

CONSTRUCTION OF THREE-CLASS ATTRIBUTES LINK SAMPLING PLAN INDEXED THROUGH ACCEPTABLE QUALITY LEVEL

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ABSTRACT

Traditional 2-class sampling plans are used to classify a lot of items as acceptable or non-acceptable by considering only the number of non-conformities found in the sample. These plans do not provide any consideration for marginal defective items. By considering the near miss item as marginal, the 3-class sampling plans are used to take a decision to accept or reject a lot based on not only the number of non-conformities; but also on the number of marginal items. Link sampling plans are the plans that provide another chance to the marginal quality lots that are not accepted based on the first sample result. This paper presents a procedure for constructing 3-class attributes Link Sampling Plan indexed through Acceptable Quality Level (AQL) and for the desired proportion of marginal items. A table is also constructed for easy selection of these plans.

1. INTRODUCTION

In acceptance sampling, single sampling plan and double sampling plan are the most commonly used plans for lot-by-lot inspection procedure. Varieties of plans have been proposed utilizing sample information from related lots to decide about the acceptance or rejection of a current lot. Dodge (1955), Wortham and Mogg (1970) and Wortham and Baker (1971) developed some of these plans. Baker and Brobst (1978) gave conditional double sampling as an alternative to the usual double sampling plan. Harishchandra and Srivenkatramana (1982) proposed some modified double sampling procedure known as Link Sampling Plan. In these plans, whenever a second sample is needed the sample information from neighboring lots is used. Even though these plans are operationally different from the usual double sampling plans, they have the OC curves identical to that of the comparable double sampling plans. The main advantage of these plans is a reduction in cost due to smaller Average Sample Number (ASN). This plan was developed with the second sample size being twice that of the first sample size. While taking second sample, the first half is obtained from the preceding lot and the second half is obtained from the succeeding lot.

These plans are used to take a decision on the lots of items in which the items can be classified as defective or non-defective. In these plans the lot quality is defined by means of the quality parameter, the proportion defective. Such plans are useful in the industries where the items are not useful when they do not meet

certain specifications. Many research papers have been published on these plans and several authors developed procedures and tables for the selection of these plans indexed through various quality levels such as AQL (Acceptable Quality Level), IQL (Indifference Quality Level), LQL (Limiting Quality Level), AOQL (Average Outgoing Quality Level), MAPD (Maximum Allowable Percent Defective) and MAAOQ (Maximum Allowable Average Outgoing Quality).

In these plans, if the decision to accept or reject a lot were based on only one sample selected from the lot, then the lots in the border quality level would also be rejected. Whereas, in many industrial testing such as food inspection and drug testing, even if the items are not meeting the specifications they can be considered as marginal item and the proportion of these marginal items are also used as the additional quality measure to define the quality of the lot. In these plans, while inspecting the sample of items, they are classified as good, marginal or bad items. These plans are known as 3-class attribute plans in which the quality of the lot is defined by means of two quality parameters, the proportions of marginal and bad items in the lot.

Bray et al (1973) developed a procedure for 3-class attributes Single Sampling Plan. Gowri Shankar *et al.* (1991) developed chain-sampling plan for three attribute classes. Newcombe and Allen (1988) suggested a three-class procedure for acceptance sampling by variables. Further, Radhakrishnan and Ravi Sankar (2008, 2009a, 2009b) developed a procedure for the selection of 3-class attributes sampling plans indexed through the parameters such as AOQL and AQL.

Schilling and Johnson (1980) have developed a table for the construction and evaluation of matched sets of single, double and multiple sampling plans. Soundararajan and Arumainayagam (1990) provided tables for easy selection of double sampling plan indexed by AQL, AOQL and LQL. Radhakrishnan and Sampath Kumar (2007a, 2007b) developed a procedure and constructed tables for the selection of mixed sampling plans indexed through MAPD and IQL and also through MAPD and AQL. Harishchandra and Srivenkatramana (1982) developed the link sampling plan for 2-class attributes sampling plans. Suresh *et al.* (1990) developed a procedure for 3-class attributes link sampling plan by considering the feature of link sampling plan. In this paper, an attempt is made to construct and select the 3-class attributes link sampling plans indexed through AQL. A table is also provided for the easy selection of these plans.

