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USE OF SOME FUNCTIONS OF SPACINGS IN THE ESTIMATION OF COMMON SCALE PARAMETER OF NORMAL AND DOUBLE EXPONENTIAL DISTRIBUTIONS

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ABSTRACT

In this paper we consider the problem of best linear unbiased estimation and best linear invariant estimation of the common scale parameter of a normal and double exponential distributions using some functions of spacings of all observations of individual samples. We have also proved a sufficient condition for the non-negativity of the common scale estimator obtained by the above method. Further we have obtained necessary and sufficient condition for the derived estimators to be constant multiple of the sum of first and last spacings of pooled sample.

1. INTRODUCTION

It is well-known that order statistics are very useful in the estimation of location and scale parameters of a distribution. For a survey of literature on the application of order statistics in estimating the location and scale parameters of distributions, see, Balakrishnan and Cohen (1991) and David and Nagaraja (2003).

For a discussion on the problem of estimation of common location and common scale parameters of several distributions using order statistics of the pooled sample is given in Sajeevkumar and Thomas (2005). Arnold et al. (1992, p.174), observed that the existence and uniqueness of the BLUE for the scale parameter of a distribution do not guarantee that it is always non-negative. Balakrishnan and Papadatos (2002) have given a simpler formula for the *BLUE* of the scale parameter of a distribution using spacings. Further they have proved a sufficient condition for their estimator to be positive. Sajeev kumar and Thomas (2010) have extended the method proposed by Balakrishnan and Papadatos (2002) to estimate the common scale parameter σ of several distributions by spacings of order statistics of inid random variables.

2. MOTIVATION FOR ESTIMATING THE COMMON SCALE PARAMETER OF SEVERAL SYMMETRIC DISTRIBUTIONS BY USING SOME FUNCTIONS OF SPACINGS

Let $X_{i1}, X_{i2}, ..., X_{in_i}$ be n_i independent observations drawn from a population with distribution function denoted by $F_i\left(\frac{x_i - \mu}{\sigma}\right)$, $i = 1,2,...,k$, $-\infty < \mu < \infty$, $\sigma > 0$. Let $X_{1:n}, X_{2:n}, ..., X_{n:n}$ be the corresponding order statistics of the pooled sample of all $n = n_1 + ... + n_k$ observations. Now if we write $Y_{r:n} = \left(\frac{X_{r:n} - \mu}{\sigma}\right)$, $r = 1,2,...,n$, then $(Y_{1:n}, Y_{2:n}, ..., Y_{n:n})$ is distributed identically same as the order statistics of the pooled sample of k independent random samples of which n_i are drawn from the distribution with distribution function $F_i(y)$, $i = 1,2,...,k$. Then the distribution theory of $(Y_{1:n}, Y_{2:n}, ..., Y_{n:n})$ can be dealt with using the result of Vaughan and Venables (1972).

Now we consider the case when the basic distributions are all symmetrically distributed about the common location parameter μ, Then from Bepat and Beg (1989) the distributions of $\underline{Y} = (Y_{1:n}, Y_{2:n}, ..., Y_{n:n})$ and $(-Y_{n:n}, -Y_{n-1:n}, ..., -Y_{1:n})$ are identical.

Now define $W_j = X_{j+1:n} - X_{j:n}$, and $W_j^* = Y_{j+1:n} - Y_{j:n}$, $j = 1,2,...,n-1$, $R_i = W_i + W_{n-i}$ and $R_i^* = W_i^* + W_{n-i}^*$, $i = 1,2,...,[n/2]$, where [.] is the usual greatest integer function. Clearly $W_j^* \stackrel{d}{=} W_{n-j}^*$, $j = 1,2,...,n-1$.

Thus if we write $\underline{W}^* = (W_1^*, W_2^*, ..., W_{n-1}^*)'$ then $\underline{W}^* \stackrel{d}{=} J\underline{W}^*$, where

$$J = \begin{bmatrix} 0 & 0 & . & . & . & 0 & 1 \\ 0 & 0 & . & . & . & 1 & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 1 & 0 & . & . & . & 0 & 0 \end{bmatrix}$$

is a square matrix of order n-1.

