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ON PROBLEM OF NONRESPONSE IN MULTIVARIATE STRATIFIED SAMPLE SURVEYS

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ABSTRACT

In a multivariate stratified population where more than one characteristics are defined on each unit of the population the use of individual optimum allocations is not feasible. A compromise criterion is needed to workout an allocation which is optimum for each characteristic in some sense. If the problem of nonresponse is also there then the precision of the estimate is further reduced. Khare (1987) discussed the problem of optimum allocation in stratified sampling in presence of nonresponse for fixed cost as well as for fixed precision of the estimate for univariate case. Khan *et al*. (2008) extended the same for multivariate population and worked out a compromise allocation using Lagrange Multiplier technique. Most of the authors minimize some function of the variances of the estimators under cost restrictions. If the units of measurement of various characteristics are not same the minimization of some function of variances does not make sense. In the present paper we minimize the weighted sum of squared coefficient of variations under the cost and other restriction. The resulting problem of working out a compromise allocation in presence of nonresponse turns out to be an All Integer Nonlinear Programming Problem. Under certain assumption this problem is solved by the Lagrange Multipliers Technique and explicit formulas are obtained for sample sizes for the first and the second attempts. A numerical illustration is given to demonstrate the suitability of the proposed criterion.

1. INTRODUCTION

The problem of nonresponse occurs in almost all surveys. The extent of nonresponse depends on various factors such as type of the survey, time of the survey, type of the target population etc. For details Särndal and Lundström (2005) may be consulted.

In nonresponse the desired data are not obtained for all the selected units in the sample. For example, if the sampling unit is an individual then the selected person may not be willing to provide the required information or he may not be at home when the interviewer called. In a questionnaire survey the sensitive questions normally have a poor response rate. Whatever the reason may be, if nonresponse exists then the sampler has an incomplete sample data that affects the quality of estimates of the unknown population parameters. Thus if stratified sampling is to be used we may assume that the strata are virtually divided into two mutually exclusive and exhaustive groups of respondents and nonrespondents.

Hansen and Hurwitz (1946) were the first who dealt with the problem of nonresponse in mail surveys. They selected a preliminary sample and mailed the questionnaires to all the selected units. After identifying the nonrespondents a second attempt was made by interviewing a subsample of nonrespondents. They constructed the estimate of the population mean by combining the data from the two attempts and derived the expression for the sampling variance of the estimate. The optimum sampling fraction among the nonrespondents is also obtained. El-Badry (1956) extended the Hansen and Hurwitz's technique by sending waves of questionnaire to the nonrespondents units to increase the response rate. Foradori (1961) has generalized El-Badry's approach and studied the use of Hansen and Hurwitz's technique under different models. Warner (1965) gave the randomized response technique to handle the sensitive questions. Srinath (1971) discussed a rule for the selection of subsample of nonrespondents under which the subsampling rate was not kept constant but taken as the sample nonresponse rates.

Stratified sampling design is widely used for estimating the population parameters of a heterogeneous population. Estimates are constructed from a stratified random sample with optimum choice of the sizes of the samples selected from various strata either to maximize the precision of the constructed estimate for a fixed cost or to minimize the cost of the survey for a fixed precision of the estimate. The sample sizes allocated according to either of the above criteria is called an "Optimum Allocation". Khare (1987) discussed the problem of optimum allocation in stratified sampling in presence of nonresponse for fixed cost as well as for fixed precision of the estimate.

The 'optimum allocation' in stratified random sampling is well known for a univariate population (Cochran (1977) and Sukhatme *et al*. (1984)). But when more than one characteristics are defined on each and every unit of the population, it is not feasible to use the individual optimum allocations to the strata unless there is a strong positive correlation between the characteristics under study. Thus, usually, one has to use an allocation that is optimum in 'some sense' for all the characteristics. Such an allocation is known as a compromise allocation in sampling literature. Neyman (1934), Peter and Bucher (undated), Geary (1949), Dalenius (1957), Ghosh (1958), Yates (1960), Aoyama (1963), Folks and Antle (1965), Chatterjee (1967, 1968), Kokan and Khan (1967), Ahsan (1975-76, 1978), Ahsan and Khan (1977), Bethel (1985, 1989), Schittkowski (1985-86), Chromy (1987), Jahan *et al*. (1994, 2001), Jahan and Ahsan (1995), Khan *et al*. (1997), Khan *et al*. (2003, 2008), Singh (2003), Díaz-García and Cortez (2006, 2008), Kozak (2006a, 2006b), Khan *et al*. (2010) and many others discussed the problem of optimum allocation in multivariate stratified surveys and suggested various compromise criteria.

Most of the above authors worked out the compromise allocation by minimizing some function of the variances of the estimates of various characteristics. As the variances are not unit free to combine the variances of different characteristics is not feasible. Hence in this manuscript the coefficient of variations are considered instead of the variances.

