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DESIGNS PARTIALLY BALANCED FOR NEIGHBOR EFFECTS

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ABSTRACT

In this paper, partially neighbor balanced designs are constructed in circular binary blocks for several configurations. Three infinite series of these designs are also developed. Some designs are obtained by hit and trial. A catalogue of proposed designs is also compiled.

1. INTRODUCTION

Neighbor designs introduced by Rees (1967), were initially used in Serology. A neighbor design is a collection of circular blocks in which any two distinct treatments appear as neighbors equally often. Experiments in agriculture, horticulture, forestry and industry often show neighbor effects and neighbor designs are useful to neutralize these effects. Most of the neighbor designs require large number of replications which experimenter cannot afford. In such situations, partially neighbor balanced or generalized neighbor designs should be recommended. A design is called partially neighbor balanced by Wilkinson *et al.* (1983) if each experimental treatment has other treatment as a neighbor, on either side, at most once. To make neighbor designs more useful and flexible, Misra *et al.* (1991) suggested dropping the condition of the constancy of λ' (number of times each pair of treatments appears as neighbors) and proposed generalized neighbor designs. Azais *et al.* (1993) constructed partially neighbor balanced designs in few complete blocks. Chaure and Misra (1996) constructed generalized neighbor designs for different cases. Chan and Eccleston (1998) developed a method to construct a class of partial nearest neighbor balanced designs. Nutan (2007) constructed families of proper generalized neighbor designs. Kedia and Misra (2008), Ahmed *et al.* (2009), Zafaryab *et al*. (2010) and Akhtar *et al*. (2010) constructed generalized neighbor designs for several configurations. Ahmed and Akhtar (2010) introduced some methods to reduce the number of blocks. Ahmed *et al*. (2011) gives the brief review of neighbor, generalized neighbor and partially neighbor balanced designs.

Effect among the adjacent observations, on either side, is called nearest neighbor effect. In this situation, following model is suggested.

$$Y = X_0\mu + X_1\tau + X_2\beta + \varepsilon$$

Where

Y is the $bk \times 1$ vector of response.

X_0 is the $bk \times 1$ vector of 1's.

X_1 is the $bk \times v$ incidence matrix for treatment effects.

X_2 is the $bk \times v$ incidence matrix for nearest neighbor effects.

μ is overall mean effect.

τ is the $v \times 1$ vector of treatment effects.

β is the $v \times 1$ vector of nearest neighbor effects.

ε is the $bk \times 1$ vector of random errors.

Off-diagonal values of $X_2'X_2$ are the number of times a distinct pair of treatments appears as nearest neighbors. If all off-diagonals values of $X_2'X_2$ are same, say λ' times then design is called neighbor balanced. If $\lambda' = 1$, then it is the most economical neighbor balanced design. If all off-diagonals values of $X_2'X_2$ are $\lambda_i'(i = 1, 2, ..., t)$, designs are called generalized neighbor (GN_t) designs. If $\lambda_i'(i = 1, 2)$ then designs are GN_2- designs, where $\lambda_1' \neq \lambda_2' \neq 0$, see Misra et al. (1991).

If all off-diagonals values of $X_2'X_2$ are either 0 or 1, design is called partially neighbor balanced (PNB) design, see Wilkinson et al. (1983). If all values of only one off-diagonal of $X_2'X_2$ are zero up to $[v/2]$, PNB designs relax the neighbor balance property at minimum level, therefore, these designs are closest to the neighbor designs. It happens in a design only if (i) v pairs of treatments do not appear as nearest neighbors for v odd, and (ii) $v/2$ pairs do not appear as nearest neighbors for v even.

In this paper, those PNB designs are constructed which are closest to the neighbor designs. In Section 2, proposed designs are constructed for $v = 2(k+1)$ and $k = 4s$, where s is integer. In Section 3, these designs are constructed for $v = 2(k+1)$ and $k = 11 + 2s^*$, where s^* is even. Proposed designs are constructed in Section 4 for $v = 4m+1$ and $k = m-1$. Section 5 gives the construction of proposed designs for some cases by hit and trial. A catalogue of these designs is presented in Section 6. All this construction is in binary blocks which means no treatment is repeated in the same block.

2. PNB DESIGNS FOR $v = 2(k+1)$ & $k = 4s$

Theorem 2.1. If $v = 2(k+1)$, $k = 4s$, where s is a natural number, and the sum of any two, three, … , $(k-1)$ successive elements of $(1,2,...,k-1)$ is not zero mod v (otherwise, rearrange these elements) then circular binary block *PNBD* can be generated by developing the following initial block cyclically mod v.

$$(0,-1,-3,-6,...,k(k-1)/2) \qquad (M_1)$$

Proof. Combined set of forward and backward differences between neighboring elements takes all the values once from 1 to $v-1$ except $v/2$ which does not appear. Absence of $v/2$ makes them partially neighbor balanced designs. Hence the theorem.
This method provides binary block *PNB* designs which save experimental material at least $[100(v-3)/(v-1)]\%$ at the cost of $[100/(v-1)]\%$ relaxation in neighbor balance property.