Let the expectation of the vector $\underline{W}^* = (W_1^*, W_2^*, ..., W_{n-1}^*)'$ and its variance-covariance matrix $D(\underline{W}^*)$ be given by

$$E(\underline{W}^*) = \underline{\alpha}, \tag{2.1}$$

(where E(W_i^*) = α_i , $j = 1,2,...,n-1$ and

$$D(\underline{W}^*) = B \tag{2.2}$$

Also

$$S = E\left(\underline{W}^* \underline{W}^{*'}\right) \tag{2.3}$$

Clearly B and S are square matrices of order $n-1$. Also the elements of $\underline{\alpha}$, B and S are free of μ and σ. Clearly

$$E(J\underline{W}^*) = J\underline{\alpha} \tag{2.4}$$

And hence we have

$$\underline{\alpha} = \text{J } \underline{\alpha} \ , \tag{2.5}$$

Also $D(J\underline{W}^*) = JBJ$ and hence

$$B = JBJ \tag{2.6}$$

Left multiplication of B by J changes the $i-th$ row into $(n-i)-th$ row and right multiplication of B by J changes the $j-th$ column into $(n-j)-th$ column. Then from (2.6) it is seen that if the $j-th$ column vector of B is $(c_1, c_2, ..., c_{n-1})'$, then its $(n-j)-th$ column vector is $(c_{n-1}, c_{n-2}, ..., c_1)'$. Similarly if the $i-th$ row of B is $(r_1, r_2, ..., r_{n-1})$ then its $(n-i)-th$ row is $(r_{n-1}, r_{n-2}, ..., r_1)$.

Also from the identity $J = J^{-1} = J'$ we get

$$B^{-1} = JB^{-1}J \tag{2.7}$$

Thus we see that the property described for B is true for B^{-1} also.

Now the best linear unbiased estimator for the common scale parameter σ using spacings of order statistics of pooled sample of all observations are given by (see Sajeevkumar and Thomas 2010)

$$L_U = (\underline{\alpha}' B^{-1} \underline{\alpha})^{-1} \underline{\alpha}' B^{-1} \underline{W} \tag{2.8}$$

And

$$\text{Var}(L_U) = \left(\underline{\alpha}' B^{-1} \underline{\alpha}\right)^{-1} \sigma^2 , \tag{2.9}$$

where $\underline{W} = (W_1, W_2, ..., W_{n-1})$.

Also the best linear invariant estimator for the common scale parameter σ using spacings of order statistics of the pooled sample of all observations and corresponding MSE are given by (see, Sajeevkumar and Thomas 2010)

$$L_I = \underline{\alpha}' S^{-1} \underline{W} \tag{2.10}$$

and corresponding MSE is given by

$$MSE(L_I) = \left(1 - \underline{\alpha}' S^{-1} \underline{\alpha}\right)\sigma^2 \tag{2.11}$$

The connection between $BLUE$ and $BLIE$ of σ is given by (See,Sajeevkumar and Thomas 2010)

$$L_I = aL_U , \tag{2.12}$$

Where a is a constant $(0 < a < 1)$ and is given by $a = \underline{\alpha}' S^{-1} \underline{\alpha}$.

Clearly the form of the vector $\underline{\alpha}$ is $\underline{\alpha}' = \left(\alpha_1, \alpha_2, ..., \alpha_{[n/2]}, \alpha_{[n/2]-1}, ..., \alpha_1\right)$ for n even and $\underline{\alpha}' = \left(\alpha_1, \alpha_2, ..., \alpha_{\left[n/2\right]}, \alpha_{\left[n/2\right]}, ..., \alpha_1\right)$ for n odd , where $[.]$ is the usual greatest integer function. Using this property of $\underline{\alpha}$ we see that the coefficients of W_j and W_{n-j} in the estimator L_U defined in (2.8) are identically equal. This property of L_U proves that $BLUE$ of σ reduce to a linear function of spacings namely $R_i = W_i + W_{n-i}, i = 1,2,...,[n/2]$. Using (2.12) clearly $BLIE$ of σ also reduce to a linear function of spacings of pooled sample namely $R_i = W_i + W_{n-i}$, $i = 1,2,...,[n/2]$. Thus there is some advantage in using $R_i, i = 1,2,...,[n/2]$ in estimating σ by an estimate which involves a covariance matrix of order $[n/2]$ only where as in (2.8) an equivalent estimate involves a covariance matrix of order n-1. In section 3 we derived an estimator of σ and its variance based on $R_i, i = 1,2,...,[n/2]$.

