*},..., *X_{nm}* be the vector of order statistics of a random sample
 V nm (1). Let $\Gamma = (\Gamma_{1m}, \$ e consider the family *G* of all absolutely continu
depend on a location parameter - and a scale param
known. Then any distribution belongs to *G* has a $p.d.f$
 $\frac{1}{l} f_0\left(\frac{x-\infty}{d-\infty}\right), \infty > 0, d > 0.$
..., $X_{n,n}$ be the v where d is known. Then any distribution belongs to G has a pd.f. of
 $x: \neg A \rightarrow = \frac{1}{d-} f_0 \left(\frac{x-\alpha}{d-} \right), \neg > 0, d > 0.$ (1)
 $\qquad = (X_{1\alpha}, X_{2\alpha}, ..., X_{n\alpha})$ be the vector of order statistics of a random sample
 n drawn from (1) drawn from (1). Let $\underline{\Gamma} = (\Gamma_{1n}, \Gamma_{2n}, ..., \Gamma_{n|n})$ and $V = ((v_{r,sn}))$ be the
means and dispersion matrix of the vector of order statistics of a
umple of size n arising from $f(x:0,1)$. Then by considering ~ as the
parameter of $f(x, z, u^2) = \frac{d}{dz} J_0 \left(\frac{d}{dz} \right)^{-1} > 0.0, u > 0.$

Let $\underline{X} = (X_{1a}, X_{2a}, ..., X_{na})$ be the vector of order statistics of a random sample

of size *n* drawn from (1). Let $\underline{r} = (r_{1a}, r_{2a}, ..., r_{na})$ and $V = ((v_{r, ra})$ be the

vec raw from (1). Let $\underline{r} = (\underline{r}_{1x}, \underline{r}_{2x}, ..., \underline{r}_{nx})$ and $V = [\underline{r}_{1x}, \underline{r}_{2x}, ..., \underline{r}_{nx}]$ and $\underline{r}_{2x}, \underline{r}_{2x}, \underline{r}_{2x}, \underline{r}_{2x},$ and dispersion matrix of the vector of order statistic
size n arising from $f(x:0,1)$. Then by considering -
c of (1), a linear unbiased estimator of - based on
y (see, Balakrishnan and Rao (1998), p.13)
 $\left(\frac{[r] - r_1]}{2}\right$ *V Wm* from (1). Let $\underline{r} = (r_{1n}, r_{2n},...,r_{nn})'$ and $V = ((v_{r,sn})')$ be the
 V and *N* (*V C*). Then by considering \sim as the
 V and dispersion matrix of the vector of order statistics of a
 V and *N* (*So ii C* means and dispersion matrix of the vector of order statistics of
pple of size n arising from $f(x:0,1)$. Then by considering \sim as the
rameter of (1), a linear unbiased estimator of \sim based on orde
given by (see, Bala *,...,* X_{nn} *)* be the vector of order statistics of a random

rom (1). Let $\underline{r} = (r_{1n}, r_{2n}, ..., r_{nn})$ and $V = ((v_{r,sn}))$

and dispersion matrix of the vector of order statistic

size n arising from $f(x:0,1)$. Then by conside drawn from (1). Let $\underline{r} = (r_{1n}, r_{2n}, \dots, r_{nn})$ and $V = ((v_{r,sn}))$ be the means and dispersion matrix of the vector of order statistics of ample of size n arising from $f(x:0,1)$. Then by considering \sim as the parameter of

$$
\hat{c} = -\frac{\Gamma V^{-1}(\Gamma^{-1} - \Gamma)}{(\Gamma V^{-1} - \Gamma)(\Gamma^{-1} - \Gamma)} \frac{X}{Y^{-1}} \tag{2}
$$
\n
$$
V(\hat{c}) = \frac{(\Gamma V^{-1} - \Gamma)(\Gamma V^{-1} - \Gamma)}{(\Gamma V^{-1} - \Gamma)(\Gamma V^{-1} - \Gamma)} \tag{3}
$$
\n
$$
V(\hat{c}) = \frac{(\Gamma V^{-1} - \Gamma)(\Gamma V^{-1} - \Gamma)}{(\Gamma V^{-1} - \Gamma)(\Gamma V^{-1} - \Gamma)} \tag{3}
$$
\n
$$
V(\hat{c}) = \frac{(\Gamma V^{-1} - \Gamma)(\Gamma V^{-1} - \Gamma)}{(\Gamma V^{-1} - \Gamma)} \tag{4}
$$
\n
$$
V(\hat{c}) = \frac{(\Gamma V^{-1} - \Gamma)(\Gamma V^{-1} - \Gamma)}{(\Gamma V^{-1} - \Gamma)(\Gamma V^{-1} - \Gamma)} \tag{4}
$$

and

$$
V(\hat{z}) = \frac{\left(\underline{\Gamma}^{\top}V^{-1}\underline{\Gamma}\right)d^{2}z^{2}}{\left(\underline{\Gamma}^{\top}V^{-1}\underline{\Gamma}\right)\left(\underline{\Gamma}^{\top}V^{-1}\underline{1}\right) - \left(\underline{\Gamma}^{\top}V^{-1}\underline{1}\right)^{2}},\tag{3}
$$

where 1 is a column vector of n ones. Also by considering d_z as the scale given by (see, Balakrishnan and Rao (1998), p.13)

isities is given by (see, Balakrishnan and Rao (1998), p.13)
\n
$$
\hat{=} = -\frac{\underline{\Gamma} V^{-1} (\underline{\Gamma} \underline{\Gamma} - \underline{\Gamma} \underline{\Gamma}) V^{-1} \underline{X}}{(\underline{\Gamma} V^{-1} \underline{\Gamma}) (\underline{\Gamma} V^{-1} \underline{\Gamma}) - (\underline{\Gamma} V^{-1} \underline{\Gamma})^2}
$$
\n(2)
\n
$$
V(\hat{=} = -\frac{(\underline{\Gamma} V^{-1} \underline{\Gamma}) (\underline{\Gamma} V^{-1} \underline{\Gamma})}{(\underline{\Gamma} V^{-1} \underline{\Gamma}) (\underline{\Gamma} V^{-1} \underline{\Gamma}) - (\underline{\Gamma} V^{-1} \underline{\Gamma})^2},
$$
\n(3)
\n
$$
V(\hat{=} = \frac{(\underline{\Gamma} V^{-1} \underline{\Gamma}) (\underline{\Gamma} V^{-1} \underline{\Gamma}) - (\underline{\Gamma} V^{-1} \underline{\Gamma})^2}{(\underline{\Gamma} V^{-1} \underline{\Gamma}) (\underline{\Gamma} V^{-1} \underline{\Gamma}) - (\underline{\Gamma} V^{-1} \underline{\Gamma})^2},
$$
\n(3)
\n
$$
T = \frac{\underline{\Gamma} V^{-1} (\underline{\Gamma} - \underline{\Gamma} \underline{\Gamma}) V^{-1} \underline{X}}{(\underline{\Gamma} V^{-1} \underline{\Gamma}) (\underline{\Gamma} V^{-1} \underline{\Gamma}) - (\underline{\Gamma} V^{-1} \underline{\Gamma})^2}
$$
\n(4)
\n
$$
V(T) = \frac{(\underline{\Gamma} V^{-1} \underline{\Gamma}) (\underline{\Gamma} V^{-1} \underline{\Gamma}) - (\underline{\Gamma} V^{-1} \underline{\Gamma})^2}{(\underline{\Gamma} V^{-1} \underline{\Gamma}) (\underline{\Gamma} V^{-1} \underline{\Gamma}) - (\underline{\Gamma} V^{-1} \underline{\Gamma})^2}.
$$
\n(5)
\n
$$
m (4) we can obtain another linear unbiased estimator ~* of ~ based on
\ner statistics is given by
$$

and

$$
V(T) = \frac{\left(1^{[V^{-1}]} \right) d^{2} \sim^{2}}{\left(\sum_{i=1}^{[V^{-1}]} \left(1^{[V^{-1}]} \right) - \left(\sum_{i=1}^{[V^{-1}]} \right)^{2}}.
$$
\n(5)

From (4) we can obtain another linear unbiased estimator \sim * of \sim based on order statistics is given by

Estimating the parameter ……by order statistics 35

using the parameter
$$
\dots
$$
 by order statistics

\n
$$
z^* = \frac{1}{d} \left(\frac{\underline{1}[\underline{V}^{-1}(\underline{1}\underline{r}] - \underline{r}\underline{1}]}{\underline{r}[\underline{V}^{-1}\underline{r}](\underline{1}[\underline{V}^{-1}\underline{1}] - (\underline{r}[\underline{V}^{-1}\underline{1}])^2} \right)
$$
\n(6)

\n
$$
V(z^*) = \frac{\underline{(\underline{1}[\underline{V}^{-1}\underline{r}] - (\underline{r}[\underline{V}^{-1}\underline{1}])^{-2}}}{\underline{(\underline{r}[\underline{V}^{-1}\underline{r}] - (\underline{r}[\underline{V}^{-1}\underline{1}])^{-1}(\underline{r}[\underline{V}^{-1}\underline{1}])^2}}.
$$
\n(7)

\nWe derive the *BLUE* z of z based on order statistics is given in the

\nwing theorem.

\nGreen 2.1: Let $\underline{X} = (X_{1:n}, X_{2:n}, ..., X_{nn})$ be the vector of order statistics of a

\ncom sample of size *n* drawn from a distribution with *p.d.f.* defined in (1).

and

$$
V(\sim^*) = \frac{\left(\underline{1}^{\dagger}V^{-1}\underline{1}\right)\sim^2}{\left(\underline{\Gamma}^{\dagger}V^{-1}\underline{\Gamma}\right)\left(\underline{1}^{\dagger}V^{-1}\underline{1}\right) - \left(\underline{\Gamma}^{\dagger}V^{-1}\underline{1}\right)^2}.
$$
\n(7)

Now we derive the *BLUE* \approx of \sim based on order statistics is given in the following theorem.