Example 2.1. If $v = 10$ and $k = 4$ then *PNB* design generated by developing the initial block $(0,1,3,6)$ cyclically mod 10 is:

B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
0	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	0
3	4	5	6	7	8	9	0	1	2
6	7	8	9	0	1	2	3	4	5

In this design, the pairs (0,5), (1,6), (2,7), (3,8) and (4,9) do not appear as nearest neighbors. Among 45 possible pairs, five pairs do not appear as neighbors, therefore, this design relaxes 11.11% property of neighbor balance but saves 77.78% experimental material.

3. PNB DESIGNS FOR $v = 2(k+1)$ & $k = 11 + 2s^*$

Theorem 3.1. *If $v = 2(k+1)$ and $k = 11 + 2s^*$, where s^* is even and the sum of any two, three,… , ($k-1$) successive elements of (1, 2, …, $k-1$) is not zero mod v after replacing the number $(k-1-s^*/2)$ by $v-(k-1-s^*/2)$ (otherwise, rearrange these resultant elements). Let the resultant elements are $a_1, a_2,...,a_{k-1}$ then following initial block will generate the require design.*

$$(0,\ a_1,\ a_1 + a_2,\ …,\ a_1 + a_2 + ... + a_{k-1}) \text{ mod } v \qquad (M_2)$$

Proof. Combined set of forward and backward differences between neighboring elements takes all the values once from 1 to $v-1$ except $v/2$ which does not appear. Hence the theorem.

This method provides *PNB* designs which save $[100(v-2)/(v-1)]\%$ experimental material at the cost of $[100/(v-1)]\%$ relaxation in neighbor balance property.

Example 3.1. For $v=24$ and $k=11$, replace 10 by 14 in (1,2,3,...,10) and rearrange the resultant elements as (1,2,3,4,5,7,6,8,9,14) to get binary block design. The required *PNB* design will be generated by developing following initial block cyclically mod 24.

$$(0,1,3,6,10,15,22,4,12,21,11)$$

4. PNB DESIGNS FOR $v=4m+1$ and $k=m-1$

Theorem 4.1. Let $v=4m+1$, $k=m-1$ and S be sum of series $\{1,2,...,(m-2)\} \bmod v$. If S does not belong to $\{m-1, m, m+1, m+2\}$ replace one or more numbers of this series by their complements so that the S' (sum of the new series mod v) will be $m-1$, m, $m+1$, or $m+2$ where $(v-a)$ is the complement of a. Let the elements of new series are $a_1, a_2, ..., a_{k-1}$ such that the sum of its any two, three, … , $(k-1)$ successive elements is not zero mod v (otherwise, rearrange it) then following initial block will provide the required *PNB* design.

$$(0,\ a_1,\ a_1+a_2,\ \ldots,\ a_1+a_2+\ldots+a_{k-1}) \bmod v \qquad (M_3)$$

If c is common divisor of k and $v(v-1)/2$, then these designs save at least $[100(v-1-2c)/(v-1)]\%$ experimental material.

Proof. Combined set of forward and backward differences between neighboring elements takes all the values once from 1 to $(v-1)$ except one value which does not appear.

Hence the theorem.

Example 4.1. Consider the numbers (1,2,3) to get binary block *PNB* design for $v=11$ and $k=4$ in 11 blocks. Sum of these numbers is equal to $m+1$, and sum of any two or three numbers is not zero mod 11, therefore, required design can be generated by developing the initial block (0,1,3,6) cyclically mod 11.

Example 4.2. Consider the numbers (1,2,3,4) to get binary block PNB design for $v=13$ and $k=5$ in 13 blocks. Sum of these numbers is 10 which does not belong to $\{m-1, m, m+1, m+2\}$. Replacing 2 by its complement 11, sum becomes 6 mod 13 which is equal to m then required design is obtained by developing the initial block $(0,1,12,2,6)$ cyclically mod 13.

5. PNB DESIGNS BY HIT AND TRIAL

Let q initial blocks are required to generate the proposed design. Proceed as follows to get q initial blocks by trial and error.

Select q sets, each of $k-1$ values from the series $\{1,2,...,v-1\}$ such that:

1. Sum of any two, three, …, or ($k-1$) successive elements of the selected set should not be zero (mod v).
2. Each element of $S*$ must be the distinct, where $S*$ contains:
   (i) Selected values of each set along with their complements, where ($v-a$) is the complement of a,
   (ii) Sum of each set (mod v) along with their complements.