3. BLUE AND BLIE FOR THE SCALE COMMON PARAMETER USING SOME FUNCTIONS OF SPACINGS

In section 2 we have proved that if the parent distributions are symmetric about the common location parameter μ then the $BLUE$ of common scale parameter σ reduces to some function of spacings. Hence there exit constants c_i^* $i = 1,2,...,[n/2]$, such that

$$L = \sum_{i=1}^{[n/2]} c_i^* R_i \overset{d}{=} \sigma \sum_{i=1}^{[n/21} c_i^* R_i^* \,, \tag{3.1}$$

Let $\underline{R}^* = \left(R_1^*, R_2^*, ..., R_{[n/2]}^*\right)'$ be the random vectors of functions of spacings of the pooled sample from the completely known $F_0(y)$ and use the notation

$$M = E(\underline{R}^*)\,, H = D(\underline{R}^*)\,, Q = E(\underline{R}^* \underline{R}^{*'})\,, \tag{3.2}$$

where $D(\xi)$)denotes the dispersion matrix of the random vector ξ . Clearly

$$H = Q - \underline{M}\,\underline{M}', \qquad H > 0,\ Q > 0 \tag{3.3}$$

Now we have the following

Proposition 3.1: Under the above assumptions and the notation of this section, the BLUE of the common scale parameter σ and its variance are given by

$$L_u = \frac{\underline{M}' H^{-1}}{\underline{M}' H^{-1} \underline{M}} \underline{R} \quad \text{with} \quad Var(L_u) = \frac{\sigma^2}{\underline{M}' H^{-1} \underline{M}} \tag{3.4}$$

where $\underline{R} = \left(R_1, R_2, ..., R_{[n/2]}\right)'$.

Proof: Since the form of the most general location-invariant estimator of σ is $L = \underline{c}'\underline{R}$, where $\underline{c} = \left(c_1, c_2, ..., c_{[n/2]}\right)'$, it follows that it is unbiased for σ if and only if

$$\underline{c}'\underline{M} = 1 \tag{3.5}$$

On the other hand its variance is given by

$$\text{Var(L)= }(\underline{c}' \text{H} \underline{c})\sigma^2 . \tag{3.6}$$

Thus we wish to minimize (3.6) under restriction (3.5).Taking into the account the Lagrangian ($Q(\underline{c}:\lambda)$) $= \underline{c}' H \underline{c} - 2\lambda(\underline{c}'\underline{M})$,it is easy to seen that the optimal value is $\underline{c} = \lambda\left(H^{-1}\underline{M}\right)$ and the restriction yields $\lambda = \frac{1}{\underline{M}' H^{-1} \underline{M}}$; this completes the proof.

Now the BLIE of the common scale parameter using function of spacings is given by the following theorem.

Proposition 3.2: Under the above assumptions and the notation of this section, the BLIE of the common scale parameter σ and its MSE are given by,

$$L_I = \underline{M}'Q^{-1}\underline{R} \quad \text{with } MSE(L_1) = (1 - \underline{M}'Q^{-1}\underline{M})\sigma^2. \tag{3.7}$$

The proof follows by the same arguments as in proposition 3.1,except that we do not have to use restriction(3.5).

The connection between $BLUE$ and $BLIE$ of the common scale parameter σ is given in the following proposition.

Proposition 3.3. There exist a constant a, $0 < a < 1$, independent of μ and σ such that

$$L_I = aL_u \tag{3.8}$$

This constant is given by

$$a = \underline{M}'Q^{-1}\underline{M} = \frac{\underline{M}'Q^{-1}\underline{M}}{1 + \underline{M}'Q^{-1}\underline{M}}. \tag{3.9}$$

Proof: First note that $Q = H + \underline{M}\,\underline{M}'$ (See equation 3.3).Thus , by theorem 8.9.3 in Graybill(1969) implies that $Q^{-1} = H^{-1} - \frac{1}{1 + \underline{M}'H^{-1}\underline{M}}\left(H^{-1}\underline{M}\right)\left(H^{-1}\underline{M}\right)'$ and thus $\underline{M}'Q^{-1}\underline{M} = \underline{M}'H^{-1}\underline{M} - \frac{\left(\underline{M}'H^{-1}\underline{M}\right)^2}{1 + \underline{M}'H^{-1}\underline{M}}$, proving the second equality in (3.9).Consider now the class of estimators of the form $L_\lambda = \lambda L_u, \lambda \in R$ .It is easy to see that they are linear invariant estimators and ,moreover, that $MSE(L_\lambda) = E(\lambda L_u - \sigma)^2 = \left(\frac{\lambda^2}{\underline{M}'H^{-1}\underline{M}} + (1-\lambda)^2\right)\sigma^2$. Therefore ,minimizing the last expression with respect to λ ,we get $\lambda = \frac{\underline{M}'H^{-1}\underline{M}}{1 + \underline{M}'H^{-1}\underline{M}} = a \in (0,1)$. For this value of $\lambda = a$ it follows that

$$MSE(L_a) = \frac{\sigma^2}{1 + \underline{M}'H^{-1}\underline{M}} = \left(1 - \underline{M}'Q^{-1}\underline{M}\right)\sigma^2 = MSE(L_I).$$ Hence the proof.

