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DOUBLE STAGE SHRINKAGE TESTIMATOR OF THE MEAN OF AN EXPONENTIAL LIFE MODEL UNDER ASYMMETRIC LOSS FUNCTION

Rakesh Srivastava and Vilpa Tanna

ABSTRACT

The present paper proposes a double stage shrinkage testimator $(\hat{\theta}_s)$ for the mean (average life) of an exponential life testing model. The risk properties of the proposed testimator have been studied using an asymmetric loss function. If the available guess value is accepted on the basis of the outcome of a preliminary test of significance (PTS), one can propose a shrinkage testimator, otherwise an additional sample is taken and a pooled estimator (based on n_1 and n_2 is proposed. Risks of the conventional estimator $(\overline{X}_1)$ and the double stage estimator $(\overline{X}_p)$ have been derived under the asymmetric loss function. It has been observed that $(\hat{\theta}_s)$ dominates both $(\overline{X}_1)$ and $(\overline{X}_p)$ in the sense of having smaller risk. A study of relative risks shows that for different levels of significance (preferably $\alpha = 1\%$) and varying degrees of overestimation or underestimation the proposed testimator fairs better than the conventional ones.

1. INTRODUCTION

The exponential distribution plays the same role in life testing experiment which the normal distribution does for agricultural and other experiments. A number of life test data have been examined (Davis, 1952) and it was observed that the exponential distribution fits well in most of the cases. This distribution occurs in many contexts such as waiting time problems, time intervals between mining accidents, the life span of electric bulbs, etc. (Maguire et al., 1952; Bartholom

1987: Epstein, 1958; Lawless, 1983). Epstein and Sobel (1953) considered the life testing problem and suggested for one – parameter exponential density,

$$f(X;\theta) = \frac{1}{\theta} e^{-\frac{X}{\theta}}; \qquad X > 0, \theta > 0 \tag{1.1}$$

The unbiased estimator $\hat{\theta}$ of θ is given by

$$\hat{\theta} = \frac{\sum_{i=1}^{r} t_i + (n-r)t_r}{r}$$

Where $t_1 \le t_2 \le ... \le t_r$ was the set of first r "ordered" observations of the time of failure of the n radio tubes put to test for their life period. Here, it is supposed that the experiment was stopped after observing the $r-th$ observation (Type II censoring) to save the time and life of the physical apparatus or animals. Thus, this estimator can be used only in the situations where one expects 'ordered' observations from the experiment. But, there are situations such as sampling from the income – distribution, waiting time for telephonic conversation or waiting time for scooter services etc., where one does not get "ordered" observations. In such situations and also when there is no apriori knowledge available for parameter θ, the sample mean, $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, is the best linear unbiased estimator ($BLUE$) of θ based on complete set of observations.

When a guess θ_0 of any parameter θ is available to the experimenter either due to past studies or his familiarity with the behaviour of the population, then this guess may be utilized to improve the estimation procedure. In order to use this information in constructing an estimator of θ, the use of a preliminary test of significance of hypothesis $H_p : \theta = \theta_0$ has been suggested by Bancroft (1944) in estimation of β_1 in the regression model $Y = \beta_1 X_1 + \beta_2 X_2 + \varepsilon$ using the preliminary test of significance $H_0 : \beta_2 = 0$. Since then the procedure has been used in many problems of estimation and testing of hypothesis and an extensive bibliography is provided by Han and Bancroft (1977) and Han, Rao and Ravichandran (1988). The procedure in this case is to collect a sample from the population under study, compute a statistic, say T, to test the preliminary hypothesis $H_p : \theta = \theta_0$ at some pre – assigned level of significance, say α. The guess θ_0 is used if the hypothesis $H_p : \theta = \theta_0$ is accepted at $\alpha -$ level and is not used if H_p is rejected.

Another approach for using guess θ_0 in the estimation of parameter θ is suggested by Katti (1962) double stage scheme. He defined the scheme for estimating the parameter θ in a normal distribution $N(\theta,\sigma^2)$, σ^2 known, when a guess θ_0 of θ is available. He has suggested a region R in the sample space using known values of the guess estimate θ_0 , σ^2, first and second sample sizes n_1 and n_2 respectively. The region R was constructed by minimizing the mean square error (MSE) of the estimator when the true value of θ was θ_0 . His estimator consisted of two parts. One, the sample mean $\bar{X}_1$ based on the first sample if it belongs to R and the other was pooled mean $\bar{X}_p = \dfrac{n_1 \bar{X}_1 + n_2 \bar{X}_2}{n_1 + n_2}$, if the first sample did not belong to R, when a second sample of size n_2 was also used. Estimators using the guess value and the full sample observations has been used by Thompson (1968 a) and Arnold and Al – Bayyati (1970) in different contexts.