If $a_1, a_2,...,a_{k-1}$ are the selected elements of first set, $b_1, b_2,...,b_{k-1}$ are that of second, …, and $q_1, q_2,...,q_{k-1}$ are the selected elements of the q^{th} set then required design is obtained by developing the following q initial blocks cyclically mod v.

$I_1 = (0,\ a_1,\ a_1 + a_2,\ \dots,\ a_1 + a_2 + ... + a_{k-1})$ mod v,

$I_2 = (0,\ b_1,\ b_1 + b_2,\ \dots,\ b_1 + b_2 + ... + b_{k-1})$ mod v,

$I_q = (0,\ q_1,\ q_1 + q_2,\ \dots,\ q_1 + q_2 + ... + q_{k-1})$ mod v, (M_4)

5.1 *PNB* designs for $v = 2qk + 2$ and $k = 4 + s*$

If $v = 2qk + 2$ and $k = 4 + s*$, where $s*$ is an even number then proposed *PNB* designs can be constructed by developing q initial blocks (using M_4) cyclically mod v. $(M_{4.1})$

Example 5.1.1. For $v = 18$ and $k = 4$, select two sets $(1,2,3)$ and $(5,7,14)$. Then develop the following two initial blocks from these sets to generate required *PNB* design.

$I_1 = (0,1,3,6)$, $I_2 = (0,5,12,8)$

5.2 *PNB* designs for $v=2m+1$ and $k=(m-1)/k$

Let $v=2m+1$ and $k=(m-1)/k$; k and q are integers then proposed *PNB* designs can be constructed by developing q initial blocks (using M_4) cyclically mod v. $(M_{4.2})$

Example 5.2.1. For $v=19$ and $k=4$, select two sets of numbers (1,2,3) and (5,7,15) then develop the following two initial blocks from these sets to generate required *PNB* design.

I_1 = (0,1,3,6), I_2 = (0,5,12,8) .

Example 5.2.2. For $v=27$ and $k=4$, select three sets (1,2,3), (4,5,7) and (9,12,19). Then develop the following three initial blocks from these sets to generate required *PNB* design.

I_1 = (0,1,3,6),, I_2 = (0,4,9,16), I_3 = (0,9,21,13)

6. CATALOGUE OF PNB DESIGNS GENERATED THROUGH THE METHODS DESCRIBED IN ABOVE SECTIONS.

v	k	b	**Initial Blocks**	**Methods**
18	4	36	(0,1,3,6),(0,5,12,8)	$M_{4.1}$
26	4	78	(0,1,3,6),(0,4,9,16),(0,9,20,12)	$M_{4.1}$
34	4	136	(0,1,3,6),(0,4,9,16),(0,8,33,23),(0,22,1,15)	$M_{4.1}$
35	4	140	(0,1,3,6),(0,4,9,16),(0,8,23,13),(0,11,23,14)	$M_{4.2}$
42	4	210	(0,1,3,6),(0,4,9,16),(0,8,17,27),(0,11,41,28), (0,18,37,20)	$M_{4.1}$
43	4	215	(0,1,3,6),(0,4,9,16),(0,8,17,28),(0,12,29,19),(0,14,34, 13) .	$M_{4.2}$
50	4	300	(0,1,3,6),(0,4,9,16),(0,8,17,27),(0,11,23,36),(0,17,35, 20), (0,21,43,24)	$M_{4.1}$
51	4	306	(0,1,3,6),(0,4,9,16),(0,8,17,27),(11,23,36),(0,17,35,2 1), (0,22,45,19)	$M_{4.2}$
58	4	406	(0,1,3,6),(0,4,9,16),(0,8,17,27),(0,11,23,36),(0,15,32, 18), (0,20,43,24),(0,25,51,30)	$M_{4.1}$
59	4	472	(0,1,3,6),(0,4,9,16),(0,8,17,27),(0,11,23,36), (0,22,48,17),(0,14,39,15),(0,14,39,15), (0,19,39,18)	$M_{4.2}$
66	4	528	(0,29,60,32),(0,1,3,6),(0,4,9,16),(0,8,17,27),(0,11,23, 36), (0,14,29,46),(0,19,40,22),(0,24,49,26)	$M_{4.1}$
67	4	536	(0,1,3,6),(0,4,9,16),(0,8,17,27),(0,11,23,36),(0,14,29, 46), (0,19,41,23),(0,24,49,29),(0,30,62,28)	$M_{4.2}$
74	4	666	(0,1,3,6),(0,4,9,16),(0,8,17,27),(0,11,23,36),(0,14,29, 46),(0,19,39,21),(0,23,47,25),(0,29,59,33), (0,32,66,35)	$M_{4.1}$
75	4	675	(0,1,3,6),(0,4,9,16),(0,8,17,27),(0,11,23,36),(0,14,29, 46),(0,24,58,18), (0,30,67,25),(0,28,2,23),(0,56,1,32)	$M_{4.2}$