Reviewing the work of Khan *et al*. (2008) we chose to minimize the weighted sum of the squared coefficient of variations (CV) of the estimates of the various characteristics to obtain a compromise allocation. This compromise criterion is more justified for the reason stated earlier. The problem of obtaining a compromise allocation is formulated as an All Integer Nonlinear Programming Problem $(AINLPP)$. A solution procedure using Lagrange Multiplier technique has been presented under certain assumptions. A numerical example is also worked out to illustrate the computational details.

2. FORMULATION OF THE PROBLEM

This section is devoted to the formulation of the problem of finding a compromise allocation that can be used as a common allocation for all characteristics defined on the units of the multivariate stratified population under study in the presence of nonresponse as an $AINLPP$. The notations of Cochran (1977) are used unless specified otherwise.

Consider a population of size N stratified into L strata. Let for the $h-$th stratum $N_h, \bar{Y}_h, S_h^2$ and $W_h = N_h/N$ denote the stratum size, stratum mean, stratum variance and stratum weight respectively. It is assumed that every stratum is virtually divided into two mutually exclusive and exhaustive groups one comprising the respondents and the other comprising the nonrespondents. Let N_{h1} and $N_{h2} = N_h - N_{h1}$ be the sizes of the respondents and nonrespondents groups respectively in the $h-$th stratum. Obviously the true values of N_{h1} and N_{h2} or their estimates are not known prior to the sample observations are obtained. Let n_h, $h=1,2,...,L$ units are drawn from the $h-$th stratum. Further, let out of n_h, n_{h1} units belong to the respondents group and the remaining $n_{h2} = n_h - n_{h1}$ units belong to the nonrespondents group, where the total sample size $n = \sum_{h=1}^{L} n_h$. Let at the second attempt, subsamples of sizes

$$r_h = n_{h2}/k_h; \qquad h=1,2,...,L \tag{2.1}$$

be drawn from n_{h2} nonrespondent group of the $h-$th stratum, where $k_h > 1$ and $1/k_h$, denote the sampling fraction among the nonrespondents. As N_{h1} and N_{h2} are random variables their unbiased estimates are given by $\hat{N}_{h1} = n_{h1} N_h/n_h$ and $\hat{N}_{h2} = n_{h2} N_h/n_h$ respectively.

Let for the $h-$th stratum $\bar{y}_{jh1}$ and $\bar{y}_{jh2(r_h)}$, $j=1,2,...,p$ denote the sample means of $j-$th characteristic measured on the n_{h1} respondents at the first attempt and the r_h subsampled units from nonrespondents at the second attempt. Using Hansen and Hurwitz (1946) estimator, the stratum mean $\bar{Y}_{jh}$ for $j-$th characteristic in the $h-$th stratum may be estimated by

$$\bar{y}_{jh(w)} = \frac{n_{h1}\bar{y}_{jh1} + n_{h2}\bar{y}_{jh2(r_h)}}{n_h} \tag{2.2}$$

It can be seen that $\bar{y}_{jh(w)}$ defined in (2.2) is an unbiased estimate of the stratum mean $\bar{Y}_{jh}$ of the $h-$th stratum for the $j-$th characteristic with a variance

$$V\left(\bar{y}_{jh(w)}\right) = \left(\frac{1}{n_h} - \frac{1}{N_h}\right)S_{jh}^2 + \left(\frac{1}{r_h} - \frac{1}{n_h}\right)W_{h2}^2 S_{jh2}^2 \tag{2.3}$$

where S_{jh}^2 is the stratum variance of $j-$th characteristic in the $h-$th stratum; $j=1,2,...,p$, $h=1,2,...,L$, given by

$$S_{jh}^2 = \frac{1}{(N_h-1)}\sum_{i=1}^{N_h}(y_{jhi} - \bar{Y}_{jh})^2 ,$$

y_{jhi} denote the value of the $i-$th of the $h-$th stratum for $j-$th characteristic and $\bar{Y}_{jh} = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{jhi}$ is the stratum mean of y_{jhi}.

S_{jh2}^2 is the stratum variance of the $j-$th characteristic in the h-th stratum among nonrespondents, given by

$$S_{jh2}^2 = \frac{1}{(\hat{N}_{h2}-1)}\sum_{i=1}^{\hat{N}_{h2}}(y_{jhi} - \bar{Y}_{jh2})^2 ,$$

$\bar{Y}_{jh2} = \frac{1}{\hat{N}_{h2}}\sum_{i=1}^{\hat{N}_{h2}} y_{jhi}$ is the stratum mean of y_{jhi} among nonrespondents.

$W_{h2} = \frac{N_{h2}}{N_h}$ is stratum weight of nonrespondents in $h-$th stratum. (Khan *et al.* (2008)).