2. GLOSSARY OF SYMBOLS

The symbols used in the 3-class attributes link sampling plans are as follows:

n_1 : first sample size ($= n$)

n_2 : second sample size ($= 2n$)

- α : Producer's risk, the probability of rejecting a good lot (usually fixed as 0.05)
- β : Consumer's risk, the probability of accepting a bad lot (usually fixed as 0.10)
- p : proportion of bad items in case of 2-class plan
- p_g : proportion of good quality items
- p_M : proportion of marginal quality items
- p_b : proportion of bad quality items
- p_1 : acceptable quality level (AQL), the sum of proportions of marginal and bad items ($p_{M1} + p_{b1}$) with $(1 - \alpha) = 0.95$.
- b_1, b_2 : constants (> 0)
- d_{i1} : total number of marginal and bad items found in the i -th sample ($i = 1, 2$)
- d_{i2} : number of bad items found in the i -th sample ($i = 1, 2$)
- c_1 : acceptance number for the marginal and bad items in the first sample
- c_2 : acceptance number for bad items in the first sample
- $c_1 + b_1$: acceptance number for the marginal and bad items in the combined sample
- $c_2 + b_2$: acceptance number for bad items in the combined sample
- $Pa(p_M, p_b)$: probability of acceptance for the given quality (p_M, p_b).
- $LSP2(n, c_1, c_2)$: 2-class attributes link sampling plan with parameters n, c_1 and c_2
- $LSP3(n, c_1, b_1, c_2, b_2)$: 3-class attributes link sampling plan with parameters n, c_1, b_1, c_2 and b_2

In the acceptance sampling literature, for the construction of 2-class attributes sampling plans the notations p_0, p_1 and p_2 respectively denote IQL, AQL and LQL. The 3-class attributes sampling plans suggested by Bray *et. al.* (1973) used the same notations p_0, p_1 and p_2 respectively for the proportion of good, proportion of marginal and proportion of bad items. In this paper, as both the concepts of AQL and proportions of good, marginal and bad items are used; the

new notations p_g, p_M and p_b respectively are used in place of p_0, p_1 and p_2 in order to increase the understandability of the readers.

3. OPERATING PROCEDURE OF $LSP2(n, c_1, c_2)$

In a 2-class attributes link sampling plan, the lot acceptance procedure is characterized by the parameters n, c_1 and c_2 . The operating procedure of this link sampling plan is as follows:

Step 1: From lot i select a random sample of size ($n_1 = n$).

Step 2: Inspect all the articles in the sample. Let d_i be the number of defectives in the sample.

Step 3: If $d_i \leq c_1$, accept the lot i ; if $d_i > c_2$, reject the lot.

Step 4: If $c_1 < d_i \leq c_2$, then defer the decision until the result of the next lot $i + 1$ is obtained.

Take a second sample of size ($n_2 = 2n$) by selecting the first n items from the preceding lot (lot $i - 1$) and the next n items from the succeeding lot (lot $i + 1$).

Step 5: Count the number of defectives in the combined sample and let $D_i = d_{i-1} + d_i + d_{i+1}$. If $D_i \leq c_2$, accept the lot i ; if $D_i > c_2$, reject the lot i .

4. OPERATING PROCEDURE OF $LSP3(n, c_1, b_1, c_2, b_2)$

Theoretically Suresh *et al.* (1990) suggested the following operating procedure for three-class attributes link sampling plan $LSP3(n, c_1, b_1, c_2, b_2)$ defined by the five parameters n, c_1, b_1, c_2 and b_2 :

Step 1: Inspect a random sample of size n taken from lot i .

Step 2: Count the total number of marginal and bad items (d_{i1}) and the number of bad items (d_{i2}) in the first sample.

Step 3: If $d_{i1} \leq c_1$ and $d_{i2} \leq c_2$, accept the lot i ;

If $d_{i1} > c_1 + b_1$ or $d_{i2} > c_2 + b_2$ reject the lot i ;

If either (a) $c_1 < d_{i1} \leq c_1 + b_1$ and $d_{i2} \leq c_2$

(or) (b) $d_{i1} \leq c_1 + b_1$ and $c_2 < d_{i2} \leq c_2 + b_2$, then defer the decision until the result of the next lot (lot $i + 1$) is obtained; go to the next step.

Step 4: Take a second sample of size $n_2 = 2n$ by selecting the first n items from the preceding lot (lot $i-1$) and the next n items from the succeeding lot (lot $i+1$).

Step 5: Count the number of defectives in the combined sample.

Let $D_{i1} = d_{(i-1)1} + d_{i1} + d_{(i+1)1}$, the total number of marginal and bad items in the combined sample.

Let $D_{i2} = d_{(i-1)2} + d_{i2} + d_{(i+1)2}$, the total number of bad items in the combined sample.