82	4	820	(0,1,3,6),(0,4,9,16),(0,8,17,27),(0,11,23,36),(0,14,29,47),(0,17,36,56),(0,22,50,29),(0,24,1,40),(0,31,63,38), (0,33,67,37)	$M_{4.1}$
83	4	830	(0,1,3,6),(0,4,9,16),(0,8,17,27),(0,11,23,36),(0,14,29,66),(0,18,37,63),(0,62,1,24),(0,58,3,32)(0,53,1,34),(0,45,1,41)	$M_{4.2}$
90	4	990	(0,1,3,6),(0,4,9,16),(0,8,17,27),(0,11,23,36),(0,14,29,46), (0,18,37,57),(0,21,43,66),(0,26,54,29),(0,31,65,35),(0,37,75,43),(0,40,81,42)	$M_{4.1}$
91	4	1001	(0,1,3,6),(0,4,9,16),(0,8,17,27),(0,11,23,36),(0,14,29,47),(0,17,36,56),(0,21,43,66),(0,24,49,1),(0,28,80,29), (0,30,79,33),(0,31,85,32)	$M_{4.2}$
98	4	1176	(0,1,3,6),(0,4,9,16),(0,8,17,27),(0,11,23,36),(0,14,29,46),(0,18,37,57),(0,21,43,66),(0,25,53,29),(0,30,61,35), (0,34,71,38),(0,40,82,43),(0,45,92,48)	$M_{4.1}$
99	4	1188	(0,1,3,6),(0,4,9,16),(0,8,17,27),(0,11,23,36),(0,14,29,47),(0,17,36,56),(0,21,43,66),(0,24,70,25),(0,73,2,42), (0,70,1,32), (0,39,87,37),(0,64,3,44)	$M_{4.2}$
23	5	46	(0,1,3,6,10),(0,5,11,4,12)	$M_{4.2}$
33	5	99	(0,1,3,6,10),(0,5,11,25,7),(0,8,17,28,16)	$M_{4.2}$
43	5	172	(0,1,3,6,10),(0,5,11,18,26),(0,9,20,39,16),(0,12,25,39,21)	$M_{4.2}$
53	5	265	(0,1,3,7,17),(0,5,11,18,29),(0,18,27,50,22),(0,13,27,48,15), (0,3,15,34,26)	$M_{4.2}$
63	5	378	(0,1,3,6,30),(0,5,11,18,26),(0,9,20,32,45),(0,49,1,17,34), (0,19,39,60,22),(0,10,33,60,32)	$M_{4.2}$
73	5	511	(0,1,3,7,29),(0,3,9,16,24),(0,11,23,10,36),(0,16,35,67,27), (0,17,35,65,23),(0,14,29,63,25), (0,5,14,24,52)	$M_{4.2}$
83	5	664	(0,1,3,6,10),(0,5,11,18,26),(0,9,20,32,45),(0,14,29,45,62),(0,18,57,77,34),(0,19,41,78,31), (0,23,47,80,32),(0,25,52,24,53)	$M_{4.2}$
93	5	837	(0,1,3,6,10),(0,5,11,18,26),(0,11,23,32,45),(0,14,29,45,62),(0,18,38,81,32),(0,19,40,81,30),(0,22,47,86,33), (0,23,47,81,25),(0,27,56,92,55)	$M_{4.2}$
15	6	15	(0,1,3,6,2,7)	M_3
26	6	52	(0,1,3,6,10,15),(0,6,13,21,4,14)	$M_{4.1}$
27	6	54	(0,1,3,6,10,15),(0,6,14,21,3,13)	$M_{4.2}$
39	6	117	(0,1,3,6,10,15),(0,6,13,21,37,12),(0,30,1,29,3,20)	$M_{4.2}$
50	6	200	(0,1,3,6,10,15),(0,6,13,21,30,20),(0,18,4,25,47,24),(0,11,23,36,2,19)	$M_{4.1}$
51	6	204	(0,1,3,6,10,15),(0,6,13,21,30,40),(0,12,25,39,4,21),(0,33,1,32,3,26)	$M_{4.2}$
63	6	315	(0,1,3,6,10,15),(0,6,13,21,30,40),(0,11,23,36,50,17),(0,16,34,53,10,39),(0,21,59,33,60,28)	$M_{4.2}$