We know in many real life situations the overestimation or underestimation are not of equal consequences. Several authors such as Canfield (1975), Zellner (1986), Basu and Ebrahimi (1991), Srivastava (1996), Srivastava and Tanna (2001) and others have shown that the estimators or testimators of the parameter of interest under the asymmetric loss function demonstrate their superiority over the estimators obtained under squared error loss function (SELF). With this motivation, here and attempt is made to study the double stage shrunken estimator under asymmetric loss function.

While estimating a parameter θ by $\hat{\theta}$ the asymmetric loss function is given by

$$L(\Delta) = b\left(e^{a\Delta} - a\Delta - 1\right), \quad a \neq 0,\ b > 0 \tag{1.2}$$

where $\Delta = \left(\dfrac{\hat{\theta}}{\theta} - 1\right)$ or $\Delta = \left(\hat{\theta} - \theta\right)$ depending upon whether the scale or the shape parameter is being estimated.

The sign and magnitude of a represents the direction and degree of asymmetry respectively and b is the factor of proportionality. The positive value of a is used when overestimation is more serious than underestimation, while a negative value of a is used in reverse situations. In throughout our study we take $b = 1$.

Varian (1975) proposed an asymmetric loss function, which has been found to be appropriate in situations where either overestimation is more serious than underestimation or vice – versa. Several comments are in ordered with regard to (1.2) first for $a = 1$, the function is quiet asymmetric about zero with overestimation being more serious than underestimation. Secondly for $a < 0$,

$L(\Delta)$ rises exponentially when $\Delta < 0$ (underestimation) and almost linearly when $\Delta > 0$ (overestimation).

Ojha et al (1980) have considered the problem of estimation of the mean θ of the one – parameter exponential population with p.d.f. given by (1.1) when a guess value θ_0 of θ is available to the experimenter. Their double stage estimation for θ is, use the mean of the first sample and guess value if the hypothesis $H_p : \theta = \theta_0$ is accepted; otherwise, use the pooled mean $\bar{X}_p$ of the two samples if H_p is rejected. Properties of the estimator have been studied under squared error loss function and recommendations regarding its applications have been attempted.

We have proposed the double shrunken estimator $\hat{\theta}_s$ in section 2. The risk properties of the proposed estimator $\hat{\theta}_s$ using asymmetric loss function has been derived in section 3. In section 4 we have compared the relative risks of $\hat{\theta}_s$ with conventional estimators $\bar{X}_1$ and $\bar{X}_p$. The paper concludes with section 5.

2. THE ESTIMATOR

Let $X_{11}, X_{12}, \ldots X_{1n_1}$ be the first – stage sample of size n_1 from the exponential population $f(X;\theta) = \frac{1}{\theta} e^{-\frac{X}{\theta}}$; $(X > 0,\ \theta > 0)$. Let θ_0 be a guess estimate of the mean θ. Compute the sample mean $\bar{X}_1 = \frac{1}{n_1} \sum_{i=1}^{n} X_{1i}$ and test the preliminary hypothesis $H_p : \theta = \theta_0$, using the test statistic $T = \frac{2n_1 \bar{X}_1}{\theta_0}$. This statistic follows the χ^2 – distribution with $2n_1$ degrees of freedom when H_p is true. It may be noted that

$$H_p \text{ is accepted if } r_1 \le \frac{2n_1 \bar{X}_1}{\theta_0} \le r_2$$

And

$$H_p \text{ is rejected, o.w.}$$

Where r_1 and r_2 being given by

$$\Pr.\left[\chi^2{}_{2n_1} \geq r_2\right] + \Pr.\left[\chi^2{}_{2n_1} \leq r_1\right] = \alpha \tag{2.1}$$

with α is the pre – assigned level of significance.

Now, if $H_p : \theta = \theta_0$ is accepted, take the estimator: $k\left(\bar{X}_1 - \theta_0\right) + \theta_0$, $[0 \leq k \leq 1]$ and if it is rejected, then take $n_2 = n - n_1$ additional observations $X_{21}, X_{22}, \ldots X_{2n_{21}}$ and use the pooled estimator $\bar{X}_p = \dfrac{n_1 \bar{X}_1 + n_2 \bar{X}_2}{n_1 + n_2}$ as the estimator of the mean. More precisely the estimator is defined as:

$$\hat{\theta}_s = \begin{cases} k\bar{X}_1 + (1-k)\theta_0; & \text{if } r_1 \leq \dfrac{2n_1 \bar{X}_1}{\theta_0} \leq r_2 \\ \bar{X}_p = \dfrac{n_1 \bar{X}_1 + n_2 \bar{X}_2}{n_1 + n_2}; & o.w. \end{cases}$$

where $\bar{X}_i = \dfrac{1}{n_i} \sum_{j=1}^{n_1} X_{ij}, \quad i = 1,2$,

where k is the shrinkage factor lying between 0 and 1.

Now, we derive the risk of $\hat{\theta}_s$ in the following section.