If $D_{i1} \leq c_1 + b_1$ and $D_{i2} \leq c_2 + b_2$ then accept the lot; otherwise reject the lot i .

Example:

In order to illustrate the procedure a link sampling plan $LSP3(20,2,1,3,1)$ is considered and various examples are given to illustrate the different decisions to be made according to the number of marginal and bad items obtained in the sample:

Case 1:

Step 1: Inspect a random sample of size $n = 20$ taken from the lot i .

Step 2: Count the number of marginal and bad items (d_{i1}) and the number of bad items (d_{i2}).

Step 3: Suppose $d_{i1} = 2$ and $d_{i2} = 1$. Since, $d_{i1} \leq c_1 (2 \leq 2)$ and $d_{i2} \leq c_2 (1 < 3)$ accept the lot i .

Case 2:

Step 1: Inspect a random sample of size $n = 20$ taken from the lot i .

Step 2: Count the number of marginal and bad items (d_{i1}) and the number of bad items (d_{i2}).

Step 3: Suppose $d_{i1} = 4$ and $d_{i2} = 1$.

Since, $d_{i1} > c_1 + b_1 (4 > 2 + 1)$ reject the lot i .

Case 3:

Step 1: Inspect a random sample of size $n = 20$ taken from the lot i .

Step 2: Count the number of marginal and bad items (d_{i1}) and the number of bad items (d_{i2}).

Step 3: Suppose $d_{i1} = 6$ and $d_{i2} = 5$. Since, $d_{i2} > c_2 + b_2$ ($5 > 3 + 1$) reject the lot i .

Case 4:

Step 1: Inspect a random sample of size $n = 20$ taken from the lot i .

Step 2: Count the number of marginal and bad items (d_{i1}) and the number of bad items (d_{i2}).

Step 3: Suppose $d_{i1} = 3$ and $d_{i2} = 1$.

Since, $c_1 < d_{i1} \leq c_1 + b_1$ ($2 < 3 \leq 2 + 1$) and $d_{i2} \leq c_2$ ($1 \leq 3$) defer the decision until the decisions about the next lot is obtained. Go to the next step.

Step 4: Inspect another sample of size $n_2 = 2n = 40$ (first $n = 20$ items from the lot $i - 1$ and second $n = 20$ items from the lot $i + 1$). Count the total number of marginal and bad items ($d_{(i-1)1} + d_{(i+1)1}$) and the number of bad items ($d_{(i-1)2} + d_{(i+1)2}$) in the second sample.

Step 5: Suppose $d_{(i-1)1} = 0$, $d_{(i+1)1} = 0$, $d_{(i-1)2} = 1$, $d_{(i+1)2} = 1$, then $D_{i1} = d_{(i-1)1} + d_{i1} + d_{(i+1)1} = 0 + 3 + 0 = 3$, the total number of marginal and bad items in the combined sample and $D_{i2} = d_{(i-1)2} + d_{i2} + d_{(i+1)2} = 1 + 1 + 1 = 3$, the total number of bad items in the combined sample.

Since, $D_{i1} \leq c_1 + b_1$ ($3 \leq 2 + 1$) and $D_{i2} \leq c_2 + b_2$ ($3 \leq 3 + 1$) then accept the lot i .

Case 5:

Steps 1 to 4 remain same.

Step 5: Suppose $d_{(i-1)1} = 1$, $d_{(i+1)1} = 1$, $d_{(i-1)2} = 1$, $d_{(i+1)2} = 1$ then $D_{i1} = d_{(i-1)1} + d_{i1} + d_{(i+1)1} = 1 + 3 + 1 = 5$, the total number of marginal and bad items in the combined sample and $D_{i2} = d_{(i-1)2} + d_{i2} + d_{(i+1)2} = 1 + 1 + 1 = 3$, the total number of bad items in the combined sample. Since, $D_{i1} > c_1 + b_1$ ($5 > 2 + 1$) and $D_{i2} \leq c_2 + b_2$ ($3 \leq 3 + 1$) then reject the lot i .