Theorem 2.1: Let $\underline{X} = (X_{1:n}, X_{2:n},..., X_{nn})$ be the vector of order statistics or teterby order statistics
 $\frac{1}{2} \int \left(\frac{1}{2} \left[\frac{1}{2} - \frac{1}{2} \right] \right)^{-1} \left(\frac{1}{2} \left[\frac{1}{2} \right] \right)^{-1} \left(\frac{1}{2} \left[\frac{1}{2} \right] \right)^{-2}$

(6)
 $\frac{1}{2} \int \left(\frac{1}{2} \left[\frac{1}{2} \right] \right)^{-2} \left(\frac{1}{2} \right)^{-1} \left(\frac{1}{2} \right)^{-1} \left(\$ *random sample of size n* drawn from *d* if $\left(\frac{1}{V} - \left(\frac{1}{V}\right)^{-1} \left(\frac{1}{V}\right)^{-1} + \left(\frac{1}{V}\right)^{-1} \left(\frac{1}{V}\right)^{-1}\right)$ (6)
 and
 $V(-^*) = \frac{\left(\frac{1}{V}\right)^{-1} \left(\frac{1}{V}\right)^{-1} \left(\frac{1}{V}\right)^{-1}}{\left(\frac{1}{V}\right)^{-1} \left(\frac{1}{V}\right)^{-1} \left(\frac{1}{V}\right)^{-1} \$ Let $\underline{r} = (r_{1:n}, r_{2:n}, ..., r_{nn})$ and $V = ((v_{r,sn})$ be the vector ating the parameterby order statistics
 \int \int \int \int $\left(\frac{1}{\left[\frac{V^{-1} \left(\frac{V}{\left[\frac{V}{\left[\frac{V}{\left[\frac{V}{\left[\frac{V}{\left[\frac{V}{\left[\frac{V}{\left[\frac{V}{\left[\frac{V}{\left[\frac{V}{\left[\frac{V}{\left[\frac{V}{\left[\frac{V}{\left[\frac{V}{\left[\frac{V}{\left[\frac{V}{\left[\frac{V}{\left[\frac{V}{\left[\frac{V}{\left[\frac{$ dispersion matrix respectively of the vector of order statistics of a random Estimating the parameterby order statistics
 $\therefore \frac{1}{\pi} \left(\frac{[V^{-1}([T^{-} - \underline{L}])^{V^{-1}} \underline{X}}{(\underline{V}^{-1} - \underline{V})^T \underline{Y}^{-1} - (\underline{V}^{-1} - \underline{V})^T} \right)$ (6)

and
 $V(-^*) = \frac{([V^{-1} - \underline{V}]([V^{-1} - \underline{V}]^{-2})}{(\underline{V}^{-1} - \underline{V})([V^{-1} - \underline{V}]^{-2})}$ is given by ⁻¹ \underline{r})($\underline{i}V^{-1}\underline{1}$) -($\underline{r}V^{-1}\underline{1}$)²

ne *BLUE* \approx of \sim based on order s
 $\underline{X} = (X_{1:n}, X_{2:n}, ..., X_{n:n})$ be the vectosize *n* drawn from a distribution wit

,..., $r_{n:n}$) and $V = ((v_{r,sn}))$ be the respecti erive the *BLUE* \tilde{z} of \sim based on order statistics is
heorem.

2.1: Let $\underline{X} = (X_{1:n}, X_{2:n},..., X_{n:n})$ be the vector of order

ruple of size *n* drawn from a distribution with *p.d.f.*
 $\Gamma_{1:n}, \Gamma_{2:n}, ..., \Gamma_{n:n}$ and $V = (($ $\sqrt{\left(\frac{1}{2}(V^{-1} - 1) - \left(\frac{1}{2}(V^{-1} - 1)\right)^2}\right)}$
 $\sqrt{\left(\frac{1}{2}(V^{-1} - 1) - \left(\frac{1}{2}(V^{-1} - 1)\right)^2}\right}$
 $\sqrt{\left(\frac{1}{2}(V^{-1} - 1) - \left(\frac{1}{2}(V^{-1} - 1)\right)^2}\right}$.

BLUE \approx of \sim based on order statistics is given in t
 $= (X_{1:n}, X_{2:n},..., X_{nn})$ $\underline{V}^{-1}(\underline{I}\underline{r} - \underline{r_1})V^{-1}\underline{X}$ (6)
 $\underline{V}^{-1}\underline{r}(\underline{I}V^{-1}\underline{I}) - (\underline{r}V^{-1}\underline{I})^2$ (7)
 $\underline{V}^{-1}\underline{r}(\underline{I}V^{-1}\underline{I}) - (\underline{r}V^{-1}\underline{I})^2$ (7)

e the *BLUE* $\stackrel{<}{\sim}$ of \sim based on order statistics is given in the

tem (6)
 $\vec{d} \left(\frac{\vec{r} \cdot \vec{v}^{-1} \vec{r}}{(\vec{r} \cdot \vec{v}^{-1})(\vec{v}^{-1} \cdot \vec{r})} - (\vec{r} \cdot \vec{v}^{-1} \cdot \vec{r})^2 \right)$ (6)
 $\frac{(\vec{r} \cdot \vec{v}^{-1} \vec{r}) (\vec{r} \cdot \vec{r}^{-1}) - (\vec{r} \cdot \vec{v}^{-1} \cdot \vec{r})^2}{(\vec{r} \cdot \vec{v}^{-1})(\vec{r} \cdot \vec{r}) - (\vec{r} \cdot \vec{v}^{-1} \cdot \vec{r})^2}$ $\begin{aligned}\n&= \frac{1}{d} \left(\frac{|\dot{V}^{-1}(\Gamma^{-} - \Gamma)}{|\dot{V}^{-1}\Gamma|} \right)^2 \\
&= \frac{1}{d} \left(\frac{|\dot{V}^{-1}(\Gamma^{-} - \Gamma)}{|\dot{V}^{-1}\Gamma|} \right)^2 - \left(\frac{|\dot{V}^{-1} \Gamma|}{|\dot{V}^{-1}\Gamma|} \right)^2 \\
&= \frac{|\dot{V}^{-1} \Gamma|}{|\dot{V}^{-1}\Gamma|} \left(|\dot{V}^{-1}\Gamma| - |\dot{V}^{-1}\Gamma| \right)^2 .\n\end{aligned}$ *i* we derive the (7)
 $\underline{F}(\underline{i}V^{-1}\underline{i}) - (\underline{r}V^{-1}\underline{i})^2$ (7)
 $\underline{E}UUE \stackrel{?}{\sim}$ of \sim based on order statistics is given in the
 $\underline{X} = (X_{1:n}, X_{2:n}, ..., X_{n:n})$ be the vector of order statistics of a

ze *n* drawn from a distribution with the BLUE z of \sim based on order statistics is given in the

m.

Let $\underline{X} = (X_{1:n}, X_{2:n}, ..., X_{n:n})$ be the vector of order statistics of a

of size *n* drawn from a distribution with *p.d.f.* defined in (1).
 $x_n, ..., r_{n:n}$ a ⁻¹(<u>Ir</u>⁻⁻r<u>1</u>) V^{-1}
 $\frac{1}{2}$
 $\left(\frac{1}{2}V^{-1}$] $-\left(\frac{1}{2}V^{-1}\right)\right)^2$ (7)
 $\frac{1}{2}$
 $\left(\frac{1}{2}V^{-1}$] $-\left(\frac{1}{2}V^{-1}\right)^2\right)^2$ (7)
 $BLUE \approx 0$ c – based on order statistics is given in the
 $\frac{X}{X} = \left(X_{Lx_1}, X_{2x_2}, ..., X_{Lx$, $X_{2:n},..., X_{n:n}$ be the vector
won from a distribution with
nd $V = ((v_{r,s:n}))$ be the
ly of the vector of order
 $f(x:0,1)$. Then the *BLUE*
 $\frac{V^{-1} \underline{X}}{1!}$
 $\frac{1}{1!} + i[\underline{V}^{-1} \underline{1}]$
 $\frac{2}{1} + i[\underline{V}^{-1} \underline{1}]$,
f *n* ones. (7)
 $V^{-1} \underline{\Gamma} \Big) \Big(\underline{1} V^{-1} \underline{1} \Big) - \Big(\underline{\Gamma} V^{-1} \underline{1} \Big)^2$ (7)

the *BLUE* \bar{z} of \sim based on order statistics is given in the

n.

at $\underline{X} = (X_{1:n}, X_{2:n}, ..., X_{nn})$ be the vector of order statistics of a

f size *n* (7)
 $\frac{1}{2} \left[\frac{1}{2}V^{-1} \frac{1}{2}\right] - \left[\frac{1}{2}V^{-1} \frac{1}{2}\right]^2$ (7)

the the *BLUE* = of - based on order statistics is given in the

em.

Let $\underline{X} = (X_{1n}, X_{2n}, ..., X_{nn})$ be the vector of order statistics of a

of size *n* dra ²) = $\frac{(\lfloor V^{-1} \rfloor)^{-2}}{(\lfloor V^{-1} \rfloor)(\lfloor V^{-1} \rfloor) - (\lfloor V^{-1} \rfloor)^{2}}$ (7)

derive the *BLUE* = of - based on order statistics is given in the

theorem.
 (2.1: Let $\underline{X} = (X_{1x}, X_{2x}, ..., X_{nx})$) be the vector of order statistics $\binom{r-1}{k-1} - \binom{r-1}{k-1} - \left(\frac{r}{k}\right)^{k-1}$ (7)
 $\binom{r-1}{k-1} - \left(\frac{r}{k}\right)^{k-1}$ ($\binom{r-1}{k-1} - \binom{r-1}{k-1} - \binom{r-1}{k-1}$ be the vector of order statistics of a

drawn from a distribution with $p.d.f.$ defined in (1).
 $\$ following theorem.
 Theorem 2.1: Let $\underline{X} = (X_{1x}, X_{2x}, ..., X_{nx})$ be the vector of order statistics of a

random sample of size *n* drawn from a distribution with $p.d.f$, defined in (1).