74	6	444	(0,11,23,36,50,33),(0,16,34,53,73,21),(0,25,51,4,55,31 (0,6,13,21,30,40),(0,1,3,6,10,15), (0,28,73,43,1,36)	$M_{4.1}$
75	6	450	(0,1,3,6,10,15),(0,6,13,21,30,40),(0,11,23,36,53,14), (0,16,34,53,73,32),(0,25,47,70,19,49)(0,54,6,34,5,38)	$M_{4.2}$
87	6	609	(0,1,3,6,10,15),(0,6,13,21,30,40),(0,11,23,36,50,66), (0,17,35,54,74,29),(0,22,45,69,7,48),(0,60,1,58,2,34) (0,61,7,42,5, 43)	$M_{4.2}$
98	6	784	(0,41,84,40,86,47),(0,1,3,6,10,15),(0,6,13,21,30,40), (0,11,23,36,50,66),(0,17,35,54,74,53),(0,22,45,69,94,26), (0,27,55,84,17,50),(0,36,73,13,77,42)	$M_{4.1}$
99	6	792	(0,1,3,6,10,15),(0,6,13,21,30,40),(0,11,23,36,50,66), (0,17,35,54,74,23),(0,21,43,67,92,37),(0,26,53,81,11,55), (0,31,63,97,33,69),(0,61,1,59,3,50)	$M_{4.2}$
17	7	17	(0,1,3,6,10,15,9)	M_3
31	7	62	(0,1,3,6,10,15,21),(0,8,17,28,9,22,15)	$M_{4.2}$
45	7	135	(1,3,6,10,15,21),(0,7,15,24,34,1,14),(0,29,10,21,36,8,26)	$M_{4.2}$
59	7	236	(0,1,3,6,10,15,21),(0,7,15,24,34,45,14), (0,12,25,40,56,14,32),(0,19,39,17,40,5,30)	$M_{4.2}$
73	7	365	(1,3,6,10,15,21),(0,7,15,24,34,45,57),(0,13,27,42,59,4,39), (0,54,1,23,46,72,32),(0,24,49,22,50,6,36)	$M_{4.2}$
87	7	522	(1,3,6,10,15,21),(0,7,15,24,34,45,57), (0,13,27,42,58,75,29), (0,18,37,57,79,15,39), (0,25,51,85,35,73,28),(0,27,58, 26,59,7,43)	$M_{4.2}$
18	8	18	(0,1,4,6,11,15,3,10)	M_1
19	8	19	(0,1,3,6,10,15,2,9)	M_3
34	8	68	(0,1,3,6,10,5,11,18),(0,8,17,27,4,16,29,15)	$M_{4.1}$
35	8	70	(0,1,3,6,10,15,30,11),(0,6,14,23,34,11, 24,17)	$M_{4.2}$
50	8	150	(0,1,3,6,10,15,21,28),(0,8,17,28,38,1,13,27), (0,15,31,48,16,35,5,26)	$M_{4.1}$
51	8	153	(0,1,3,6,10,15,21,28),(0,8,17,27,38,50,12,26), (0,15,31,48,16,34,3, 24)	$M_{4.2}$
66	8	264	(0,1,3,6,10,15,21,28),(0,8,17,27,38,50,63,14), (0,15,31,49,2,22,1,23),(0,42,17,43,4,33,63,32)	$M_{4.1}$
67	8	268	(0,1,3,6,10,15,21,28),(0,8,17,27,38,50,63,20), (0,14,29,45,62,13,61,25),(0,46,24,47,6,33,62,32)	$M_{4.2}$
82	8	410	(0,1,3,6,10,15,21,28),(0,8,17,27,38,50,63,21), (0,14,29,45,62,80,17,37),(0,22,45,69,12,38,76,33), (0,30,61,11,45,10,65,36)	$M_{4.1}$
83	8	415	(0,1,3,6,10,15,21,28),(0,8,17,27,38,50,63,39), (0,14,29,46,64,1,20,41),(0,22,45,20,47,73,6,46), (0,54,1,53,2,52,3,38)	$M_{4.2}$
98	8	588	(0,32,65,1,36,72,11,50),(0,40,2,59,4,48,93,47), (0,1,3,6,10,15,21,28),(0,8,17,27,38,50,63,77), (0,15,31,48,66,85,7,29),(24,49,75,4,34,65,42)	$M_{4.1}$