4. CONDITION FOR WHICH BLUE (OR BLIE) OF THE SCALE PARAMETER σ IS A MULTIPLE OF $R_1 = W_1 + W_{n-1}$ AND NON-NEGATIVE

In this section, we obtain, necessary and sufficient conditions under which the $BLUE$ (or, equivalently, the $BLIE$) of the common scale parameter σ is a constant multiple of $R_1 = W_1 + W_{n-1} = X_{2:n} - X_{1:n} + X_{n:n} - X_{n-1:n}$,(Clearly when $n=2$ then R_1 reduces to twice sample range and when $n=3$ then R_1 reduces to sample range), have also proved the non-negativity for the scale estimator L_u of σ .In particular we shall prove the following.

Theorem 4.1: The $BLUE$ (or the $BLIE$) of the common scale parameter σ is a constant multiple of R_1 if and only if any one of the following equivalent conditions hold.

(i). There exist a constant λ_1 such that $\underline{M} = \lambda_1(Q\underline{n})$, where $\underline{n} = (1,0,...,0)$, vector of order $[n/2]$.

(ii).There exist a constant λ_1 such that $E(R_i^*) = \lambda_1 E(R_i^* R_1^*)$, $i = 1,...,[n/2]$, where $R_1^* = W_1^* + W_{n-1}^* = Y_{2:n} - Y_{1:n} + Y_{n:n} - Y_{n-1:n}$.

(iii). There exist a constant λ_2 such that $\underline{M} = \lambda_2(H\underline{n})$.

(iv).There exist a constant λ_2 such that $E(R_i^*) = \lambda_2 Cov(R_i^* R_1^*)$, $i = 1,...,[n/2]$,

where R_1^* is as defined in (ii).

When (i)-(iv) hold, the $BLUE$ of σ is given by

$$L_u = \left[\frac{X_{2:n} - X_{1:n} + X_{n:n} - X_{n-1:n}}{\lambda_1\left(\underline{n}'Q\underline{n}\right)}\right] = \left[\frac{X_{2:n} - X_{1:n} + X_{n:n} - X_{n-1:n}}{\lambda_2\left(\underline{n}'H\underline{n}\right)}\right]$$

and the $BLIE$ of the common scale parameter σ is

.

$$L_I = \lambda_1\left(X_{2:n} - X_{1:n} + X_{n:n} - X_{n-1:n}\right) = \left[\frac{\lambda_2\left(X_{2:n} - X_{1:n} + X_{n:n} - X_{n-1:n}\right)}{1 + \lambda_2^2\left(\underline{n}'H\underline{n}\right)}\right].$$

Proof: First observe that (i) is equivalent to (ii) and (iii) is equivalent to (iv). If (ii) holds, then $E(R_i^*)(1-\lambda_1)E(R_1^*) = \lambda_1 Cov(R_i^* R_1^*)$, $i = 1,...,[n/2]$, which can be rewritten as $E(R_i^*) = \lambda_2 Cov(R_i^* R_1^*)$, $i = 1,...,[n/2]$ where $\lambda_2 = \dfrac{\lambda_1}{1-\lambda_1 E(R_1^*)}$ and thus, (iv) holds (observe that $\lambda_1 = \dfrac{E(R_1^*)}{E(R_1^{*2})}$,since $E(R_1^*) = \sum_{i=1}^{[n/2]} E(R_1^*) = \lambda_1 \sum_{i=1}^{[n/2]} E(R_i^* R_1^*) = \lambda_1 E(R_1^{*2})$ and hence $0 < \lambda_1 E(R_1^*) < 1$.

