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APPLICATION OF LATIN SQUARE DESIGNS IN CDC SYSTEM

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ABSTRACT

The incomplete block designs for complete diallel crosses are proposed for p parental lines, where p is a prime or power of a prime. These incomplete block designs are derived by using Latin squares and mutually orthogonal Latin squares designs of order p. The efficiency of these designs in comparison to randomized block designs is one.

1. INTRODUCTION

Latin square designs are normally used in experiments to remove the heterogeneity of experimental material in two directions. These designs require that the number of replications equals the number of treatments or varieties.
Orthogonal Latin squares are used for construction of graeco Latin squares, balanced incomplete block designs and square lattice designs etc. A set of $(p-1)$ orthogonal Latin squares of side $'p'$ can always be constructed if $'p'$ is a prime or power of prime. If $p = 4t + 2$, $t > 1$, then there exists more than one mutually orthogonal Latin squares of order p [Bose , Shrikhande and Parker (1960)] . From a practical view point, mutually orthogonal Latin squares are important and an exhaustive list of these squares is available [Fisher and Yates (1963].

Various form of diallel crosses as mating designs are used in plant and animal breeding to study the genetic properties and potential of inbred lines or individuals. Let $'p'$ denote the number of lines and let a cross between lines i and j be denoted by (i, j) , $i \neq j = 0,1,2,..., p-1$ and let $'p'$ be number of crosses. Among the four types of diallel discussed by Griffing (1956), method 4 is the most commonly used diallel in plant breeding. This type of diallel crossing includes the genotypes of one set of F_1's, but neither the parents nor reciprocals. In the diallel mating design of method 4, $v = p(p-1)/2$. We shall refer to this as a complete diallel cross (CDC). Our main interest will be in comparing the parents with respect to their general combining abilities $(g.c.a.)$.

The common practice with (CDC) is to evaluate the crosses in a completely randomized design or randomized complete block design as the environment designs. Due to limitation of homogeneous experimental units in a block to accommodate all the chosen crosses, the estimates of genetic parameters would not be precise enough if a complete block design was adopted for the large number of crosses. It is for this reason that the use of incomplete block designs as environment designs has been advocated by Bratten (1965), Aggarwal (1974), Ceranka and Mejza (1988), Agarwal and Das (1990), Divecha and Ghosh (1994), Singh and Hinkelmann (1988) and Sharma (1996).

The problem of generating optimal mating designs for experiments with diallel crosses without specific combining ability has been investigated by using nested incomplete block designs, triangular balanced incomplete block designs, group divisible partially balanced incomplete block designs and circular designs by Gupta and Kageyama (1994), Dey and Midha (1996), Mukerjee (1997), Das *et al*. (1998), Parsad *et al*. (1999), Parsad *et al*. (2005) and Sharma (2004).

In this paper, we give two simple methods to obtain optimal incomplete block designs for complete diallel crosses for $'p'$ parental lines by using Latin squares and mutually orthogonal Latin squares designs. The model considered involves only the general combining ability effects, the specific combining ability being excluded from the model. For definition regarding Latin square and mutually Orthogonal Latin square see Fisher and Yates (1963).

2. METHODS OF CONSTRUCTION

Method 1: Assume that there are $'p'$ inbred lines and it is desired to find an incomplete block design for a mating design involving $p(p-1)/2$ crosses as described in section 1. Consider a Latin square design D_1 of order p. Take any row (or column) and cross the elements of this row (or column) with corresponding elements in another row (or column). This constitutes one block of the proposed design. Similarly other blocks are obtained. Thus from design D_1, we derive a block design D_2 for (CDC). Clearly design D_2 involves $p(p-1)/2$ crosses in $(p-1)$ blocks of size p. Each cross is replicated twice in D_2. The method of construction is illustrated below.

Example 1: For obtaining $5(5-1)/2=10$ crosses for (CDC), consider a Latin square design of order 5 D_1. By crossing the elements of the first row of Latin square design with the corresponding elements of other rows and considering rows as blocks, we get mating design D_2 with parameters $v=10, k=4, b=5$ and $r=2$.