Let $\underline{r} = (r_{1x}, r_{2x}, ..., r_{nx})$ and **a**
 $\frac{X}{X} = (X_{1:n}, X_{2:n}, ..., X_{n:n})$ be the vector of order statistics of a

size *n* drawn from a distribution with *p.d.f.* defined in (1).

,,,,r_{*n;n*})</sub> and $V = ((v_{r,sn})$ be the vector of means and

respectively of the ve **Theorem 2.1:** Let $\underline{X} = (X_{1n}, X_{2n},...,X_{n|X})$ be the vector of order statistics of a
random sample of size *n* drawn from *a* distribution with $p.d.f.$ defined in (1).
Let $\underline{r} = (r_{1n}, r_{2n},...,r_{nn})$ and $V = ((v_{r,n}))$ be the v **em 2.1:** Let $\underline{X} = (X_{1:n}, X_{2:n},..., X_{nn})'$ be the vector of order statistics of a
 $r = (r_{1:n}, r_{2:n},..., r_{nn})$ and $V = ((v_{r,n:n}))$ be the vector of means and

sion matrix respectively of the vector of order statistics of a random

si 2n,..., $\Gamma_{n,n}$ and $V = ((v_{r,s:n}))$

x respectively of the vector of

drawn from $f(x:0,1)$. Then th
 $\frac{\Gamma V^{-1} \underline{X} + \underline{i} V^{-1} \underline{X}}{1 - \frac{1}{\Gamma} + 2d\Gamma V^{-1} \underline{1} + \underline{i} V^{-1} \underline{1}}$
 $\frac{d^2 z^2}{1 - \frac{1}{\Gamma} + 2d\Gamma V^{-1} \underline{1} + \underline{i} V^{-1} \underline{1}}$, *Y*_{2*m*},...,*Y*_{*nm*} *J* and *V* = ((*Y_{r,sm}*)) be the vector of
 Y and *V* = ((*Y_{r,sm}*)) be the vector of
 n drawn from *f*(*x*:0,1). Then the *BLUE* z^2 of the p
 $\frac{d\Gamma V^{-1} \underline{X} + \underline{i} V^{-1} \underline{X}}{V^{-1} \underline{I} + \underline{i$ be the value of $\left[\frac{d\mathbf{r}}{V} - \frac{1}{2} \mathbf{r} \right]$ and $V = \left(\left[\mathbf{v}_{r, x}\right]\right)$ be the vector of means and
tarix respectively of the vector of order statistics of a random
e n drawn from $f(x:0,1)$. Then the *BLUE* $\frac{z}{r}$

$$
z = \frac{\left(d\underline{\Gamma}V^{-1}\underline{X} + \underline{i}V^{-1}\underline{X}\right)}{\left(d^2\underline{\Gamma}V^{-1}\underline{\Gamma} + 2d\underline{\Gamma}V^{-1}\underline{1} + \underline{i}V^{-1}\underline{1}\right)}
$$
(8)

and its variance

$$
Var(z) = \frac{d^2 z^2}{\left(d^2 \underline{\Gamma} V^{-1} \underline{\Gamma} + 2d\underline{\Gamma} V^{-1} \underline{1} + \underline{i} V^{-1} \underline{1}\right)},
$$
\n(9)

where 1 is a column vector of *n* ones.

variance
 $\pi r(\tilde{z}) = \frac{d^2 z^2}{(d^2 \Gamma V^{-1} \Gamma + 2d\Gamma)^2}$

1 is a column vector of *n*

Given $\underline{X} = (X_{1:n}, X_{2:n}, ...$

cs of a random sample of *n*
 $r = 1, 2, ..., n$, and $Cov(Y_{r:n})$
 $\frac{r}{d} = Y_{r:n}, r = 1, 2, ..., n$ ts variance
 $Var(z) = \frac{d^2 z^2}{(d^2 \sum V^{-1} \sum t^2 2d \sum V^{-1} \sum t^2 1^2)}$,
 $P = \frac{1}{2}$ is a column vector of *n* ones.
 f: Given $\underline{X} = (X_{1:n}, X_{2:n}, ..., X_{n:n})'$ and let $Y_{1:n}$

titics of a random sample of size *n* drawn from $n, r =$ *E E C F C* **E** *E C C C E <i>C* **C** *C C <i>C C* umn vector of *n* ones.

<u>X</u> = $(X_{1:n}, X_{2:n},..., X_{n:n})'$ and

andom sample of size *n* d

.., *n*, and $Cov(Y_{r:n}, Y_{s:n}) = v_{r,s}$
 $Y_{r:n}, r = 1, 2,..., n$
 $d\Gamma_{r:n} + 1 \rangle \sim, r = 1, 2,..., n$
 $x \sim \frac{2}{v_{r,r:n}}, r = 1, 2,..., n$ $z = \frac{a_1 \cdot b_1 \cdot c_2 \cdot c_3 \cdot c_4 \cdot c_5 \cdot c_6 \cdot c_7 \cdot c_7 \cdot c_8 \cdot c_9 \cdot c_1 \cdot c_1 \cdot c_1 \cdot c_2 \cdot c_3 \cdot c_4 \cdot c_5 \cdot c_7 \cdot c_7 \cdot c_8 \cdot c_9 \cdot c_9 \cdot c_1 \cdot c_1 \cdot c_2 \cdot c_3 \cdot c_1 \cdot c_2 \cdot c_3 \cdot c_1 \cdot c_1 \cdot c_2 \cdot c_2 \cdot c_3 \cdot$

$$
\frac{X_{r:n} - \sim d}{d} = Y_{r:n}, r = 1, 2, ..., n
$$

and

$$
E(X_{rn}) = (d\Gamma_{rn} + 1) \sim r = 1, 2, ..., n
$$
\n(10)

$$
V(X_{rn}) = d^{2} \sim^{2} v_{r, rn}, \ r = 1, 2, ..., n
$$
\n(11)

and

N. K. Sajeevkumar and Irshad M. R.
\n
$$
Cov(X_{r,n}, X_{sn}) = d^2 \sim^2 v_{r,s:n}, 1 \le r < s \le n.
$$
\n(12)
\n
$$
E(\underline{X}) = (d\underline{r} + \underline{1}) \sim
$$
\n(13)

From (10) to (12) one can also write,

Now
$$
(X_{rn}, X_{sn}) = d^2 \sim {}^2v_{r,s;n}
$$
, $1 \le r < s \le n$.
\n
$$
Cov(X_{rn}, X_{sn}) = d^2 \sim {}^2v_{r,s;n}
$$
, $1 \le r < s \le n$.
\n
$$
E(\underline{X}) = (d\underline{r} + \underline{1}) \sim
$$
\n
$$
D(X) = U^{12} \quad (14)
$$

and

$$
D(\underline{X}) = Vd^2 \sim^2,
$$
\n⁽¹⁴⁾

N. K. Sajeevkumar and Irshad M. R.
 $Cov(X_{r,n}, X_{sn}) = d^2 \sim^2 v_{r, sn}$, $1 \le r < s \le n$. (12)
 $n (10) \text{ to } (12) \text{ one can also write,}$
 $E(\underline{X}) = (d\underline{r} + \underline{1})$ ~ (13)
 $D(\underline{X}) = Vd^2 \sim^2$, (14)
 $P(\underline{X}) = Vd^2 \sim^2$, (14)
 $P(\underline{X}) = Vd^2 \sim^2$, (14) *N. K. Sajeevkumar and Irshad M. R.*
 $Cov(X_{r,n}, X_{sn}) = d^2 - 2v_{r,s,n}$, $1 \le r < s \le n$. (12)

(10) to (12) one can also write,
 $E(\underline{X}) = (d\underline{\Gamma} + \underline{1}) -$ (13)
 $D(\underline{X}) = Vd^2 - 2$, (14)
 $P(\underline{X}) = (d\underline{X})^2 - 2 = (T_{1,n}, T_{2,n},...,T_{n,n})$ and $V =$ where 1 is a column vector of n ones, $\underline{r} = (r_{1n}, r_{2n},...,r_{nn})$ and $V = ((v_{r,m}))$. 36
 N , K . Sajeevkumar and Irshad M. R.
 $Cov(X_{rx}, X_{zx}) = d^2 - 2v_{r,x,n}$, $1 \le r < s \le n$.

(12)

From (10) to (12) one can also write,
 $E(\underline{X}) = (d\underline{r} + \underline{1})$.

and
 $D(\underline{X}) = Vd^2 - 2$,

where $\underline{1}$ is a column vector of n o *N. K. Sajeevkumar an*
 X_{sn}) = $d^2 \sim^2 v_{r,sn}$, $1 \le r < s \le n$.

2) one can also write,
 $\frac{r_{\Gamma} + 1}{l}$ -
 $I^2 \sim^2$,

olumn vector of n ones, $\underline{r} = (r_{1n}, r_{2n}, ..., r_{nn})$ and l

alized Gauss-Markoff theorem, the *BLUE* $\$ *N. K. Sajeevkumar and Irsha*
 n, X_{sn} = $d^2 \sim^2 v_{r,sn}$, $1 \le r < s \le n$.