3. Risk of $\hat{\theta}_s$

$\bar{X}_1$ and $\bar{X}_2$ are independently distributed with probability density function

$$g(\bar{X}_i) = \frac{n_i^{\,n_i}}{\theta^{n_i}\Gamma(n_i)} e^{-\frac{n_i X_i}{\theta}} X_i^{-n_i - 1} \text{ for } 0 \leq \bar{X}_i \leq \infty, \quad i = 1,2 \tag{3.1}$$

Now, the risk of $\hat{\theta}_s$ under asymmetric loss function $L(\Delta)$ can be defined as:

$$R_L\left(\hat{\theta}_S\right)=\left[\begin{array}{l}\displaystyle\int_{\bar{X}_1=\frac{r_1\theta_0}{2n_1}}^{\frac{r_2\theta_0}{2n_1}}\int_{\bar{X}_2=0}^{\infty}\left[e^{a\left\{\frac{k\bar{X}_1+(1-k)\theta_0}{\theta}-1\right\}}-a\left\{\frac{k\bar{X}_1+(1-k)\theta_0}{\theta}-1\right\}-1\right] \\ \qquad g\left(\bar{X}_1\right)g\left(\bar{X}_2\right)d\bar{X}_1\,d\bar{X}_2 \\ \displaystyle+\int_{\bar{X}_1=0}^{\frac{r_2\theta_0}{2n_1}}\int_{\bar{X}_2=0}^{\infty}\left[e^{a\left\{\frac{\bar{X}_p}{\theta}-1\right\}}-a\left\{\frac{\bar{X}_p}{\theta}-1\right\}-1\right]g\left(\bar{X}_1\right)g\left(\bar{X}_2\right)d\bar{X}_1\,d\bar{X}_2 \\ \displaystyle\int_{\bar{X}_1=\frac{r_2\theta_0}{2n_1}}^{\infty}\int_{\bar{X}_2=0}^{\infty}\left[e^{a\left\{\frac{\bar{X}_p}{\theta}-1\right\}}-a\left\{\frac{\bar{X}_p}{\theta}-1\right\}-1\right]g\left(\bar{X}_1\right)g\left(\bar{X}_2\right)d\bar{X}_1\,d\bar{X}_2\end{array}\right] \tag{3.2}$$

A straightforward integration of (3.2) gives us:

$$R_L\left(\hat{\theta}_S\right)=\left[\begin{array}{l}\dfrac{e^{a\{(1-k)\lambda-1\}}}{\left(1-\dfrac{ak}{n_1}\right)^{n_1}}\{I(\varpi_2;n_1)-I(\varpi_1;n_1)\}+\dfrac{e^{-a}}{\left(1-\dfrac{ak}{(n_1+n_2)}\right)^{n_1+n_2}} \\ \quad\left[1-\{I(\varpi_6;n_1)-I(\varpi_5;n_1)\}+a\left(\dfrac{n_1}{n_1+n_2}-k\right)\right] \\ \quad\{I(\varpi_4;n_1+1)-I(\varpi_3;n_1+1)\}+a\left(\dfrac{n_1}{n_1+n_2}-(1-k)\lambda\right) \\ \quad\{I(\varpi_4;n_1)-I(\varpi_3;n_1)\}-1\end{array}\right] \tag{3.3}$$

where,

$$\varpi_1 = \frac{1}{2}\lambda r_1\left(1-\frac{ak}{n_1}\right), \varpi_2 = \frac{1}{2}\lambda r_2\left(1-\frac{ak}{n_1}\right), \varpi_3 = \frac{1}{2}\lambda r_1, \varpi_4 = \frac{1}{2}\lambda r_2$$

$$\varpi_5 = \frac{1}{2}\lambda r_1\left(1-\frac{a}{(n_1+n_2)}\right), \varpi_6 = \frac{1}{2}\lambda r_2\left(1-\frac{a}{(n_1+n_2)}\right), \lambda = \frac{\theta_0}{\theta}$$

and $I(u;p) = \dfrac{\int_0^u e^{-x} x^{p-1} dx}{\int_0^\infty e^{-x} x^{p-1} dx}$; refers to the standardized incomplete gamma function.

4. RISK COMPARISON

A natural way of comparing the risk of the proposed double stage shrinkage testimator $(\hat{\theta}_s)$, is to study its performance with respect to the best available estimators $\bar{X}_1$ (single sample based on n_1 only) and $\bar{X}_p$ (based on n_1 and n_2), we defined the risks of conventional estimators under $L(\Delta)$ as:

$$R_L\left(\bar{X}_1\right) = \left[\int_{\bar{X}_1=0}^{\infty}\left[e^{a\left\{\frac{\bar{X}_1}{\theta}-1\right\}} - a\left\{\frac{\bar{X}_1}{\theta}-1\right\}-1\right] g\left(\bar{X}_1\right) d\bar{X}_1\right] \qquad (4.1)$$

A straightforward integration of (4.1) gives us:

$$R_L\left(\bar{X}_1\right) = \frac{e^{-a}}{\left(1-\frac{ak}{n_1}\right)^{n_1}} - 1 \qquad (4.2)$$