99	8	594	(0,1,3,6,10,15,21,28),(0,8,17,27,38,50,63,77), (0,15,31,48,66,85,6,27),(0,23,47,72,98,45,92,41), (0,29,59,90,23,56,2,36),(0,64,27,65,5,47,7,50)	$M_{4.2}$
21	9	21	(0,1,20,2,6,12,17,3,11)	M_3
39	9	78	(0,1,3,6,11,15,24,36,10),(0,6,13,21,32,7,22,38,18)	$M_{4.2}$
57	9	171	(0,1,3,6,10,15,21,28,36),(0,26,35,45,56,11,24,49,19), (0,14,56,15, 32,50,30,52,18)	$M_{4.2}$
75	9	300	(0,1,3,6,10,15,21,28,36),(0,9,19,30,42,55,69,10,25), (0,17,35,54,74,20,42,66,26),(0,23,50,3,32,62,18,51,1 0)	$M_{4.2}$
93	9	465	(0,1,3,6,10,15,21,28,36),(0,9,19,30,42,55,69,84,21), (0,16,33,51,70,90,19,42,66), (0,25,51,79,15,46,78,18,52), (0,35,91, 36,75,22,63,13,64)	$M_{4.2}$
23	10	23	(0,1,3,6,11,15,9,16,2,10)	M_3
42	10	84	(0,1,3,6,10,15,21,28,36,27),(0,10,21,33,5,34,8,25,1,2 0)	$M_{4.1}$
43	10	86	(0,1,3,6,10,15,21,28,36,13),(0,9,19,30,1,16,32,7,33,2 1)	$M_{4.2}$
63	10	189	(0,1,3,6,10,15,21,28,36,45), (0,10,21,33,46,60,12,28,57,24),(0,17,36,56,35,57,19, 42,5,32)	$M_{4.2}$
82	10	328	(0,1,3,6,10,15,21,28,36,45),(0,28,57,5,36,68,19,53,6, 42),(0,10,21,33,46,60,75,9,26,44),(0,19,39,60,1,23,4 7,72,16,43)	$M_{4.1}$
83	10	332	(0,1,3,6,10,15,21,28,36,45),(0,10,21,33,46,60,75,8,2 5,43)(0,19,39,60,82,22,46,71,14,41),(0,28,57,4,35,67 ,17,51,3,39)	$M_{4.2}$
25	11	25	(0,1,3,6,10,15,21,4,11,2,12)	M_3
47	11	94	(0,1,3,6,11,15,21,28,36,45,10), (0,11,24,38,6,22,39,10,29,2,23)	$M_{4.2}$
69	11	207	(0,1,3,6,10,15,21,28,36,45,55), (0,11,23,36,51,67,15,33,52,3,24), (0,47,1,26,52,10,51,11,41,3,35)	$M_{4.2}$
91	11	364	(0,1,3,6,10,15,21,28,36,45,55), (0,11,23,36,50,65,81,7,25,44,64), (0,21,43,66,90,24,50,1,42,85,34), (0,68,39,69,9,41,74,20,76,38,77)	$M_{4.2}$
26	12	26	(0,11,21,24,25,1,5,10,16,23,6,14)	M_1
27	12	27	(0,1,4,6,10,15,22,3,9,18,2,12)	M_3
50	12	100	(0,1,3,6,10,15,21,28,36,45,5,16), (0,13,28,42,10,27,46,16,37,9,36, 24)	$M_{4.1}$
51	12	102	(0,1,3,6,10,16,21,28,36,45,4,15), (0,13, 29,43,9,27,46,15,36,8,37,25)	$M_{4.2}$
74	12	222	(0,1,3,6,10,15,21,28,36,45,55,17), (0,23,47,72,46,73,27,56,12,43,1,35), (0,12,23,36,50,65,7,25,44,64,11,33)	$M_{4.1}$