(12) one can also write,
 $(d\underline{\Gamma} + 1)$ -
 $Vd^2 \sim^2$,

column vector of n ones, $\underline{\Gamma} = (\Gamma_{\text{Ln}}, \Gamma_{2n}, ..., \Gamma_{nn})$ and $V = ((\text{realized Gauss-Markoff theorem, the *BLUE* $\approx$$ *N. K. Sajeevkum*
 X_{rn}, X_{sn} = $d^2 \sim^2 v_{r,sn}$, $1 \le r < s \le n$.

to (12) one can also write,
 $= (d\underline{r} + \underline{1}) \sim$
 $= Vd^2 \sim^2$,

s a column vector of n ones, $\underline{r} = (r_{\perp n}, r_{\ge n}, ..., r_{n,n})$

igeneralized Gauss-Markoff theorem *N. K. Sajeevkumar and Irsha*
 $Cov(X_{r,n}, X_{sn}) = d^2 \sim^2 v_{r, s;n}$, $1 \le r < s \le n$.
 $E(\underline{X}) = (d\underline{\Gamma} + 1)$ $D(\underline{X}) = Vd^2 \sim^2$,
 $D(\underline{X}) = Vd^2 \sim^2$,
 $P = \frac{1}{2}$ is a column vector of n ones, $\underline{\Gamma} = (\Gamma_{1:n}, \Gamma_{2:n}, ..., \Gamma_{nn})$ and $V = ((v, v)$ *N. K. Sajeevkumar and Irshad M. R.*
 $(X_{\tau,n}) = d^{2} - 2v_{\tau,\text{ex}}, 1 \le r < s \le n.$ (12)

(12) one can also write,
 $d\underline{\Gamma} + 1$) - (13)
 $Vd^{2} - 2$, (14)
 $Vd^{2} - 2$, (14)

column vector of n ones, $\underline{\Gamma} = (\Gamma_{1x}, \Gamma_{2x}, ..., \Gamma_{nx})$ an +1) ~

-²,

mn vector of n ones, $\underline{r} = (r_{1:n}, r_{2:n}, ...$

red Gauss-Markoff theorem, the *BLU*
 $\frac{(1)^{j}V^{-1}\underline{X}}{V^{-1}(d\underline{r}+1)},$
 $\frac{(V^{-1}\underline{X}+\underline{i}V^{-1}\underline{X})}{V^{-1}+2d\underline{r}V^{-1}\underline{1}+\underline{i}V^{-1}\underline{1}}$ = $Vd^2 \sim^2$,

s a column vector of n ones, $\underline{r} = (\underline{r}_{1n}, \underline{r}_{2n}, ..., \underline{r}_{nn})$ and

eneralized Gauss-Markoff theorem, the *BLUE* \approx of $\sim \frac{(d\underline{r} + 1)^{\nu-1}X}{(\underline{r} + 1)^{\nu-1}(d\underline{r} + 1)},$
 $\frac{(d\underline{r}^{\nu-1}X + iV^{-1}X)}{(\underline{r}^$ *N. K. Sajeevkumar ana irsnaa w.*
 $= d^2 \sim^2 v_{r,s,n}$, $1 \le r < s \le n$.

(c e can also write,
 \sim (vector of n ones, $\underline{r} = (r_{1n}, r_{2n}, ..., r_{nn})$ and $V = ((v_{r,s})$

Gauss-Markoff theorem, the *BLUE* \approx of \sim is given by
 $\frac{V^{-$ *N. K. Sajeevkumar and Irshad M. K*
 $(X_{xn}) = d^2 - 2v_{r,s,n}$, $1 \le r < s \le n$. (12

12) one can also write,
 $d\underline{r} + 1$) ~ (13
 $d^2 - 2$, (14

olumn vector of n ones, $\underline{r} = (r_{1n}, r_{2n}, ..., r_{nn})$ and $V = ((v_{r,s:n}))$

alized Gauss-Mar *d* $(X_{r,n}, X_{sn}) = d^2 - 2v_{r,sn}$, $1 \le r < s \le n$.

(12)

(12) (12) one can also write,
 $\sum_{n=1}^{\infty} (d\underline{r} + 1) -$
 $\sum_{n=1}^{\infty} (d\underline{r} + 1) -$
 $\sum_{n=1}^{\infty} (d\underline{r} + 1) -$

(13)
 $\sum_{n=1}^{\infty} (d\underline{r} + 1) -$
 $\sum_{n=1}^{\infty} (d\underline$ *N. K. Sajeevkumar and Irsha*
 $Cov(X_{r,n}, X_{sn}) = d^2 \sim^2 v_{r, s:n}$, $1 \le r < s \le n$.
 $E(\underline{X}) = (d\underline{\Gamma} + \underline{1}) \sim$
 $D(\underline{X}) = Vd^2 \sim^2$,
 $P(\underline{X}) = Vd^2 \sim^2$,
 \Rightarrow $\Gamma = \frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(\underline{X})^2 dx$
 \Rightarrow $\Gamma = \frac{1}{2} \int_{-\infty}^{\infty$ 1) ~ (13)

(14)

m vector of n ones, $\underline{r} = (r_{1n}, r_{2n},...,r_{nn})$ and $V = ((v_{r,sn}))$

d Gauss-Markoff theorem, the *BLUE* \tilde{z} of \sim is given by,
 $\frac{V^{-1} \underline{X}}{V^{-1}(d\underline{r} + \underline{1})}$,
 $\frac{-1}{\underline{X} + \underline{1}V^{-1}\underline{X}}$
 $+2d\underline{r}$ t^{2-2} , (14)

lumn vector of n ones, $\underline{r} = (r_{1n}, r_{2n},...,r_{nn})$ and $V = ((v_{r,sn}))$.

lized Gauss-Markoff theorem, the *BLUE* \approx of \sim is given by,
 $\frac{(-1)^{t}V^{-1}\underline{X}}{2}$
 $\frac{1}{2}(V^{-1}\underline{X} + \underline{i}V^{-1}\underline{X})$
 $\frac{1}{2}(V^{-1}\underline{X$ *N. K. Sajeevkumar and Irshad M. R.*
 $\begin{aligned}\n&= d^{2} \sim {}^{2}v_{r,s;n}, 1 \leq r < s \leq n. \tag{12}\n\end{aligned}$

me can also write,

1)-

(13)
 $\begin{aligned}\n&\frac{1}{s}, \quad \text{(14)} \\
&\text{(15)} \\
&\text{(15)} \\
&\text{(16)} \\
&\text{(17)} \\
&\text{(18)} \\
&\text{(19)} \\
&\text{(19)} \\
&\text{(10)} \\
&\text{(14)} \\
&\text$ 1 1 1 one can also write,
 $+1$) ~ (13)
 -2 ,
 $+2$,
 $+1$) ~ (14)

times vector of n ones, $\Gamma = (\Gamma_{1m}, \Gamma_{2m}, ..., \Gamma_{mn})$ and $V = ((v_{r, \text{cm}}))$.
 $V = \frac{1}{2}V^{-1} \frac{X}{2}$
 $V^{-1}(d\Gamma + 1)$,
 $V^{-1}X + iV^{-1} \frac{X}{2}$
 $\frac{d^2-2}{2}$
 $\frac{d^2-2}{2$

$$
z = \frac{\left(d\underline{\Gamma} + 1\right)V^{-1}\underline{X}}{\left(d\underline{\Gamma} + 1\right)V^{-1}\left(d\underline{\Gamma} + 1\right)},
$$

That is

$$
z = \frac{(d\Gamma V^{-1} \underline{X} + \underline{1} V^{-1} \underline{X})}{d^2 \Gamma V^{-1} \Gamma + 2d\Gamma V^{-1} \underline{1} + \underline{1} V^{-1} \underline{1}}
$$

and

From (10) to (12) one can also write,
\n
$$
E(\underline{X}) = (d\underline{r} + \underline{1}) -
$$
\nand
\n
$$
D(\underline{X}) = Vd^2 \sim^2,
$$
\n(14)
\nwhere 1 is a column vector of n ones, $\underline{r} = (r_{\text{Ln}}, r_{\text{2n}}, ..., r_{\text{ran}})$ and $V = ((v_{r,\text{gen}}))$.
\nThen by generalized Gauss-Markoff theorem, the *BLUE* \approx of \sim is given by,
\n
$$
\approx \frac{(d\underline{r} + \underline{1})V^{-1}\underline{X}}{(d\underline{r} + \underline{1})V^{-1}(d\underline{r} + \underline{1})},
$$
\nThat is
\n
$$
\approx \frac{(d\underline{r}V^{-1}\underline{X} + \underline{1}V^{-1}\underline{X})}{d^2\underline{r}V^{-1}\underline{r} + 2d\underline{r}V^{-1}\underline{1} + \underline{1}V^{-1}\underline{1}} =
$$
\nand
\n
$$
V(\approx) = \frac{d^2\pi^2}{(d\underline{r} + \underline{1})V^{-1}(d\underline{r} + \underline{1})} = \frac{d^2\mu^2}{d^2\underline{Q}V^{-1}\underline{q} + 2d\underline{Q}V^{-1}\underline{1} + \underline{1}V^{-1}\underline{1}}.
$$
\nThis proves the theorem.
\n3. ESTIMATIVE THE PARAMETER \sim OF THE EXPONENTIAL
\n**DISTMAING THE PARAMETER \sim OF THE EXPONENTIAL
\nCOEFFICIENT OF VARIATION OF S' KRINITION
\nthe *CS* C'ATE \sim C'ATE \sim C'ATE \sim C' \sim
\n2. ESTIMATIVE THE \sim C'ATE \sim C'ATE \sim C' \sim
\n2. ESTIMATIVE THE \sim C'ATE \sim C'ATE \sim C' \sim
\n3. ESTIMATIVE THE \sim C'ATE \sim C'ATE \sim C' \sim
\n3. ESTIMATIVE THE \sim C'ATE \sim C'ATE <**

This proves the theorem.