Further,

$$R_L\left(\bar{X}_p\right)=\left[\int_{\bar{X}_1=0}^{\infty}\int_{\bar{X}_2=0}^{\infty}\left[e^{a\left\{\frac{\bar{X}_p}{\theta}-1\right\}}-a\left\{\frac{\bar{X}_p}{\theta}-1\right\}-1\right]\times g\left(\bar{X}_1\right)g\left(\bar{X}_2\right)d\bar{X}_1\,d\bar{X}_2\right] \tag{4.3}$$

A straightforward integration of (4.3) gives us:

$$R_L\left(\bar{X}_p\right)=\frac{e^{-a}}{\left(1-\frac{ak}{n_1+n_2}\right)^{n_1+n_2}}-1 \tag{4.4}$$

We now define the relative efficiency which we call the relative risk of $\bar{X}_1$ w.r.t. $\hat{\theta}_s$ under $L(\Delta)$ and that of X_p w. r. t. $\hat{\theta}_s$ under $L(\Delta)$ as follows:

$$R_{R1}=\frac{R_L\left(\bar{X}_1\right)}{R_L\left(\hat{\theta}_S\right)} \tag{4.5}$$

and

$$R_{R2}=\frac{R_L\left(\bar{X}_p\right)}{R_L\left(\hat{\theta}_S\right)} \tag{4.6}$$

We observe that relative risks are the function of $n_1, n_2, k\lambda, a$ and α. In order to study the relative risks of $\ddot{\theta}_s$ w. r. t. $\bar{X}_1$ (single sample based on n_1 only) and $\ddot{\theta}_s$ w. r. t. X_p (based on n_1 and n_2), we have computed its values for the various combinations of (n_1, n_2) as: (25,15) (25,15), (30,10), (35,5), (36,4) and (38,2), $0.8 \le \lambda \le 1.2$, $0 \le k \le 1.0$, $\alpha = 1\%$ and $\alpha = 5\%$ and for various values of degrees of asymmetry positive as well as negative $(a = \pm 1$ to $\pm 14)$, to observe the behaviour of $\ddot{\theta}_s$ under overestimation or underestimation situations. Some of the graphs of relative risks have been assembled in the appendix. From these graphs it has been observed that:

(1) Keeping $n_1 = 25, n_2 = 15$ and fixing $\alpha = 1\%$ we have allowed the variation in $a = 1,2,3,4$. Also k varies from 0 to 1 for whole range of k (specially when $k \le 0.8)$) $\ddot{\theta}_s$ performs better than and $\hat{\hat{\bar{X}}}_1$ (and $\bar{X}_p$). Highest gain is observed for $a = 1$ in the sense that in this case the values of relative risks $(R_{R1}$ and $R_{R2})$ are highest compared to those for other values of α. A higher value of relatives risks indicated a better control over the risk of the proposed estimator. So it is recommended to take $a = 1$.

(2) In the next comparison stage we have fixed the value of a at 1 and have allowed the variation for values of α s such as $\alpha = 1\%$, 5% and 10%. $\lambda = 1$ indicates the situations of maximum gain whenever the comparisons have been made under the squared error loss function, so we fix $\lambda = 1$ and again for the whole range of k (specially when $k \le 0.6$) $\hat{\theta}_s$ fairs better than $\bar{X}_1$ (and $\bar{X}_p$) for all the values of α s but specially for a lower value of α i.e. $\alpha = 1\%$ the performance of its best.

(3) To observe the effect of higher degree of asymmetry (negative) we have allowed the variation in the values of a as $-1,-2,-3,-4$, rest of the other things remaining same as above, it is observed that a very 'subtle' difference is there in the behaviour of relative risks for different negative values of a. However for $a = -4$ for R_{R1} and $a = -3$ for R_{R2}, the proposed estimator grips the relative risk values more in terms of its higher magnitude values. Almost a similar pattern is observed for $a = -1,-2,-3,-4$. The range of k is somewhat little bit reduced in the sense that relative risks values are good up to $k = 0.6$. For all this range the values of relative risks are greater than unity for $0.8 \le \lambda \le 1.2$ implying superiority of the proposed estimator over the existing ones.

(4) Again, in the situation when underestimation is more serious than overestimation (i.e. most negative values of a) we have allowed the variation in the values of α. The values considered are: $\alpha = 1\%$, 5% and 10% for $\alpha = 1\%$ and for the whole range of k (i.e. 0 to 1) the magnitude of relative risks is greater than 1 indicating the better performance of the proposed estimator. Further, for $\alpha = 5\%$, and 10% also the values are good but a very little difference in the values of relative risks. So, a lower value of

α i.e. $\alpha = 1\%$ is indicative of the best performance for negative values of a.