75	12	225	(0,1,3,6,10,15,21,28,36,45,55,16), (0,11,23,36,50,65,7,25,44,64,10,32), (0,23,74,24,50,2,30,59,14,45,3,37)	$M_{4.2}$
98	12	392	(0,1,3,6,10,15,21,28,36,45,55,66), (0,12,25,39,54,70,87,7,26,46,67,23), (0,22,47,71,97,26,54,83,15,46,79,36), (64,29,66,28,67,9,50,8,53,1,48)	$M_{4.1}$
99	12	396	(0,1,3,6,10,15,21,28,36,45,55,66), (0,12,25,39,54,70,87,6,26,45,92,41), (0,21,43,66,90,16,42,69,97,44,88,34), (0,29,59,90,58,93,30,67,6,45,85,43)	$M_{4.2}$
29	13	29	(0,1,3,7,13,10,15,22,4,12,21,2,14)	M_3
55	13	110	(0,1,3,6,10,15,21,28,36,45,2,13,23), (0,14,29,42,4,22,41,6,28,49,18,43,27)	$M_{4.2}$
81	13	243	(0,1,3,6,10,15,21,28,36,45,55,66,24), (0,12,25,39,54,70,6,24,43,63,3,26,48), (0,56,30,57,4,33,63,13,45,79,43,78,41)	$M_{4.2}$
31	14	31	(0,1,3,6,10,16,21,14,22,2,11,23,5,15)	M_3
58	14	116	(0,1,3,6,10,15,21,28,36,45,55,8,20,33), (0,14,29,45,5,22,41,3,24,46,11,35,4,30)	$M_{4.1}$
59	14	118	(0,1,3,6,10,15,21,28,36,45,55,7,19,32), (0,14,29,45,3,21,40,1,22,44,8,32,57,26)	$M_{4.2}$
87	14	261	(0,1,3,6,10,15,21,28,36,45,55,66,78,13), (0,41,55,70,86,16,34,53,73,7,30,54,79,32), (0,61,1,29,58,2,32,65,31,66,15,52,3,42)	$M_{4.2}$
32	15	32	(0,19,1,2,4,7,11,16,22,29,5,14,24,3,15)	M_2
33	15	33	(0,2,5,6,12,16,23,28,3,13,22,10,21,1, 15)	M_3
63	15	126	(0,1,4,6,10,17,22,28,36,45,55,3,15,29,42), (0,15,31,48,3,22,2,24,47,8,33,59,25,53,17)	$M_{4.2}$
93	15	279	(0,1,3,6,10,15,21,28,36,45,55,66,78,91,14), (0,15,32,50,69,89,17,39,62,86,18,44,71,6,35), (0,63,32,64,4,38,74,37,75,21,61,9,51,1,45)	$M_{4.2}$
34	16	34	(0,1,3,6,10,15,21,28,2,11,22,32,12,24,5,18)	M_1
35	16	35	(0,20,19,21,24,28,33,4,12,22,29,3,14, 26,5,18)	M_3
66	16	132	(0,1,3,6,10,15,21,28,36,45,57,2,12,25,56,22), (0,15,31,48,1,19,39,60,46,3,27,52,12,41,2,30)	$M_{4.1}$
67	16	134	(0,1,3,6,10,15,21,28,36,45,55,66,11,24,54,18), (0,14,29,45,64,17,38,60,16,40,65,24, 51,12,50,33)	$M_{4.2}$
98	16	294	(0,1,3,6,10,15,21,28,36,45,55,66,78,91,7,22), (0,66,1,35,70,8,45,84,26,67,11,54,9,53,2,48), (0,16,33,51,70,90,13,36,61,85,14,40,68,97,29,60)	$M_{4.1}$
99	16	297	(0,14,15,17,20,24,29,35,42,50,59,69,80,92,6,21), (0,16,33,51,70,90,13,36,60,85,12,39,67,96,27,58), (0,32,98,33,68,5,42,80,20,79,22,65,10,55,2,49)	$M_{4.2}$
37	17	37	(0,1,3,6,10,15,21,28,36,8,18,29,4,17,31,9,30)	M_3
71	17	142	(0,1,3,6,10,15,21,28,36,45,55,66,7,20,34,63,22), (0,15,31,48,66,14,34,55,7,32,56,11,38,10,41,2,35)	$M_{4.2}$

39	18	39	(0,16,8,9,11,14,18,23,29,36,6,19,31,2, 13, 28,3,20)	M_3
74	18	148	(0,1,3,6,10,15,21,28,36,45,55,66,4,17,31,46,62,18), (0,17,71,16,37,59,8,32,57,9,38,65,19,50,10,42,1,36)	$M_{4.1}$
75	18	150	(0,1,3,6,10,15,21,28,36,45,55,66,4,16,30,46,61,17), (0,33,51,70,50,71,18,41,65,15,43,69,21,53,8,37,2,36)	$M_{4.2}$
40	19	40	(0,1,3,6,10,15,21,28,36,5,16,26,38,11,25,9,24,2,19)	M_2
41	19	41	(0,27,28,30,33,37,1,7,14,22,31,2,12,23,36,10,26,3, 20)	M_3
79	19	158	(0,1,3,6,10,15,21,28,36,45,55,66,78,12,26,41,57,74,18),(0,60,1,22,44,68,14,40,67,16,45,75,27,59,13,47,3, 39,76)	$M_{4.2}$
42	20	42	(0,18,20,23,27,32,38,3,11,21,30,41,12,24,39,13,31,8, 22)	M_1
43	20	43	(0,24,25,27,30,34,39,2,9,17,26,36,4,16,29,1,15,31,5,23	M_3
82	20	164	(0,1,3,6,10,15,21,28,36,45,55,66,78,9,23,38,54,71,7, 26), (0,20,41,63,4,28,56,81,54,1,53,3,34,67,19,55,8,45,2, 40)	$M_{4.1}$
83	20	166	(0,1,3,6,10,15,21,28,36,45,55,66,78,8,22,37,53,70,5, 24),(0,39,19,40,62,2,27,53,80,25,54,1,32,64,14,48,13 ,49,3, 41)	$M_{4.2}$
45	21	45	(0,1,3,6,2,8,13,20,28,37,4,15,29,39,7,23,38,12,32,5,22)	M_3
87	21	174	(0,20,21,23,26,30,35,41,48,56,65,75,86,11,24,38,53, 69,1,19,36),(0,41,62,84,20,44,19,45,72,13,42,73,16,48,81,28, 63,14,51,3,43)	$M_{4.2}$
47	22	47	(0,1,36,38,41,46,3,9,16,24,33,43,7,20,34,2,18,35,6,25,45, 19)	M_3
90	22	180	(0,2,3,6,11,17,21,28,36,45,55,66,78,1,15,30,46,63,81 ,10,31,51),(0,23,45,69,5,30,57,86,24,54,85,27,60,4,39,75,22, 62,10,51,3,46)	$M_{4.1}$
91	22	182	(0,21,41,42,44,47,51,56,62,69,77,86,5,16,28,43,57,70,87,2,30,49),(0,50,2,24,47,71,5,31,58,86,25,54,85,26,59,4, 39,73,19,57,6,45)	$M_{4.2}$
48	23	48	(0,1,3,6,10,15,21,28,36,45,7,18,30,43,9,24,40,13,33, 2,20, 42,23)	M_2
49	23	49	(0,3,5,9,10,15,21,28,36,45,6,17,29,42,8,22,38,7,26,46,18, 40,23)	M_3
95	23	190	(0,1,3,6,10,15,21,28,36,45,55,66,78,91,11,25,41,58,76,2,2,44,63),(0,72,1,26,52,79,12,41,71,7,40,74,14,50, 87,30, 69,15,55,2,45,90,46)	$M_{4.2}$
50	24	50	(0,22,36,9,10,12,15,19,24,30,37,45,4,14,25,38,5,21,33, 48,17,35,6, 26)	M_1
98	24	196	(0,1,3,6,10,15,21,28,36,45,56,66,78,91,7,22,38,55,73 ,92,14,35,82,32),(0,22,45,70,96,72,1,29,58,89,21,54, 88,25, 61,2,39,77,19,60,4, 47,91,46)	$M_{4.1}$