5. OPERATING CHARACTERISTIC FUNCTION

The operating characteristic (OC) function of the plan $LSP3(n, c_1, b_1, c_2, b_2)$ is

$$p_a(p_M, p_b) = \sum_{j=0}^{c_2} \sum_{i=0}^{c_1-j} m_{ij}(n) + \left[\sum_{j=0}^{c_2} \sum_{i=c_1+1-j}^{c_1+b_1-j} m_{ij}(n) + \sum_{j=c_2+1}^{c_2+b_2} \sum_{i=0}^{c_1+b_1-j} m_{ij}(n) \right] \times \sum_{j=0}^{c_2+b_2} \sum_{i=0}^{c_1+b_1-j} m_{ij}(2n) \quad (5.1)$$

It is based on the trinomial probability distribution, which is a particular case of a multinomial probability distribution. This 3-class attribute plan is having three categories of quality proportions, p_M, p_b and p_g . The graph for the OC function of the plan $LSP3(n, c_1, b_1, c_2, b_2)$ is an OC surface with the probability of acceptance $P_a(p_M, p_b)$ plotted against the two quality parameters viz., the marginal quality (p_M) and the bad quality (p_b).

6. ACCEPTABLE QUALITY LEVEL

Sampling plans are used to make product disposition decisions. They decide which lots of product to accept and release and which lots to reject and either rework or discard. An ideal plan should reject all bad lots while accepting all good lots. There is a chance of making mistake because these plans are based on the results of a sample taken from the lot and not on the entire lot. The AQL is the proportion defective with 95% chance of acceptance.

7. CONSTRUCTION OF $LSP3(n, c_1, b_1, c_2, b_2)$ PLANS INDEXED THROUGH AQL

As the number of three-class plans is admittedly large, the most useful subset is one for which the number of bad quality items may be fixed as 1. ie., $c_1=c_2=b_2=1$. The 3-class attributes link sampling plans $LSP3(n, c_1, b_1, c_2, b_2)$ are constructed by fixing one of the quality parameters, the proportion of marginal items (p_M) at certain level, the OC function given in (5.1) is equated to 0.95 and using a C-program the other quality parameter, the proportion of bad items p_b is found out. The quality level AQL is calculated as $p_1 = p_M + p_b$. For different values of $b_1=1, 2, 3$ and 4 the LSP3 plans are constructed and presented in Table 1, Table 2, Table 3 and Table 4 respectively. These tables can be used to select the various three class attributes link sampling plans.

8. SELECTION OF $LSP3(n, c_1, b_1, c_2, b_2)$ PLANS FOR THE GIVEN p_M AND AQL

Using Table 1, Table 2, Table 3 and Table 4 one can select a suitable $LSP3(n, c_1, b_1, c_2, b_2)$ plan for the desired p_M and AQL .

Example: Suppose an engineer needs a plan for the proportion of marginal items $p_M = 0.02$, and $AQL = 0.05$. He can select the size of the sample from Table 1, Table 2, Table 3 and Table 4 corresponding to the value of $p_M = 0.02$ and $p_b = 0.03$.

- From table 1 one can get the plan $LSP3(14, 1, 1, 1, 1)$.
- From table 2 one can get the plan $LSP3(18, 1, 2, 1, 1)$.
- From table 3 one can get the plan $LSP3(20, 1, 3, 1, 1)$.
- From table 4 one can get the plan $LSP3(20, 1, 4, 1, 1)$.

The OC surface of the plan $LSP3(14, 1, 1, 1, 1)$ is presented in Figure-1.

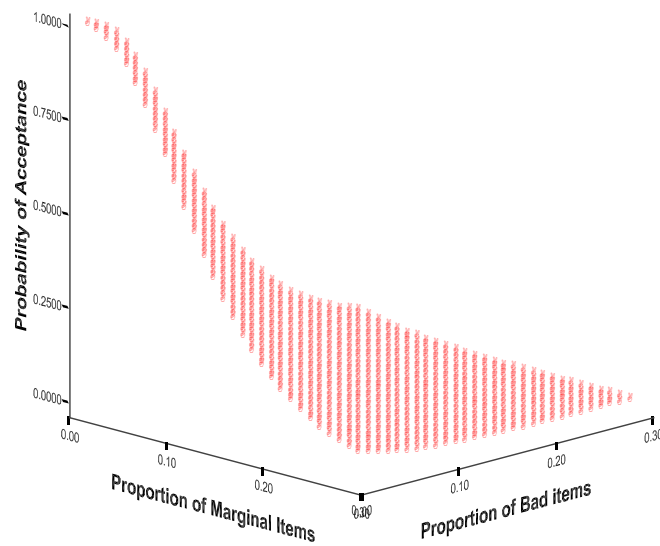


Figure 1: OC surface of the plan $LSP3(14, 1, 1, 1, 1)$

Practical Application: Suppose a medical insurance claim processing company is interested to audit whether the company settles the medical claims properly during a specified period (a month or a year). The company can use a suitable 3-class attributes link sampling plan stating the number of claims to audit, the maximum number of marginal and bad claims and the maximum number of bad claims alone to be considered. Suppose the company fixes AQL as 0.05 (50 number of marginal and bad claims out of 1000 claims audited) with $p_M = 0.02$ (2 out of 100 claims are marginal) then inspect (audit) a random sample of 14 claims from the claims of a lot i in a given month or year and count the number of marginal and bad claims (d_{i1}) and number of bad claims alone (d_{i2}) in that sample.