(5) Next, we have fixed $\alpha = 1\%$, $\lambda = 1$, $a = 1$ (because of the best values of relative risks at this value) and have allowed the variations in (n_1, n_2). Looking at the behaviour of relative risks it is observed that the magnitude of values of relative risks decreases as the pair sample size decreases i.e. it is highest for the pair $n_1 = 25$, $n_2 = 15$ then the next is for $n_1 = 30$, $n_2 = 10$ etc. and so on. Therefore, the recommend pair value is $(25,15)$. However for other pairs of (n_1, n_2) its performance is also better compared to conventional estimators.

(6) Finally we have fixed $a = -4$ for R_{R1} and $a = -3$ for R_{R2}, $\alpha = 1\%$, $\lambda = 1$ and have allowed the variation in the sample values pairs as $n_1 = 25$, $n_2 = 15$ and so on up to $n_1 = 38$, $n_2 = 2$. Here again the same pattern as in the case of positive values of a is observed. Except for $n_1 = 38$, $n_2 = 2$ which fairs better than other combinations of n_1 and n_2.

Looking at the overall performance of relative risks of $\hat{\theta}_s$, w. r. t. $\bar{X}_1$ and $\bar{X}_p$, it has been observed that the magnitude of R_{R1} is higher for all the values of α^{s} s, α^{s}, (n_1, n_2) almost the whole range of λ and k compared to the magnitude of R_{R2}. A higher magnitude indicates 'better control' over the risk of proposed estimator w. r. t. $\bar{X}_1$ when it is compared with $\bar{X}_p$. So, it can be said that $\hat{\theta}_s$ fairs better than both $\bar{X}_1$ and $\bar{X}_p$, in particular more w. r. t. $\bar{X}_1$.

5. CONCLUSION

We have studied the risk properties of a double stage shrinkage testimator of mean (scale) of exponential life model under the asymmetric loss function. A lower level of significance (i.e. $\alpha = 1\%$ or 5%) conjoined with different combinations of single/double stage samples (in particular $n_1 = 25$, $n_2 = 15$ and $n_1 = 38$, $n_2 = 2$) are recommended for various degrees of (positive/negative) asymmetry. In particular $a = +1$ (positive) is recommended w. r. t. $\bar{X}_1$ and $\bar{X}_p$, $a = -4$ (negative) is recommended w. r. t. $\bar{X}_1$ and $a = -3$ (negative) is recommended w. r. t. $\bar{X}_p$ at $\alpha = 1\%$ its performance is the best.

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Rakesh Srivastava

Department of Statistics
Faculty of Science
The M. S. University ofBaroda
Vadodara – 390 002

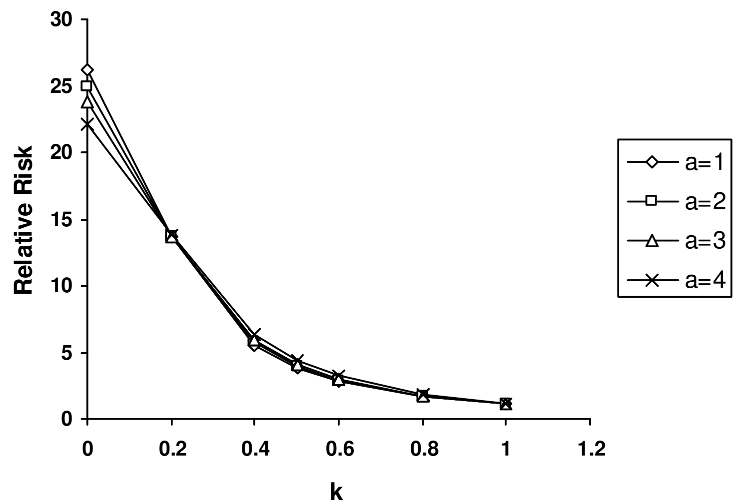
Vilpa Tanna

Department of P.S.M.
P.D.U. Medical College
Rajkot – 360 001
Email: vilpatanna@rediffmail.com

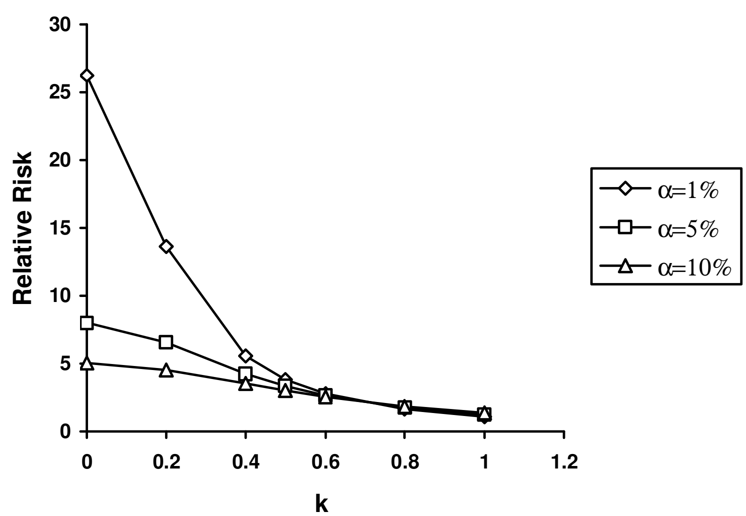
Appendix

Graphs of Relative Risk (R_{R1})