It is obvious that the values of $\bar{Y}_j$, S_{jh}^2 and S_{jh2}^2 are unknown in real surveys but can be approximated or estimated from a recent or preliminary survey (Kozak (2006b)).

The variance of $\bar{y}_{j(w)} = \sum_{h=1}^{L} W_h \bar{y}_{jh(w)}$, (ignoring fpc) is given by

$$V(\bar{y}_{j(w)}) = \sum_{h=1}^{L} W_h^2 V(\bar{y}_{jh(w)})$$

$$= \sum_{h=1}^{L} \frac{W_h^2 \left(S_{jh}^2 - W_{h2} S_{jh2}^2\right)}{n_h} + \sum_{h=1}^{L} \frac{W_h^2 W_{h2}^2 S_{jh2}^2}{r_h}; \ j = 1, 2, ..., p \qquad (2.4)$$

where $\bar{y}_{j(w)}$ is an unbiased estimate of the over all population mean $\bar{Y}_j$ of the j-th characteristic and $V(\bar{y}_{jh(w)})$ is as given in (2.3).

The total expected cost of the survey as given in Khan *et al*. (2008) is taken as

$$\hat{C} = \sum_{h=1}^{L} \left(c_{h0} + c_{h1} W_{h1}\right) n_h + \sum_{h=1}^{L} c_{h2} r_h \qquad (2.5)$$

The coefficient of variation of $\bar{y}_{j(w)}$, is $CV_j(\bar{y}_{j(w)}) = \dfrac{\sqrt{V(\bar{y}_{j(w)})}}{\bar{Y}_j}$.

Thus the squared coefficient of variation of $\bar{y}_{j(w)}$, that is $\{CV_j(\bar{y}_{j(w)})\}^2 = \dfrac{V(\bar{y}_{j(w)})}{\bar{Y}_j^2}$; $j = 1, 2, \ldots, p$.

The required compromise allocation will then be the solution to the *AINLPP*

$$\text{Minimize} \quad \sum_{j=1}^{p} a_j \{CV_j(\bar{y}_{j(w)})\}^2 \qquad (2.6)$$

$$\text{subject to} \quad \sum_{h=1}^{L} \left(c_{h0} + c_{h1} W_{h1}\right) n_h + \sum_{h=1}^{L} c_{h2} r_h \le C_0 \qquad (2.7)$$

$$\text{and} \quad \left.\begin{array}{l} 2 \le n_h \le N_h \\ 2 \le r_h \le n_h \\ n_h, r_h \ \text{integers}; \ h = 1, 2, \ldots, L \end{array}\right\} \qquad (2.8)$$

The restrictions in (2.8) are imposed to have estimates of the strata standard deviations and to avoid the oversampling.

The objective function of the *AINLPP* (2.6) – (2.8) may be expressed as

$$\sum_{j=1}^{p} a_j \{CV_j(\bar{y}_{j(w)})\}^2 = \sum_{h=1}^{L} \frac{W_h^2 (A_h^2 - W_{h2} B_h^2)}{n_h} + \sum_{h=1}^{L} \frac{W_h^2 W_{h2}^2 B_h^2}{r_h} \tag{2.9}$$

where $A_h^2 = \sum_{j=1}^{p} \frac{a_j S_{jh}^2}{\bar{Y}_j^2}$ and $B_h^2 = \sum_{j=1}^{p} \frac{a_j S_{jh2}^2}{\bar{Y}_j^2}$, (2.10)

and $a_j > 0$ is the weight assigned to the $j-$th characteristic according some measure of its importance. Without loss of generality we can assume that $\sum_{j=1}^{p} a_j = 1$.

Note that expressions (2.9) and (2.10) are different from the expressions for A_h^2 and B_h^2 in Khan *et al*. (2008).

The *AINLPP* (2.6) – (2.8) may now be restated as

$$\text{Minimize} \quad \sum_{h=1}^{L} \frac{W_h^2 (A_h^2 - W_{h2} B_h^2)}{n_h} + \sum_{h=1}^{L} \frac{W_h^2 W_{h2}^2 B_h^2}{r_h} \tag{2.11}$$

$$\text{subject to} \quad \sum_{h=1}^{L} \left(c_{h0} + c_{h1} W_{h1}\right) n_h + \sum_{h=1}^{L} c_{h2} r_h \le C_0 \tag{2.12}$$

$$\text{and} \quad \left.\begin{array}{l} 2 \le n_h \le N_h \\ 2 \le r_h \le n_h \\ n_h, r_h \text{ integers}; h = 1, 2, \ldots, L \end{array}\right\} \tag{2.13}$$