3. ESTIMATING THE PARAMETER OF THE EXPONENTIAL DISTRIBUTION BY ORDER STATISTICS WHEN THE COEFFICIENT OF VARIATION C IS KNOWN

In this section we consider an exponential distribution $E(-,d^2-^2)$ with

$$
z = \frac{(d\underline{r} + \underline{1})'V^{-1}\underline{X}}{(d\underline{r} + \underline{1})'V^{-1}(d\underline{r} + \underline{1})},
$$

\nt is
\n
$$
z = \frac{(d\underline{r}'V^{-1}\underline{X} + \underline{1}V^{-1}\underline{X})}{d^2\underline{r}'V^{-1}\underline{r} + 2d\underline{r}'V^{-1}\underline{1} + \underline{1}V^{-1}\underline{1}}
$$

\n
$$
V(z) = \frac{d^2z^2}{(d\underline{r} + \underline{1})'V^{-1}(d\underline{r} + \underline{1})}
$$

\n
$$
= \frac{d^2\mu^2}{d^2\underline{\alpha}'V^{-1}\underline{\alpha} + 2d\underline{\alpha}'V^{-1}\underline{1} + \underline{1}V^{-1}\underline{1}}.
$$

\ns proves the theorem.
\n**3. ESTIMATING THE PARAMETER - OF THE EXPONENTIAL**
\n**DISTRIBUTION BY ORDER STATISTICS WHERE**
\n**COEFFICIENT OF VARIATION C IS KNOWN**
\nthis section we consider an exponential distribution $E(-d^2z^2)$ with
\n f .
\n
$$
f(x:-,d-)=\frac{1}{d-z}e^{\frac{(x-z)}{d-z}}, \quad -d>0, x \ge -1.
$$
 (15)
\n
$$
f(x) = \frac{1}{d-z}e^{\frac{(x-z)}{d-z}}, \quad -d>0, x \ge -1.
$$

 $\zeta = \frac{\left(d\sum V^{-1}\underline{X} + \underline{1}V^{-1}\underline{X}\right)}{d^2\sum V^{-1}\underline{r} + 2d\underline{r}V^{-1}\underline{1} + \underline{1}V^{-1}\underline{1}}$
 $V(z) = \frac{d^2z^{-2}}{(d\underline{r} + \underline{1})V^{-1}(d\underline{r} + \underline{1})}$
 $= \frac{d^2\mu^2}{d^2\underline{\alpha'}V^{-1}\underline{\alpha} + 2d\underline{\alpha'}V^{-1}\underline{1} + \underline{1}V^{-1}\underline{1}}$

proves the theorem.
 EST $\left(d\underline{r}V^{-1}\underline{X} + iV^{-1}\underline{X}\right)$
 $\frac{d^2}{dx^2}V^{-1}\underline{r} + 2d\underline{r}V^{-1}\underline{1} + iV^{-1}\underline{1}$
 $= \frac{d^2r^2}{(d\underline{r} + 1)V^{-1}(d\underline{r} + 1)}$
 $= \frac{d^2\mu^2}{d^2\underline{\alpha}'V^{-1}\underline{\alpha} + 2d\underline{\alpha}'V^{-1}\underline{1} + iV^{-1}\underline{1}}$

So the theorem.
 EXPLAINATING THE $V^{-1}X + iV^{-1}X$
 $\frac{d^2-2}{(d\Gamma V^{-1} + iV^{-1} + iV^{-1}$
 EXECUTE PARAMETER - OF THE EXPONENTIAL REPORTER TAINS TO and $V(z) = \frac{d^2 z^2}{(d\mathbf{r} + \mathbf{i})\mathbf{V}^{-1}(d\mathbf{r} + \mathbf{i})}$
 $= \frac{d^2 \mathbf{h}^2}{d^2 \mathbf{g}' \mathbf{V}^{-1} \mathbf{g} + 2d \mathbf{g}' \mathbf{V}^{-1} \mathbf{i} + \mathbf{j}' \mathbf{V}^{-1} \mathbf{i}}$.

This proves the theorem.

3. **ESTIMATING THE PARAMETER - OF THE EXPONENTI** in (15) is

$$
c = \frac{s \tan darddeviation}{arithmeticmean} = \frac{d}{1+d}.
$$

s tan *darddeviation* $= \frac{d}{1+d}$.
 $= (X_{1:n}^E, X_{2:n}^E, ..., X_{nn}^E)$ be the vector of order statistics of a

of size *n* drawn from (15). Let $\underline{\Gamma}^E = (\Gamma_{1:n}^E, \Gamma_{2:n}^E, ..., \Gamma_{nn})$ *arithmeticmean* $\frac{d}{dt} = \frac{d}{dt}$
 arithmeticmean $\frac{d}{dt} = \frac{1}{1 + d}$
 arithmeticmean $\frac{d}{dt} = \left(X_{1:n}^E, X_{2:n}^E, ..., X_{n:n}^E\right)$ be the vector of order statistics of a random
 a of size *n* drawn from (15). Let $\sum_{i=$ Let $\underline{X}^E = (X_{1:n}^E, X_{2:n}^E, ..., X_{nn}^E)$ be the vector of order statistics of a ra ating the parameterby order statistics
 $\frac{37}{25}$
 $\frac{1}{25} = \frac{\sin \frac{dr}{dt}}{\sinh \frac{dr}{dt}} = \frac{1}{1 + d}.$
 $\frac{X^E}{2} = \left(X_{En}^E, X_{2:n}^E, ..., X_{nn}^E\right)'$ be the vector of order statistics of a random

le of size *n* drawn from (15) sample of size *n* drawn from (15). Let $\underline{\Gamma}^E = (\Gamma_{1n}^E, \Gamma_{2n}^E, ..., \Gamma_{n}^E)$ and 37

der statistics of a random
 $\underline{\Gamma}^{E} = (\Gamma^{E}_{1:n}, \Gamma^{E}_{2:n}, ..., \Gamma^{E}_{nn})$ and

atrix of the vector of order

m the standard exponential

location parameter of (15),

utting $V = V^{E}$ in (2) and is ting the parameterby order statistics

= $\frac{s \tan \text{darddeviation}}{\text{arithmeticmean}} = \frac{d}{1+d}$.
 $\underline{X}^E = (X_{1n}^E, X_{2n}^E, ..., X_{nn}^E)$ be the vector of order statistic

e of size *n* drawn from (15). Let $\underline{r}^E = (r_{1n}^E, ...)$
 $((v_{r,sn}$ *Estimating the parameterby order statistics* 37
 $c = \frac{s \tan \text{darddeviation}}{\text{arithmeticmean}} = \frac{d}{1+d}$.

Let $\underline{X}^E = (\overline{X}_{Ln}^E, \overline{X}_{2n}^E, ..., \overline{X}_{nn}^E)$ be the vector of order statistics of a random

sample of size *n* drawn from statistics of a random sample of size n drawn from the standard exponential Estimating the parameterby order statistics
 $c = \frac{\text{stan} \text{dard} \text{deviation}}{\text{arithmeticmean}} = \frac{d}{1+d}$.

Let $\underline{X}^E = \left(X_{En}^E, X_{En}^E, ..., X_{En}^E\right)$ be the vector of order statistics of a random

sample of size *n* drawn from (15). L Estimating the parameterby order statistics
 $c = \frac{\text{stan}\,d\text{arideveitation}}{\text{arithmeticmean}} = \frac{d}{1+d}$.

Let $\underline{X}^E = \left(X_{\text{In}}^E, X_{\text{In}}^E, \dots, X_{\text{In}}^E\right)$ be the vector of order statistics of a random

sample of size *n* drawn f given by $\frac{a}{\text{tan}} = \frac{d}{1+d}.$..., X_{nn}^E be the vector of order statistics of a randor

drawn from (15). Let $\underline{r}^E = (\Gamma_{1:n}^E, \Gamma_{2:n}^E, ..., \Gamma_{nn}^E)$ an

mean vector and dispersion matrix of the vector of order

1 sample of size $Y_{2:n}^{E}, ..., X_{nn}^{E}$ be the vector of order statistics of a random
 n drawn from (15). Let $\underline{r}^{E} = (\Gamma_{1:n}^{E}, \Gamma_{2:n}^{E}, ..., \Gamma_{nn}^{E})$ and

the mean vector and dispersion matrix of the vector of order

dom sample of size n dr order statistics
 $\frac{d}{dt}$

be the vector of order statistics of a random

from (15). Let $\Gamma^E = (\Gamma^E_{1:n}, \Gamma^E_{2:n}, ..., \Gamma^E_{nn})$ and

ctor and dispersion matrix of the vector of order

of size n drawn from the standard exponent ating the parameterby order statistics
 $\frac{1}{2} = \frac{\sin \frac{dr}{dt}$
 $\frac{1}{2} = \frac{\sin \frac{dr}{dt}$
 $\frac{1}{2} = \left(X_{1a}^F, X_{2a}^F, \ldots, X_{n,a}^F \right)$ be the vector of order statistics of a random

le of size *n* drawn from (15). Let $E_{m,\text{max}}$
 E_{n} ,..., X_{n}^{E} $\Big)$ be the vector of order statistics of a random
 n drawn from (15). Let $\underline{r}^{E} = (\Gamma_{E_{n}}^{E}, \Gamma_{E_{n}}^{E}, ..., \Gamma_{n}^{E})$ and

the mean vector and dispersion matrix of the vector of order
 K_{2n}^{E} ,..., $X_{n,n}^{E}$ be the vector of order statistics of a random

ze *n* drawn from (15). Let $\underline{\Gamma}^{E} = (\Gamma_{1n}^{E}, \Gamma_{2n}^{E}, ..., \Gamma_{nn}^{E})$ and

be the mean vector and dispersion matrix of the vector of order

andom sa *vm* $\frac{1}{2}$ *v v <i>V x k_m* $\left| \frac{d}{dx} \right|$ *v x k_m* $\left| \frac{d}{dx} \right| \left| \frac{d}{dx} \right| \left| \frac{d}{dx} \right| \left| \frac{d}{dx} \right| \left|$ *veralition* $\frac{1}{2}$ *verality of* $\frac{1}{2}$ *<i>verality of* $\frac{1}{2}$ *<i>verality* $X_{n,j}^{(E)}$ be the vector of order statistics of a random
 n drawn from (15). Let $\frac{1}{2}E = (\Gamma_{n,j}^{E}, \Gamma_{n,j}^{E}, ..., \Gamma_{nn}^{E})$ and

the mean vect *meter**by order statistics*
 deviation $= \frac{d}{1+d}$.
 n tiemean $= \frac{d}{1+d}$.
 n drawn from (15). Let $\mathbb{L}^{E} = (\Gamma_{\text{Ln}}^{E}, \Gamma_{\text{Ln}}^{E}, ..., \Gamma_{\text{Ln}}^{E})$ and
 n drawn from (15). Let $\mathbb{L}^{E} = (\Gamma_{\text{Ln}}^{E}, \Gamma_{\$ *ating the parameterby order statistics*
 $c = \frac{\sin \frac{ar{d}{dt}}{\ar{d}{dt}}$
 $c = \frac{\sin \frac{ar{d}{dt}}{\ar{d}{dt}} = \frac{d}{1+d}$.
 $\underline{X}^E = \left(X_{En}^E, X_{En}^F, \dots, X_{En}^F\right)$ be the vector of order statistics of a random

ble of size *n* drawn f $=\frac{d}{1+d}$.