Conversely, if (iv) holds, then

$$E(R_1^*) = \sum_{i=1}^{[n/2]} E(R_i^*) = \lambda_2 \sum_{i=1}^{[n/2]} Cov(R_i^* R_1^*) = \lambda_2 Var(R_1^*)$$

showing that $\lambda_2 = \dfrac{E(R_1^*)}{Var(R_1^*)} \in (0,\infty)$ and

$E(R_i^*) + \lambda_2 E(R_i^*) E(R_1^*) = \lambda_2 E(R_i^* R_1^*)$

$i = 1,...,[n/2]$ the last expression can be rewritten as $E(R_i^*) = \lambda_1 E(R_i^* R_i^*)$,

$i = 1,...,[n/2]$ with $\lambda_1 = \dfrac{\lambda_2}{1+\lambda_2 E(R_1^*)}$ which is (ii).There fore all conditions (i)-(iv) are equivalent.

Assume now that (i) holds. Then from (3.7), the *BLIE* of σ is

$L_I = \underline{M}' Q^{-1} \underline{R} = \lambda_1 (\underline{n}' \underline{R}) = \lambda_1 (X_{2:n} - X_{1:n} + X_{n:n} - X_{n-1:n})$ and the other formula follows from (3.9) and (3.4).

In order to prove the necessity, assume that $L_\lambda = \lambda R_1 = \lambda(\underline{n}' \underline{R})$ is the *BLIE* of σ.Then $E(\lambda R_1 - \sigma)^2 = \lambda^2 (\underline{n}' Q \underline{n}) + 1 - \lambda) \underline{n}' \underline{M}) \sigma^2$, must be minimum with respect to λ, showing that $\lambda = \dfrac{\underline{n}' \underline{M}}{\underline{n}' Q \underline{n}}$. Since for this value of λ we must have $E(\lambda R_1 - \sigma)^2 = \left(1 - \dfrac{(\underline{n}' \underline{M})^2}{\underline{n}' Q \underline{1}}\right) \sigma^2 = MSE(L_I) = (1 - \underline{M}' Q^{-1} \underline{M})^2$, we conclude that $(\underline{n}' \underline{M})^2 = (\underline{n}' Q \underline{n})(\underline{M}' Q^{-1} \underline{M})$.This is Cauchy-Schwarz inequality written as an equality and, therefore, this equality is attained only if there exist a constant λ_1 such that $\underline{M} = \lambda_1 (Q \underline{n})$, completing the proof.

The non-negativity nature of the estimator L_u and L_I of the common scale parameter σ is given in the following theorem.

Theorem 4.2: If either $n=2$ or 3 then the *BLUE* (and the *BLIE*) of the common scale parameter σ is nonnegative.

Proof: If $n=2$ or 3, then by theorem 4.1 the $n=2$ or 3 of σ is a constant multiple of the pooled sample range and the result is obvious.

Theorem 4.3: If the known distribution functions, $F_i(y), i=1,..2,....,k$ are such that

$$Cov(R_i^* R_j^*) \le 0 \ , \ i \ne j, \ \ i,j=1,...,[n/2] \qquad (4.1)$$

then the *BLUE* (and the *BLIE*) of the common scale parameter σ is non-negative.

Proof: Under (4.1) the positive definite matrix H has non-positive off-diagonal elements. Therefore from theorem 12.2.9 in Graybill(1969) it follows that the positive definite matrix H^{-1} has all its elements non-negative, this shows that $H^{-1}\underline{M} > 0$ component wise, and the assertion follows from expression (3.4).

5. ESTIMATION OF COMMON SCALE PARAMETER OF NORMAL AND DOUBLE EXPONENTIAL DISTRIBUTIONS

In this section we consider a random sample of size n_1 drawn from a normal distribution with probability density function

$$f_1(x,\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \ , -\infty < x < \infty , -\infty < \mu < \infty , \sigma > 0$$

and sample of size n_2 drawn from double exponential distribution with probability density function

$$f_2(x,\mu,\sigma) = \frac{1}{\sqrt{2}\sigma} e^{-\sqrt{2}\left|\frac{x-\mu}{\sigma}\right|} \quad , \quad -\infty < x < \infty \ \text{ for } 1 \le n_1 \le 5 \ , 1 \le n_2 \le 5$$

and illustrate the method proposed in section 3 to estimate the common scale parameter σ of the above two distributions by function of spacings of pooled sample of observations. To obtain the estimators we have evaluated means, variance and co variances of function of spacings of the combined sample ,by using the means, variances and covariances of order statistics of the combined sample consisting of observations drawn from standard normal distribution with probability density function $f_1(x,0,1)$ and standard double exponential distribution with probability density function $f_2(x,0,1)$ for $n=2(1)5$ given in sajeevkumar and Thomas(2005), and are presented in Table 1.