D_1

0	1	2	3	4
1	2	3	4	0
2	3	4	0	1
3	4	0	1	2
4	0	1	2	3

D_2

B_1	0×1	1×2	2×3	3×4	4×0
B_2	0×2	1×3	2×4	3×0	4×0
B_3	0×3	1×4	2×0	3×1	4×0
B_4	0×4	1×0	2×1	3×2	4×0

Method 2: It is known that when $'p'$ is a prime or a power of a prime, it is possible to construct $(p-1)$ orthogonal mutually Latin squares in such a way that they differ only in cyclical interchange of the rows from $II-$nd to $p-$th. Such squares are taken for the construction of incomplete block designs for diallel crosses. The only exception is that for $p=6$, such squares can not be constructed**.**

Assume that there are $'p'$ inbred lines and it is desired to find an incomplete block design for a mating design involving $p(p-1)/2$ crosses. Consider two mutually orthogonal Latin squares of semi-standard form of order p and superimposed one over the other. We get a design D_3 where every symbol (i,j) occurs exactly once, $i,j=0,1,2,...,(p-1)$. From the design D_3, after removing the pair $i=j$ and considering rows as a block, we obtain a block design D_4 for diallel crosses containing $p(p-1)/2$ crosses with parameters $v=p(p-1)/2$, $k=p-1, b=p$ and $r=2$. The method of construction is illustrated below.

Example 2: For obtaining $5(5-1)/2=10$ crosses for CDC, consider two mutually orthogonal mutually Latin squares L_1 and L_2 of semi-standard form of order 5 and superimpose one over the other, we get D_3.

L_1

0	1	2	3	4
1	2	3	4	0
2	3	4	0	1
3	4	0	1	2
4	0	1	2	3

L_2

0	1	2	3	4
2	3	4	0	1
4	0	1	2	3
1	2	3	4	0
3	4	0	1	2

D_3

(0,0)	(1,1)	(2,2)	(3,3)	(4,4)
(1,2)	(2,3)	(3,4)	(4,0)	(0,1)
(2,4)	(3,0)	(4,1)	(0,2)	(1,3)
(3,1)	(4,2)	(0,3)	(1,4)	(2,0)
(4,3)	(0,4)	(1,0)	(2,1	(3,2

D_4

B_1	1×2	2×3	3×4	4×0	0×1
B_2	2×4	3×0	4×1	0×2	1×3
B_3	3×1	4×2	0×3	1×4	2×0
B_4	4×3	0×4	1×0	2×1	3×2

3. ANALYSIS AND OPTIMALITY

For the data obtained from both the designs D_2 and D_4, we postulate the following model. Let Y be $n\times 1$ vector of observations from an experiment in which $(v = p(p-1)/2)$ treatments (crosses) are applied to 'n' plots arranged in $b(= p-1)$ blocks of size $k(= p)$ each.

The linear model for data obtained in such a block design may be expressed as given below:

$$\mathrm{Y} = \mu\, 1 + \mathrm{X}\,\tau + \mathrm{D}\,\beta + \mathrm{e} \tag{3.1}$$

where 1 is the $n\times 1$ of ones, X is the $n\times p$ design matrix for gca effects, D is an $n\times b$ design matrix for blocks, so that $N = X'D$ is the $p\times b$ incidence matrix of the design, μ is a general mean, τ is a $p\times 1$ vector of gca parameters, β is a $b\times 1$ vector of block effects and e is an $n\times 1$ vector of residuals.

It is assumed that e is a random and normally distributed with $E(e) = 0$, $V(e) = \sigma^2 I$.

It can be easily seen that each of the lines appears in each of the blocks exactly 2 times. Further following Dey and Midha (1996), Das *et al.* (1998) and Parsad *et al.* (1999), the information matrix for estimating the linear function of general combining ability (gca) effects given by the above model is

$$C = r(p-2)(I_p - 1/p 1_p 1'_p) \tag{3.2}$$

where I_p is an identity matrix of order p and 1_p is a $p\times 1$ column vector of all ones. Clearly C is a completely symmetric matrix and generalized inverse of C is

$$C^{-} = 1/r(p-2) \tag{3.3}$$

Hence it is not hard to see that each elementary contrast among gca effects is estimated with a variance $2\sigma^2 / r(p-2)$.

4. EFFICIENCY FACTOR

If one adopts a randomized complete block design wit $r = 2$, each block having all the $p(p-1)/2$ crosses, the variance of best linear unbiased estimate *BLUE* of any contrast among the *gca* effects [Griffing (1956)] is $2\sigma_1^2 / r(p-2)$, where σ_1^2 is the per observation variance. Thus the efficiency factor of the proposed designs relative to randomized block design under the assumption of equal intra- block variance is equal to 1.

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