99	24	198	(0,1,3,6,10,15,21,28,36,45,55,66,78,91,7,23,37,54,72,92,2,33,80,29),(0,77,1,25,50,76,4,32,62,93,26,59,94,29,65, 3,41,80,21,63,5,61,6,51)	$M_{4.2}$
56	27	56	(0,1,3,6,10,15,21,28,36,45,55,12,24,35,49,9,26,41,5,23, 42,7,30,54,32,2,27)	M_2
58	28	58	(0,26,53,54,56,1,5,10,16,23,31,40,50,3,15,28,42,57,17, 34,55,13,33,52,18,41,8,30)	M_1
64	31	64	(0,1,3,6,10,15,21,28,36,45,55,2,14,27,41,56,8,25,43,62, 18,39,61, 20,44,19,47, 9,38,4,31)	M_2
66	32	66	(0,30,58,59,61,64,2,7,13,20,28,37,47,60,6,17,31,46,62, 15,33,50,4, 25,49,5,32,55, 14,40,3,34)	M_1
72	35	72	(0,1,3,6,10,15,21,28,36,45,55,66,7,19,33,48,64,9,27,46, 67,17,37, 60,12,38,63, 18,47,5,49,8,40,2,35)	M_2
74	36	74	(0,32,65,66,68,71,1,6,12,19,27,36,46,57,69,8,22,37,53, 70,14,33,54,4,24,47,73,21, 48,2,31,56,13,43,3,38)	M_1
80	39	80	(0,1,3,7,10,15,21,28,37,45,55,67,78,12,25,40,56,73,11,30,50,71,13,36,60,5,31,58,6,35,65,34,66,19,53,8,44,2,39)	M_2
82	40	82	(0,1,6,12,14,17,21,28,37,45,58,68,80,13,24,38,57,73,8,26,46,67,7,30,54,79,23,50,78,25,55,4,36,69,22,56,10,47,3, 42)	M_1
88	43	88	(0,4,5,7,10,15,21,28,37,45,58,68,80,3,17,32,48,65,83,14,4,55,77,12,36,61,87,26,54,84,25,56,1,33,69,35,70,19, 57,8,50,2,43)	M_2
90	44	90	(0,2,5,12,13,19,23,28,37,45,56,66,79,1,16,32,50,64,83,10,31,51,74,6,30,57,82,18,48,76,17,49,78,22,58,3,36,73,21, 60,11,54,4,46)	M_1
96	47	96	(0,1,3,6,10,15,21,28,36,45,55,66,78,91,9,24,40,57,75,94,18,39,61,84,12,37,63,90,22,51,81,16,48,82,19,54,17,53, 93,35,74,20,64,11, 52,2,47)	M_2
98	48	98	(0,1,5,12,18,20,23,28,37,45,59,69,81,96,9,22,39,58,74,92,15,35,60,82,8,31,57,84,14,43,73,6,38,71,7,42,78,17, 55,94,36,77,21,64,10,56,3,50)	M_1

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