n) be the vector of order statistics of a random

wn from (15). Let $\underline{r}^E = (\underline{r}_{\ln}^E, \underline{r}_{2n}^E, ..., \underline{r}_{n:n}^E)$ and

vector and dispersion matrix of the vector of order

ple of size n drawn from the standarby order statistics
 $\lim_{m} = \frac{d}{1+d}$.
 $(X_{n,n}^{E})$ be the vector of order statistics of a random

drawn from (15). Let $\underline{r}^{E} = (\underline{r}_{E}^{E}, \underline{r}_{E,n}^{E}, \dots, \underline{r}_{nn}^{E})$ and

nean vector and dispersion matrix of the vec be parameterby order statistics
 $\frac{1}{2}$
 $\frac{1}{2}$

(b) be the mean so parameterby order statistics
 $X_{L,n}^F, X_{L,n}^F, \ldots, X_{n,n}^F$ be the vector of order statistics of a random
 $X_{L,n}^F, X_{L,n}^F, \ldots, X_{n,n}^F$ be the vector of order statistics of a random

size *n* drawn from (15). L on matrix of the vector of c

from the standard expone

us the location parameter of (

by putting $V = V^E$ in (2) ar
 $\left(V^E\right)^{-1} \underline{X}^E$
 $\left(\underline{r}^E\right)^{\dagger} \left(V^E\right)^{-1} \underline{1}^2$
 $\left(\underline{r}^E\right)^{\dagger} \left(V^E\right)^{-1} \underline{1}^2$ diamontal internal in the vector of order statistics of a random

of size *n* drawn from (15). Let $\underline{r}^E = (r_{1n}^E, r_{2n}^E, ..., r_{nn}^E)$ and

of size *n* drawn from (15). Let $\underline{r}^E = (r_{1n}^E, r_{2n}^E, ..., r_{nn}^E)$ and

of si ctor and dispersion matrix of the vector of

of size n drawn from the standard expon

considering ~ as the location parameter of
 $\frac{1}{\left(\frac{\Gamma^E}{\rho}\right)^2 - \left(\frac{\Gamma^E}{\rho}\right)!} \left(\frac{V^E}{\rho}\right)^{-1} \frac{X^E}{\sqrt{\left(\frac{1}{\rho}\left(V^E\right)^{-1}1\right)^2}}$
 the vector of order statistics of a random

om (15). Let $\underline{r}^E = (\Gamma_{\text{Ln}}^E, \Gamma_{\text{2n}}^E, ..., \Gamma_{nn}^E)$ and

and dispersion matrix of the vector of order

size n drawn from the standard exponential

sidering - as the locatio *V V V* $\left(\frac{d}{1+d}\right)$

b e the vector of order statistics of a random

m from (15). Let $\underline{r}^E = \left(r_{\text{lin}}^E, r_{\text{lin}}^E, \dots, r_{\text{min}}^E\right)$ and

vector and dispersion matrix of the vector of order

le of size n drawn from the stand $\frac{\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} x_i} = \left(X_{1:n}^E, X_{2:n}^E, \ldots, X_{nn}^E \right)$ be the vector of order statistics of a random

of size *n* drawn from (15). Let $\underline{r}^E = \left(r_{1:n}^E, r_{2:n}^E, \ldots, r_{nn}^E \right)$ and

of size *n* drawn from (15). *lard deviation* = $\frac{d}{1+d}$.
 whetichean
 E, $\sum_{2n} E_n$, X_{2n}^E , X_{2n}^E , Y_{2n}^E be the vector of order statistics of a random

size *n* drawn from (15). Let $\Gamma^E = (\Gamma_{\text{tan}}^E \cdot \Gamma_{2n}^E, ..., \Gamma_{nn}^E)$ and

be the the tiches and $1 + d$
 $\int_{E_1, X}^{E} \sum_{2,n}^{E} \ldots X_{n,n}^{E}$ be the vector of order statistics of a random

size *n* drawn from (15). Let $\underline{\Gamma}^{E} = (\Gamma_{En}^{E} \cdot \Gamma_{Zev}^{E} \ldots \Gamma_{ex}^{E})$ and

be the mean vector and dispersion matri *Example of size n* drawn from the standard exponent of $f(x:0,1)$. Then by considering \sim as the location parameter of (1) biased estimator of \sim is obtained by putting $V = V^E$ in (2) and $\left[\left(\underline{r}^E \right) \left(V^E \right)^{-1}$ $\binom{E}{r,s,n}$) be the mean vector and dispersion matrix of the vector of order

of a random sample of size n drawn from the standard exponential

on $f(x:0,1)$. Then by considering - as the location parameter of (15),

unb

stics of a random sample of size n drawn from the standard exponential
\nribution
$$
f(x; 0,1)
$$
. Then by considering \sim as the location parameter of (15),
\nnear unbiased estimator of \sim is obtained by putting $V = V^E$ in (2) and is
\n
$$
\hat{\sigma}_E = -\frac{\left(\underline{r}^E\right) \left(V^E\right)^{-1} \left(1 \left(\underline{r}^E\right) - \left(\underline{r}^E\right) \right) \left(V^E\right)^{-1} \underline{X}^E}{\left(\left(\underline{r}^E\right) \left(V^E\right)^{-1} \left(\underline{r}^E\right) \right) \left(1 \left(V^E\right)^{-1} 1\right) - \left(\left(\underline{r}^E\right) \left(V^E\right)^{-1} 1\right)^2}
$$
\n
$$
V(\hat{\sigma}_E) = \frac{\left(\underline{r}^E\right) \left(V^E\right)^{-1} \left(\underline{r}^E\right) \left(1 \left(V^E\right)^{-1} 1\right) - \left(\left(\underline{r}^E\right) \left(V^E\right)^{-1} 1\right)^2}{\left(\left(\underline{r}^E\right) \left(V^E\right)^{-1} \left(\underline{r}^E\right) \right) \left(1 \left(V^E\right)^{-1} 1\right) - \left(\left(\underline{r}^E\right) \left(V^E\right)^{-1} 1\right)^2},
$$
\n
$$
V(\hat{\sigma}_E) = \frac{1}{n} \text{ is a column vector of n ones.}
$$
\nUsing the results of Sarhan (1954), p.322, then (16) and (17) reduces to,
\n
$$
\hat{\sigma}_E = \frac{1}{n-1} \left[nX^E_{1:n} - \overline{X}^E\right]
$$
\n
$$
V(\hat{\sigma}_E) = \frac{d^2 \times^2}{n(n-1)}
$$
\n
$$
V(\hat{\sigma}_E) = \frac{d^2 \times^2}{n(n-1)}
$$
\n
$$
V(\hat{\sigma}_E) = \frac{1}{n(n-1)}
$$
\n
$$
V(\hat{\sigma}_E) = \frac{1}{n(n-1)}
$$
\n
$$
V(\hat{\sigma}_E) = \frac{1}{n(n-1)}
$$
\n
$$
V(\hat{\sigma}_E) = \frac{1}{
$$

and

$$
\hat{\epsilon}_E = -\frac{(\mathbf{r}^E)(\mathbf{v}^E)^{-1}(\mathbf{r}^E)}{((\mathbf{r}^E)^{-1}(\mathbf{r}^E))^2} \tag{16}
$$
\n
$$
V(\hat{\epsilon}_E) = \frac{(\mathbf{r}^E)(\mathbf{v}^E)^{-1}(\mathbf{r}^E)(\mathbf{v}^E)^{-1}(\mathbf{r}^E)^{-1}(\
$$

where 1 is a column vector of n ones.

Using the results of Sarhan (1954), p.322, then (16) and (17) reduces to,

$$
\hat{C}_E = \frac{1}{n-1} \left[nX_{1:n}^E - \overline{X}^E \right] \tag{18}
$$

and

$$
V\left(\hat{z}_E\right) = \frac{d^2 z^2}{n(n-1)}
$$
\n(19)

where \overline{X}^E is the sample mean. Also by considering d as the scale parameter, another linear unbiased estimator \sim_E^* of \sim corresponding to (6) is given by.