If $d_{i1} \leq c_1 (=1)$ and $d_{i2} \leq c_1 (=1)$, accept the lot of claims of period i ;

If $d_{i1} > c_1 + b_1 (=1+1)$ or $d_{i2} > c_2 + b_2 (=1+1)$ reject the lot i and inform the management to initiate corrective action.

If either (a) $c_1 (=1) < d_{i1} \leq c_1 + b_1 (=1+1)$ and $d_{i2} \leq c_2 (=1)$

(or) (b) $d_{i1} \leq c_1 + b_1 (=1+1)$ and $c_2 (=1) < d_{i2} \leq c_2 + b_2 (=1+1)$, defer the decision until the decisions about the next lot is obtained.

Inspect another sample of size $n_2 = 2n = 28$ (first $n = 14$ items from the lot $i-1$ and next $n = 14$ items from the lot $i+1$). Count the total number of marginal and bad items ($d_{(i-1)1} + d_{(i+1)1}$) and the number of bad items ($d_{(i-1)2} + d_{(i+1)2}$) in the second sample of size 28.

Let $D_{i1} = d_{(i-1)1} + d_{i1} + d_{(i+1)1}$ and $D_{i2} = d_{(i-1)2} + d_{i2} + d_{(i+1)2}$.

If $D_{i1} \leq c_1 + b_1 (=1+1)$ and $D_{i2} \leq c_2 + b_2 (=1+1)$ then accept the lot i ; otherwise, reject the lot i and inform the management for further investigation.

Table 8.2: $LSP3(n,1,2,1,1)$ for $\alpha = 0.05$

| p_M | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.1 |
|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|------------|
| p_b | | | | | | | | | | |
| 0.01 | | | 24 | 20 | 17 | 14 | | 11 | 10 | |
| 0.02 | | 24 | 19 | 16 | 14 | | 11 | 10 | | |
| 0.03 | 21 | 18 | 16 | 14 | 12 | 11 | 10 | 9 | 8 | 8 |
| 0.04 | 16 | 15 | 13 | 12 | 11 | 10 | 9 | | | |
| 0.05 | 13 | 12 | 11 | | | 9 | | | | |
| 0.06 | 11 | | 10 | 9 | | 8 | | 7 | | |
| 0.07 | 10 | 9 | 8 | | 8 | | 7 | | | 6 |
| 0.08 | | 8 | 8 | | 7 | | | 6 | 6 | |
| 0.09 | | | 7 | 7 | | | 6 | | | |
| 0.10 | 7 | | | | 6 | | | | | 5 |
| 0.11 | | | 6 | 6 | | | | 5 | 5 | |
| 0.12 | 6 | | | | | | 5 | | | |
| 0.13 | | | | 5 | 5 | | | | | |
| 0.14 | | 5 | 5 | | | | | | | |
| 0.15 | 5 | | | | | | | | 4 | 4 |
| 0.16 | | | | | | | 4 | 4 | | |
| 0.17 | | | | | 4 | 4 | | | | |
| 0.18 | | 4 | 4 | 4 | | | | | | |
| 0.19 | 4 | | | | | | | | | |
| 0.22 | | | | | | | | | | 3 |
| 0.23 | | | | | | | | 3 | 3 | 3 |
| 0.24 | | | | | | 3 | 3 | 3 | | |
| 0.25 | | 3 | 3 | 3 | 3 | 3 | | | | |

9. CONCLUSION

It is concluded from the study that it is possible to select a 3-class attributes link sampling plan for a given proportion of marginal items and a specified acceptable quality level. These plans are designed in such a way that the OC surfaces pass through the desired quality level and can be used in place of 2-class sampling plans in the industries such as food industry, pharmaceutical industry and health insurance companies and also in the industries where the quality of the product or service is categorized into three classes good, bad and marginal. This concept results in more probability of acceptance of the lots than in 2-class attributes sampling plans with a smaller sample size (Radhakrishnan and Ravi Sankar (2009a)). These plans can be compared with the plans indexed through other parameters for their efficiency.

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