The coefficients $c_1, c_2, ..., c_{[n/2]}$ of the *BLUE* $L_u = \sum_{i=1}^{[n/2]} c_i R_i$, ($Var(L_u)$) and the coefficients $d_1, d_{2,...,} d_{[n/2]}$ of the *BLIE* $L_I = \sum_{i=1}^{[n/2]} d_i R_i$, ($MSE(L_I)$) have been also evaluated and are presented in Table 2.From Table 1 ,it is noted that all $Cov(R_i^*, R_j^*) \leq 0, for \quad i \neq j$ and hence the *BLUE* and *BLIE* for σ is positive ,which is evident in Table 2.

Table 1: Expected value, Variances and co-variances of function of spacings arising from Standard normal and double exponential distributions.

Sample size (n)	n_1	n_2	Choice of i for $1 \leq i \leq [n/2]$	$E(R_i^*)$	Choice of i, j for $1 \leq i \leq j \leq [n/2]$	$Cov(R_i^*, R_j^*)$
2	1	1	1	2.20048	1,1	3.15720
3	1	2	1	1.63056	1,1	0.98402
	2	1	1	1.66422	1,1	0.88264
4	1	3	1	1.47626	1,1	0.92948
			2	1.02692	1,2	-0.04464
					2,2	0.84008
	2	2	1	1.47550	1,1	0.83992
			2	1.08172	1,2	-0.07628
					2,2	0.88256
	3	1	1	1.47148	1,1	0.75402
			2	1.13568	1,2	-0.11012
					2,2	0.93376
5	1	4	1	1.42118	1,1	0.91860
			2	0.84668	1,2	-0.01894
					2,2	0.29218
	2	3	1	1.40312	1,1	0.83306
			2	0.88320	1,2	-0.03396
					2,2	0.29750
	3	2	1	1.38264	1,1	0.75080
			2	0.91944	1,2	-0.04996
					2,2	0.30422
	4	1	1	1.36010	1,1	0.67176
			2	0.95514	1,2	-0.06730
					2,2	0.31268

Table 2: Coefficients of the functions of spacings in the *BLUE* L_u and *BLIE* L_1 of σ, $\sigma^{-2}Var(L_u)$ and $\sigma^{-2}MSE(L_I)$.

Sample Size (n)	n_1	n_2	Estim-ators	$L_U = \sum_{i=1}^{[n/2]} c_i R_i \,/\, L_I = \sum_{i=1}^{[n/2]} d_i R_i$		$\sigma^{-2}Var(L_U)$ / $\sigma^{-2}MSE(L_I)$
				c_1/d_1	c_2/d_2	
2	1	1	L_U	0.45445.		0.65203
			L_I	0.60532		0.39468
3	1	2	L_U	0.61329		0.37011
			L_I	0.70110		0.72987
	2	1	L_U	0.60088		0.31869
			L_I	0.75833		0.24167
4	1	3	L_U	0.43648	0.34633	0.26434
			L_I	0.34522	0.27392	0.20907
	2	2	L_U	0.43991	0.32440	0.23365
			L_I	0.35660	0.26296	0.18940
	3	1	L_U	0.44583	0.30287	0.20579
			L_I	0.36974	0.25118	0.17067
5	1	4	L_U	0.33323	0.62174	0.20710
			L_I	0.27606	0.51507	0.17157
	2	3	L_U	0.33903	0.59363	0.18692
			L_I	0.28564	0.50014	0.15749
	3	2	L_U	0.34732	0.56532	0.16818
			L_I	0.29732	0.48393	0.14397
	4	1	L_U	0.35836	0.53667	0.15044
			L_I	0.31150	0.46650	0.13077

Remark: To compare the efficiency of our estimator L_U we can take the *BLUE* $\sigma^* = \delta U_1 + (1-\delta)U_2$ of σ based on U_1 and U_2, where U_1 is the Lloyd's *BLUE* of σ based on a sample of size n_1 arising from normal distribution alone and U_2 is the Lloyd's *BLUE* of σ based on a sample of size n_2 arising from double exponential distribution alone. Here again σ^* is the *BLUE* based on U_1 and U_2, when $\delta = \frac{Var(U_2)}{Var(U_1)+Var(U_2)}$. The efficiency comparison of L_U relative to σ^* for $n = 2(1)5, n_1, n_2 < 5, n_1 + n_2 = 5$ are given in Sajeev Kumar and Thomas(2005). From Sajeev Kumar and Thomas (2005) it is clear that $Var(L_U)$ is much less than variance of σ^*.

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