$$
\left(\left(\underline{\Gamma}^{E}\right)\left(V^{E}\right) \left(\underline{\Gamma}^{E}\right)\right) \left(1 \left(V^{E}\right) 1\right) - \left(\left(\underline{\Gamma}^{E}\right)\left(V^{E}\right) 1\right)
$$
\n
$$
V\left(\hat{E}_{E}\right) = \frac{\left(\underline{\Gamma}^{E}\right)\left(V^{E}\right)^{-1}\left(\underline{\Gamma}^{E}\right)\left(1 \left(V^{E}\right)^{-1}1\right) - \left(\left(\underline{\Gamma}^{E}\right)\left(V^{E}\right)^{-1}1\right)^{2}}{\left(\left(\underline{\Gamma}^{E}\right)\left(V^{E}\right)^{-1}\left(\underline{\Gamma}^{E}\right)\right)\left(1 \left(V^{E}\right)^{-1}1\right) - \left(\left(\underline{\Gamma}^{E}\right)\left(V^{E}\right)^{-1}1\right)^{2}}, \qquad (17)
$$
\n
$$
\text{re } \frac{1}{\text{ is a column vector of n ones.}
$$
\n
$$
\text{Using the results of Sarhan (1954), p.322, then (16) and (17) reduces to,}
$$
\n
$$
\hat{E}_{E} = \frac{1}{n-1} \left[nX^{E}I_{12} - \overline{X}^{E}\right] \qquad (18)
$$
\n
$$
V\left(\hat{E}_{E}\right) = \frac{d^{2}-2}{n(n-1)} \qquad (19)
$$
\n
$$
\text{re } \overline{X}^{E} \text{ is the sample mean. Also by considering } d - \text{ as the scale parameter,}
$$
\n
$$
\text{her linear unbiased estimator } \hat{E}_{E}^{*} \text{ of } \hat{E} \text{ corresponding to (6) is given by.}
$$
\n
$$
\hat{E}_{E} = \frac{\left(1\right)\left(V^{E}\right)^{-1}\left(1 \left(\underline{\Gamma}^{E}\right)^{-1}\left(\underline{\Gamma}^{E}\right)\right)\left(V^{E}\right)^{-1}\underline{X}^{E}}{d\left[\left(\left(\underline{\Gamma}^{E}\right)\left(V^{E}\right)^{-1}\left(\underline{\Gamma}^{E}\right)\right)\left(1 \left(V^{E}\right)^{-1}1\right) - \left(\left(\underline{\Gamma}^{E}\right)\left(V^{E}\right)^{-1}1\right)^{2}\right]}\right] \qquad (20)
$$

and

$$
N. K. Sajeevkumar and Irsha d M. R.
$$

\n
$$
V\left(-\frac{1}{E}\right) = \frac{\left(1\left(V^{E}\right)^{-1}1\right)^{-2}}{\left(\left(\frac{1}{E}\right)\left(V^{E}\right)^{-1}\left(\frac{1}{E}\right)\right)\left(1\left(V^{E}\right)^{-1}1\right) - \left(\left(\frac{1}{E}\right)\left(V^{E}\right)^{-1}1\right)^{2}}.
$$
\n(21)
\nng the results of Sarhan (1954), p. 322, then (20) and (21) reduces to,
\n
$$
-\frac{1}{E} = \frac{n}{d(n-1)}\left[\overline{X}^{E} - X_{\text{Ln}}^{E}\right]
$$
\n(22)
\n
$$
V(-\frac{1}{E}) = \frac{-2}{(n-1)}
$$
\n(23)
\ng theorem 2.1, the *BLUE* of ~ corresponding to (8) for the exponential
\nibution is given by
\n
$$
z_{E} = \frac{d\left(\frac{1}{E}\right)\left(V^{E}\right)^{-1}X^{E} + 1\left(V^{E}\right)^{-1}X^{E}}{d^{2}\left(\frac{1}{E}\right)\left(V^{E}\right)^{-1}\left(\frac{1}{E}\right) + 2d\left(\frac{1}{E}\right)\left(V^{E}\right)^{-1}1 + 1\left(V^{E}\right)^{-1}1}.
$$
\n(24)
\n
$$
V(z_{E}) = \frac{d^{2}-2}{d^{2}\left(\frac{1}{E}\right)\left(V^{E}\right)^{-1}\left(\frac{1}{E}\right) + 2d\left(\frac{1}{E}\right)\left(V^{E}\right)^{-1}1 + 1\left(V^{E}\right)^{-1}1}.
$$
\n(25)

Using the results of Sarhan (1954), p.322, then (20) and (21) reduces to,

$$
-\frac{1}{E} = \frac{n}{d(n-1)} \left[\overline{X}^E - X_{1:n}^E \right]
$$
 (22)

and

$$
V(\sim_E^*) = \frac{2}{(n-1)}
$$
\n⁽²³⁾

Using theorem 2.1, the $BLUE$ of \sim corresponding to (8) for the exponential distribution is given by

$$
\mathcal{L}\left(\frac{\mathcal{L}^E}{E}\right) = \frac{1}{\left(\left(\frac{\mathcal{L}^E}{E}\right)^2 \left(\frac{V^E}{V}\right)^{-1}\left(\frac{\mathcal{L}^E}{V}\right)\right)} \left(\frac{1}{2}\left(V^E\right)^{-1}1\right) - \left(\left(\frac{\mathcal{L}^E}{E}\right)^2 \left(V^E\right)^{-1}1\right)^2}.
$$
\n
\n*g* the results of Sarhan (1954), p.322, then (20) and (21) reduces to,
\n
$$
\mathcal{L}^* = \frac{n}{d(n-1)} \left[\overline{X}^E - X^E_{\text{Ln}}\right]
$$
\n
$$
V(-\frac{\ast}{E}) = \frac{2}{(n-1)}
$$
\n*g* theorem 2.1, the *BLUE* of ~ corresponding to (8) for the exponential
\nibution is given by\n
$$
\frac{d\left(\frac{\mathcal{L}^E}{V}\right)\left(V^E\right)^{-1} \underline{X}^E + \frac{1}{2}\left(V^E\right)^{-1} \underline{X}^E}{d^2\left(\frac{\mathcal{L}^E}{V}\right)\left(V^E\right)^{-1}\left(\frac{\mathcal{L}^E}{V}\right)^2} = \frac{d^2 - 2}{d^2\left(\frac{\mathcal{L}^E}{V}\right)\left(V^E\right)^{-1}\left(\frac{\mathcal{L}^E}{V}\right)^2} = \frac{d^2 - 2}{d^2\left(\frac{\mathcal{L}^E}{V}\right)\left(V^E\right)^{-1}\left(\frac{\mathcal{L}^E}{V}\right)^2} = \frac{d^2 - 2}{d^2\left(\frac{\mathcal{L}^E}{V}\right)\left(V^E\right)^{-1}\left(\frac{\mathcal{L}^E}{V}\right)^2} = \frac{d^2 - 2}{d^2\left(\frac{\mathcal{L}^E}{V}\right)\left(V^E\right)^{-1}\left(\frac{\mathcal{L}^E}{V}\right)^2} = \frac{d^2 - 2}{d^2\left(\frac{\mathcal{L}^E}{V}\right)\left(V^E\right)^{-1}\left(\frac{\mathcal{L}^E}{V}\right)^{-1}} = \frac{d^2 - 2}{d^2\left(\frac{\mathcal{L}^E}{V}\right)\left(V^E\right)^{-
$$

and

$$
V\left(\tilde{z}_E\right) = \frac{d^2 \tilde{z}^2}{d^2 \left(\underline{\Gamma}^E\right) \left(V^E\right)^{-1} \left(\underline{\Gamma}^E\right) + 2d\left(\underline{\Gamma}^E\right) \left(V^E\right)^{-1} \underline{1} + \underline{1} \left(V^E\right)^{-1} \underline{1}}
$$
(25)

By using the results of Sarhan (1954), p.322, we have found out for the exponential distribution given in (15) ,the following results

and
\n
$$
V(-\frac{1}{E}) = \frac{-2}{(n-1)}
$$
\n(23)
\nUsing theorem 2.1, the *BLUE* of ~ corresponding to (8) for the exponential
\ndistribution is given by
\n
$$
z_{E} = \frac{d(\underline{r}^{E})^{2}}{d^{2}(\underline{r}^{E})^{2}} (\underline{r}^{E})^{2} (\underline{r}^{E})^{
$$

Using the above results, (24) and (25) reduces to,

$$
z_{E} = \frac{n}{2d + d^{2} + n} X_{1:n}^{E} + \frac{d}{2d + d^{2} + n} \overline{X}^{E}
$$
 (26)

$$
= e_1 X_{1:n}^E + e_2 \overline{X}^E,
$$
\t(27)

and \overline{X}^E is the sample mean of a random sample of size *n* taken from (15), and

mating the parameter by order statistics

\n
$$
V(z_{E}) = \frac{d^{2} - 2}{n(2d + d^{2} + n)},
$$
\n(28)

\nFor $d = \frac{c}{n}$, a is the known coefficient of variation. The main solution of

parameterby order statistics
 $\frac{d^2 z^2}{n(2d+d^2+n)}$,
 $\frac{n}{2d+d^2+n}$,
 $\left(\frac{1}{2d} + d^2 + n\right)$,
 $\frac{n}{2d}$, c is the known coefficient of variation. The main
 $\frac{n}{2d}$ is that, one can obtain the *B* electation param *eterby order statistics* 39
 $\frac{l^2 - 2}{l^2 + n}$, (28)

is the known coefficient of variation. The main advantage of

1 (27) and (28) is that, one can obtain the *BLUE* and its

tion parameter \sim of the exponentia where $d = \frac{c}{1}$ $1-c$ and $1-c$ $d = \frac{c}{1-c}$, *c* is the known coefficient of variation , *c* is the known coefficient of variation. The main advantage of the results given in (27) and (28) is that, one can obtain the *BLUE* and its variance of the location parameter \sim of the exponential distribution with known coefficient of variation without knowing the values of means, variances and covariances of the entire order statistics arising from the standard exponential distribution.

4. MOMENT ESTIMATOR OF THE PARAMETER OF THE EXPONENTIAL DISTRIBUTION WHEN d IS KNOWN

In this section we consider an exponential distribution $E(-,d^2-^2)$ with p.d.f. ³⁹

2 (28)
 n. The main advantage of

tain the *BLUE* and its

d distribution with known

of means, variances and

the standard exponential
 ETER ~ **OF THE**
 N d IS KNOWN
 $E(-A^2-2^2)$ with p.d.f.
 \vdots size n dr Estimating the parameterby order statistics
 $V(\tilde{z}_E) = \frac{d^2 - 2}{n(2d + d^2 + n)}$,

where $d = \frac{c}{1 - c}$, c is the known coefficient of variation.

the results given in (27) and (28) is that, one can obtoraince of the given in (15). Let $X_1, X_2, ..., X_n$ are random sample of size n drawn from the *p p p p d* $\frac{d^2 - 2}{n(2d + d^2 + n)}$, (28)
 p $V(\tilde{z}_E) = \frac{d^2 - 2}{n(2d + d^2 + n)}$, (28)

where $d = \frac{c}{1 - c}$, *c* is the known coefficient of variation. The main advantage of

the results given in (27) and (28) is exponential distribution given in (15) and let $X =$ 1 $1\sum_{i=1}^{n} V_i$ be the first new. *n* $=\frac{1}{n}\sum_{i=1}^{n}X_i$ be the first raw *i*=1 (28)

ariation. The main advantage of

can obtain the *BLUE* and its

mential distribution with known

ralues of means, variances and

from the standard exponential
 **XRAMETER ~ OF THE

WHEN d IS KNOWN**

pution $E(-A^2-2^2$ moment of the sample. Equating the first population raw moment and first raw moment of the sample, we get the moment estimator of \sim , namely et $X_1, X_2, ..., X_n$ are random

1 (15). Let \sim be the first

ibution given in (15) and lample. Equating the first po

imple, we get the moment es
 $\frac{2}{\sqrt{1 + d}}$

5 NIIMERICAL ILL **MENT ESTIMATOR OF THE PARAMETER**
MENT ESTIMATOR OF THE PARAMETER
RPONENTIAL DISTRIBUTION WHEN d IS F
n we consider an exponential distribution $E(-a^2)$.