1) $n_1 = 25, n_2 = 15, \lambda = 1.0, \alpha = 0.01$

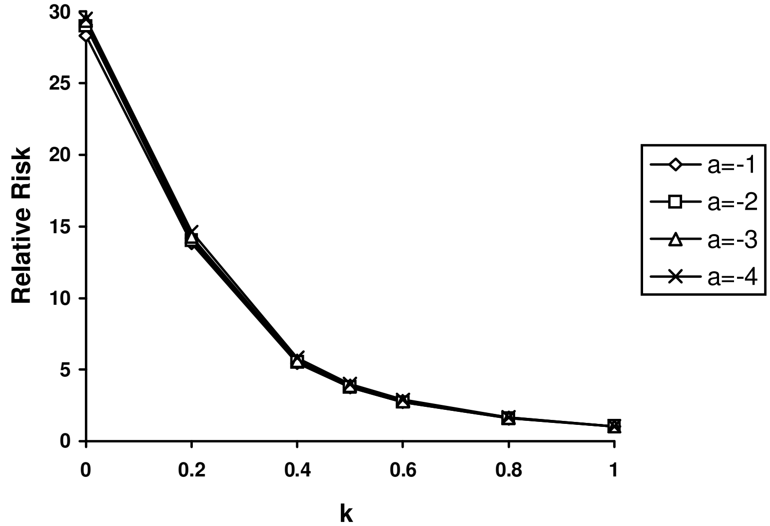


2) $n_1 = 25, n_2 = 15, \lambda = 1.0, a = 1$

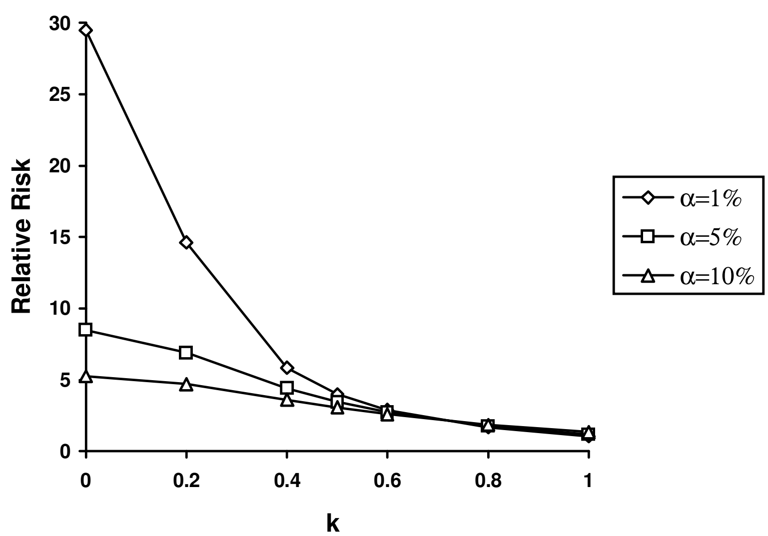


Graphs of Relative Risk (R_{R1})