3. THE SOLUTION

Taking equality in (2.12) and ignoring the restrictions in (2.13) Khan *et al.* (2008) used the Lagrange multiplier technique and obtained the explicit expressions for n_h and r_h; $h = 1, 2, \ldots, L$ as

$$n_h = \frac{C_0 \sqrt{W_h^2 (A_h^2 - W_{h2} B_h^2) \big/ (c_{h0} + c_{h1} W_{h1})}}{\sum_{h=1}^{L} \sqrt{W_h^2 (A_h^2 - W_{h2} B_h^2)(c_{h0} + c_{h1} W_{h1})} + \sum_{h=1}^{L} W_h W_{h2} B_h \sqrt{c_{h2}}} \tag{3.1}$$

$$\text{and } r_h = \frac{C_0 W_h W_{h2} B_h \big/ \sqrt{c_{h2}}}{\sum_{h=1}^{L} \sqrt{W_h^2 (A_h^2 - W_{h2} B_h^2)(c_{h0} + c_{h1} W_{h1})} + \sum_{h=1}^{L} W_h W_{h2} B_h \sqrt{c_{h2}}} \tag{3.2}$$

The values of A_h^2 and B_h^2 in (3.1) and (3.2) are different from A_h^2 and B_h^2 given in Khan *et al*. (2008).

If the rounded off n_h and r_h; $h = 1, 2, \ldots, L$ satisfy restrictions (2.13) and the cost constraint (2.12) the *AINLPP* (2.11) – (2.13) is solved. If any or both of (2.12) and (2.13) are violated then an appropriate All Integer Nonlinear Programming Technique may be used to obtain the required solution.

4. A NUMERICAL ILLUSTRATION

The following data are from the Khan *et al.* (2008). The values of N_h, S_{1h}^2, S_{2h}^2, W_{h1}, W_{h2}, c_{h0}, c_{h1} and c_{h2} for four strata and two characteristics are given in Table 4.1.

Table 4.1: Data for 4 strata and 2 characteristics

h	N_h	S_{1h}^2	S_{2h}^2	W_{h1}	W_{h2}	c_{h0}	c_{h1}	c_{h2}
1	1214	4817.72	8121.15	0.70	0.30	1	2	3
2	822	6251.26	7613.52	0.80	0.20	1	3	4
3	1028	3066.16	1456.40	0.75	0.25	1	4	5
4	786	6207.25	6977.72	0.72	0.28	1	5	6

In addition to the above we assume that $\overline{Y}_1 = 165$ and $\overline{Y}_2 = 237$. Let the total amount available for the survey be $C_0 = 3000$ units and the characteristics are equally important therefore $a_1 = a_2 = 0.5$, so that $a_1 + a_2 = 1$.

Using the given values of the parameters the values of A_h^2 and B_h^2; $h = 1, 2, 3 \& 4$ as given by (2.10) are tabulated in Table 4.2.

Table 4.2: Values of A_h^2 and B_h^2 for 4 strata

h	A_h^2	B_h^2
1	0.160771807	0.040192952
2	0.182580673	0.045645168
3	0.069275925	0.017318981
4	0.176112703	0.044028176

Using (3.1) and (3.2) the optimum values of n_h and r_h that is n_h^* and r_h^*; $h = 1, 2, 3 \& 4$ are obtained as

$n_1^* = 307.5078278$, $n_2^* = 188.9247519$, $n_3^* = 133.2929626$, $n_4^* = 150.9203289$

$r_1^* = 42.89647065$, $r_2^* = 17.87049247$, $r_3^* = 15.39134557$, $r_4^* = 19.18393062$

Rounding off to the nearest integer we get the integer compromise allocations as

$n_1^* = 308$, $n_2^* = 189$, $n_3^* = 133$, $n_4^* = 151$

$r_1^* = 43$, $r_2^* = 18$, $r_3^* = 15$, $r_4^* = 19$

with an objective value of 0.00019557.

It can be seen that the above values of n_h^* and r_h^*; $h = 1, 2, \ldots, 4$ satisfy the restrictions in (2.13).

The total cost incurred given by (2.12) comes out to be $2998.4 < 3000$.

5. CONCLUSION

Table 5.1 gives the percentage increase in the coefficient of variations of various characteristics under different individual optimum allocations.

Table 5.1: Percentage increase in the coefficient of variations

Characteristics	Percentage increase with respect to characteristics $j=1$	Percentage increase with respect to characteristics $j=2$	Proposed compromise allocation
$j=1$	0 %	2.76 %	0.28 %
$j=2$	2.05 %	0 %	0.95 %

It can be seen that the percentage increase under the proposed compromise allocation is considerably less than the percentage increase when individual optimum allocation for one characteristic is used for both the characteristics.

Thus we conclude that the proposed compromise criterion is a suitable criterion for working out a usable compromise allocation for multivariate stratified surveys with nonresponse.

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