Let $X_1, X_2, ..., X_n$ are random sample of size n

in (15). Let

$$
\sim'' = \frac{\overline{X}}{1+d}
$$

and

$$
V\left(\sim\right') = \frac{d^2\sim^2}{n(1+d)^2}
$$

5. NUMERICAL ILLUSTRATION

Now we have evaluated the coefficients of X^{E} _{*n*} and \overline{X}^{E} of the *BLUE* z_{E} z_{E} given in (27) for $n = 2(1)20$ and for $c = 0.15$ and 0.2, where c is the coefficient of variation and are given in table 5.1. Also we have evaluated $V(\hat{\sigma}_E), V(\hat{\sigma}_E), V(\hat{\sigma}_E), V(\hat{\sigma}_E),$ the relative efficiency $RE_1 = RE(\hat{\sigma}_E / \hat{\sigma}_E)$ of given in (15). Let $X_1, X_2, ..., X_n$ are random sample of size n drawn fi
 $p.d.f.$ given in (15). Let \sim be the first population raw moment

exponential distribution given in (15) and let $\overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$ be the given in (15). Let $X_1, X_2, ..., X_n$ are random sample of size n drawn from the
 $p.d.f.$ given in (15). Let \sim be the first population raw moment of the

exponential distribution given in (15) and let $\overline{X} = \frac{1}{n} \sum_{i=1}^{$ z_{E} relative to z_{E} , the relative efficiency $RE_2 = RE(z_{E} - z_{E})$ of z_{E} relative (15). Let \sim be the first population raw moment of the

ution given in (15) and let $\overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$ be the first raw

pple. Equating the first population raw moment and first raw

pple, we get the moment es to \sim_E^* , the relative efficiency $RE_3 = RE \left(\frac{Z_E}{E} / \frac{E}{E}\right)$ of $\left(\frac{Z_E}{E} \right)$ relative to \sim_E^* , for (b) and let $X = \frac{1}{n} \sum_{i=1}^{n} X_i$ be the first raw
first population raw moment and first raw
ment estimator of \sim , namely
ment estimator of \sim , namely
LILLUSTRATION
ents of $X^{E_{Ln}}$ and \overline{X}^{E} of the *BLUE* $n = 2(1)20$ and $c = 0.15$ and 0.2, and are presented in table 5.2. It may be

noted that in all the cases our estimator z_{E} is much better than that of z_{E} , z_{E}^{*} , *N. K. Sajeevkumar and Irshad M. R.*
 z_{E} is much better than that of \hat{z}_{E} , z_{E}^{*} ,
in the *BLUE*, \tilde{z}_{E} , for different values and \sim_E ["]_E.

		\boldsymbol{n}			$c = 0.15$						$c = 0.2$		
		e ₁		e ₂		e ₁		e ₂					
		2		0.83890		0.07402			0.78049		0.09756		
		3		0.88650			0.05215		0.84211		0.07018		
		$\overline{4}$		0.91239		0.04025			0.87671	0.05479			
		5		0.92866		0.03278			0.89888		0.04494		
		6		0.93984		0.02764			0.91429		0.03810		
		7		0.94799		0.02390			0.92562		0.03306		
	8		0.95419		0.02105			0.93431	0.02920				
	9		0.95907		0.01881			0.94118	0.02614				
	10		0.96301		0.01699			0.94675		0.02367			
	11		0.96626		0.01550			0.95135	0.02162				
	12		0.96899		0.01425			0.95522	0.01990				
	13		0.97130		0.01319			0.95853	0.01843				
	14		0.97330		0.01227			0.96137	0.01717				
		15		0.97503		0.01147			0.96386	0.01606			
		16		0.97656		0.01077			0.96604	0.01509			
		17		0.97791		0.01015			0.96797	0.01423			
		18		0.97911		0.00960			0.96970	0.01347			
		19			0.98019		0.00910		0.97125	0.01278			
		20		0.98116			0.00866		0.97264		0.01216		
									Table 5.2: Variances of the estimators, $\hat{z}_E, \hat{z}_E, \hat{z}_E, \hat{z}_E$ and the relative efficiencies RE_1, RE_2 and RE_3 for $c = 0.15$				
	\boldsymbol{n}	$V(\hat{\zeta}_E)$		$V\left(\mathbb{a}^*_{E}\right)$		$V\left(\sim_E^{\scriptscriptstyle\leftarrow}\right)$	$V(\tilde{z}_E)$		RE_1		RE ₂		RE ₃
	2	0.01557		1.00000	0.01125		0.00653		2.38438		153.13936		1.72282
	3	0.00519		0.50000	0.00750		0.00307		1.69055		162.86645		2.44300
	$\overline{4}$	0.00260		0.33333	0.00563		0.00178		1.46067		187.26404		3.16292
	$\overline{ }$	0.0017		0.2500	0.00150		0.0011		1.21102		015.7170		2.07021

Table 5.1: Coefficients of $X_{1:n}^E$ and \overline{X}^E in the *BLUE*, z_E , for different values of *n* and c .

 \bar{E}_E , \bar{E}_E , \bar{E}_E and the relative efficiencies RE_1, RE_2 and RE_3 for $c = 0.15$

\boldsymbol{n}	$V(\hat{\zeta}_E)$	\sim_E^*	V \sim_E	$V(\tilde{\sigma}_E)$	RE_1	RE ₂	RE ₃
2	0.01557	1.00000	0.01125	0.00653	2.38438	153.13936	1.72282
3	0.00519	0.50000	0.00750	0.00307	1.69055	162.86645	2.44300
$\overline{4}$	0.00260	0.33333	0.00563	0.00178	1.46067	187.26404	3.16292
5	0.00156	0.25000	0.00450	0.00116	1.34483	215.51724	3.87931
6	0.00104	0.20000	0.00375	0.00081	1.28395	246.91358	4.62963
7	0.00074	0.16667	0.00321	0.00060	1.23333	277.78333	5.35000
8	0.00056	0.14286	0.00281	0.00046	1.21739	310.56522	6.10870
9	0.00043	0.12500	0.00250	0.00037	1.16216	337.83784	6.75676
Ω	0.00035	0.11111	0.00225	0.00030	1.16667	370.36667	7.50000
11	0.00028	0.10000	0.00205	0.00025	1.12000	400.00000	8.20000
12	0.00024	0.09091	0.00188	0.00021	1.14286	432.90476	8.95238

REFERENCES

Arnholt, A.T. and Hebert, J.L. (1995): Estimating the mean with known coefficient of variation, *Amer. Statist.*, **49**, 367-369.

Balakrishnan, N. and Rao, C. R. (1998): *Hand Book of Order Statistics: Theory and Methods*, **16**, Elsevier, New York.

Ghosh, M. and Razmpour, A. (1982): Estimating the location parameter of an exponential distribution with known coefficient of variation, *Calcutta Statist. Assoc. Bull.*, **31**, 137-150.

Gleser, L. J. and Healy, J. D. (1976): Estimating the mean of a normal distribution with known coefficient of variation, *J. Amer. Statist. Assoc.*, 7**1**, 977-981.

Guo, H. and Pal, N. (2003): On a normal mean with known coefficient of variation, *Calcutta Statist. Assoc. Bull.*, **54**, 17-29.

Khan, R. A. (1968): A note on estimating the mean of a normal distribution with known coefficient of variation, *J. Amer. Statist. Assoc.*, **63**, 1039-1041.

Kunte, S. (2000): A note on consistent maximum likelihood estimation for N

N. K. Sajeevkum

Gleser, L. J. and Healy, J. D. (1976): Estimating the m

distribution with known coefficient of variation, *J. Amer. S*

777-981.

Guo, H. and Pal, N. (2003): On a normal mean with knovariation, *Calcutt* Sajeevkumar, N. K and Thomas, P. Y. (2005): *Estimating the Mean of Logistic Distribution with Known Coefficient of Variation by Order Statistics*. Recent advances in statistical theory and applications, ISPS proceedings, **1**, 170-176.

Samanta, M. (1984): Estimation of the location parameter of an exponential distribution with known coefficient of variation, *Comm. Statist. Theory Methods*, **31**, 1357-1364.

Sarhan, A. E. (1954): Estimation of the mean and standard deviation by order statistics, *Ann. Math. Stat.*, **25**, 317-328.

Searls, D. T. (1964): The utilization of known coefficient of variation in the estimation procedure, *J. Amer. Statist. Assoc*., **59**, 1225-1226.

Thomas, P. Y. and Sajeevkumar, N. K. (2003): Estimating the mean of normal distribution with known coefficient of variation by order statistics, *J. Kerala Statist. Assoc.,* **14**, 26-32.

Received: 30.10.2011 N. K. Sajeevkumar

Department of Statistics Government college Kariavattom Trivandrum -695 581 Email: sajeevkumarnk@gmail.com

Irshad M. R.

Kerala University Library centre University of Kerala Trivandrum Email: irshadm24@gmail.com