3) $n_1 = 25, n_2 = 15, \lambda = 1.0, \alpha = 0.01$

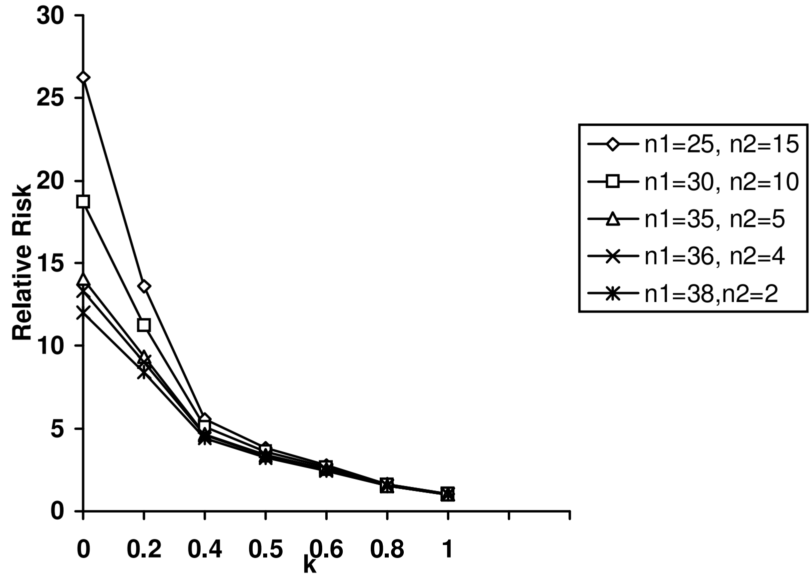


4) $n_1 = 25, n_2 = 15, \lambda = 1.0, a = -4$

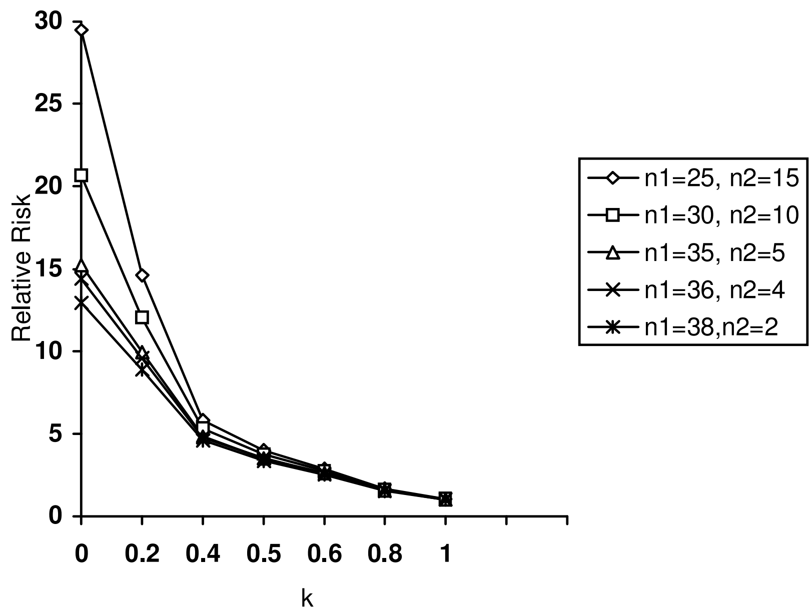


Graphs of Relative Risk (R_{R1})

5) $a=1,\ \lambda=1.0,\ \alpha=0.01$

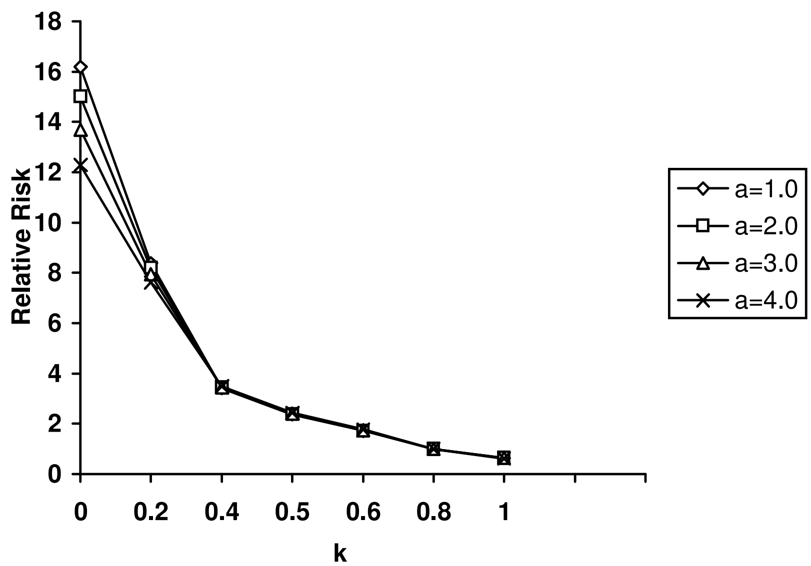


6) $a=-4.0,\ \lambda=1.0,\ \alpha=0.01$

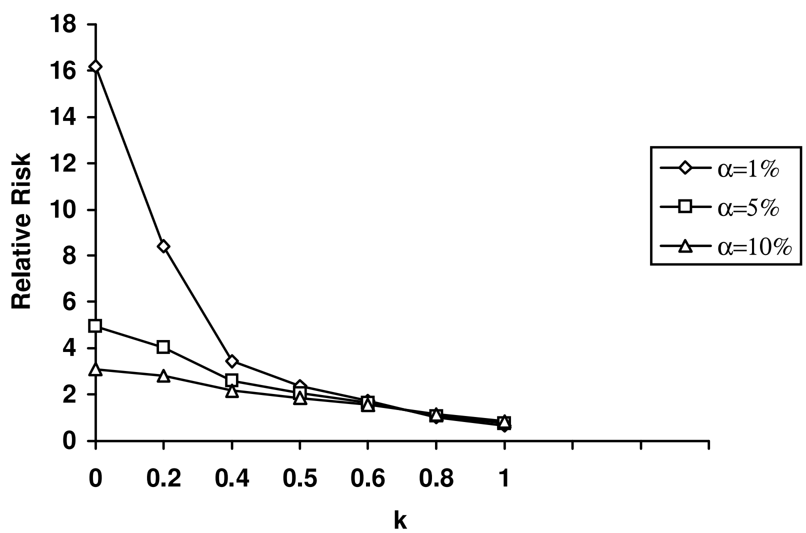


Graphs of Relative Risk (R_{R2})

7) $n_1 = 25,\ n_2 = 15,\ \lambda = 1.0,\ \alpha = 0.01$

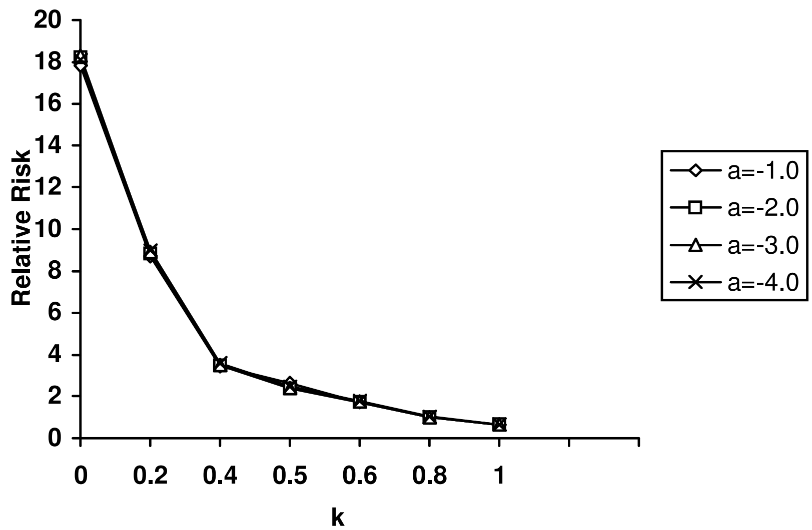


8) $n_1 = 25,\ n_2 = 15,\ \lambda = 1.0,\ a = 1$

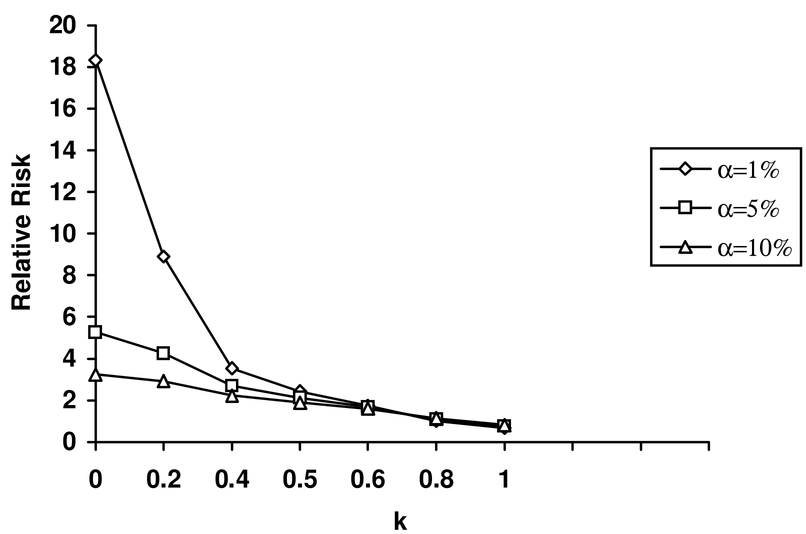


Graphs of Relative Risk (R_{R2})

9) $n_1 = 25,\ n_2 = 15,\ \lambda = 1.0,\ \alpha = 0.01$

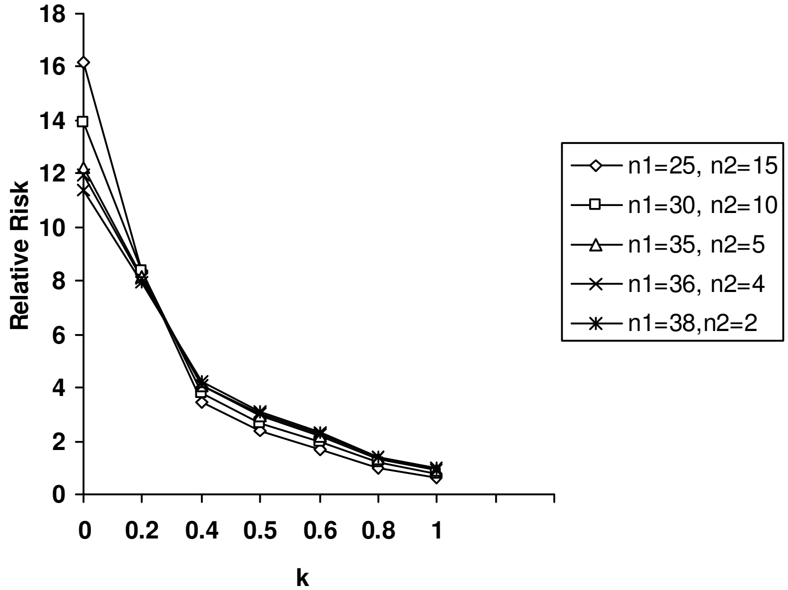


10) $n_1 = 25,\ n_2 = 15,\ \lambda = 1.0,\ a = -3$



Graphs of Relative Risk (R_{R2})

11) $a=1,\ \lambda=1.0,\ \alpha=0.01$



12) $a=-3.0,\ \lambda=1.0,\ \alpha=0.01$