

BAYES ESTIMATE OF THE PARAMETERS OF GUMBEL'S BIVARIATE EXPONENTIAL DISTRIBUTIONS

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ABSTRACT

For the Bivariate exponential distributions of type I and II are given by Gumbel (1960) we have obtained the Bayes estimate for the parameters θ and α respectively. In this case we do not get the difficulty of exponential integrals. At the end, we have given the tables for the bias and mean squared error of the Bayes estimators.

1. INTRODUCTION

Bivariate exponential distributions can be designed for the life testing of a two component system, which functions even on failure of one of the component. Two battery cells supporting an electronic device, the paired organs of human body are some examples where a bivariate exponential distribution can be applied.

As Lindley (1961) remarks, since they avoid integration over the sample space and the resulting distributional problems, Bayesian methods are often easy to apply than the usual ones, but they do bring in their wake the choice of the prior distribution. Savage (1954) believes that no prior distribution is more correct than any other in the sense that the prior distribution reflects only the beliefs of the person making the inference or decision. According to Gumbel (1960) the correlation coefficient of the variables X and Y of type I distribution is never positive and lies in the interval -0.4 to 0 .

Barnett (1985) had given the estimate of the parameter θ based on coefficient of correlation. Serious difficulties were encountered in the estimation of θ . Dixit and Karadkar (1999) considered the maximum likelihood and method of moments to estimate θ . Here, we consider the prior distribution of θ as a Beta distribution since θ lies between 0 and 1 .

2. JOINT DISTRIBUTION OF $(X_1, Y_1, X_2, Y_2, \dots, X_n, Y_n)$ FOR GUMBEL'S TYPE I DISTRIBUTION

$$f(\underline{x}, \underline{y}, \theta) = \prod_{i=1}^n f(x_i, y_i, \theta)$$

$$\begin{aligned}
&= \prod_{i=1}^n [(1 + \theta x_i)(1 + \theta y_i) - \theta] e^{-\sum_{i=1}^n x_i - \sum_{i=1}^n y_i - \theta \sum_{i=1}^n x_i y_i} \\
&= P e^{-\sum_{i=1}^n x_i - \sum_{i=1}^n y_i - \theta \sum_{i=1}^n x_i y_i}, \tag{2.1}
\end{aligned}$$

where

$$P = \prod_{i=1}^n [(1 + \theta x_i)(1 + \theta y_i) - \theta].$$

Let $A_i = x_i + y_i - 1$ and $B_i = x_i y_i$, $i = 1, 2, \dots, n$.

Therefore,

$$P = \prod_{i=1}^n (1 + A_i \theta + B_i \theta^2). \tag{2.2}$$

Case 1): Let $n = 2$, $C_{1,1} = A_1$ and $C_{1,2} = B_1$. Then

$$\begin{aligned}
P &= (1 + A_1 \theta + B_1 \theta^2)(1 + A_2 \theta + B_2 \theta^2) \\
&= (1 + C_{1,1} \theta + C_{1,2} \theta^2)(1 + A_2 \theta + B_2 \theta^2) \\
&= 1 + (C_{1,1} + A_2) \theta + (B_2 + C_{1,1} A_2 + C_{1,2}) \theta^2 \\
&\quad + (C_{1,1} B_2 + C_{1,2} A_2) \theta^3 + C_{1,2} B_2 \theta^4 \\
&= 1 + C_{2,1} \theta + C_{2,2} \theta^2 + C_{2,3} \theta^3 + C_{2,4} \theta^4. \tag{2.3}
\end{aligned}$$

This can be written as:

$$(C_{2,0} \quad C_{2,1} \quad C_{2,2} \quad C_{2,3} \quad C_{2,4}) \underline{\theta} = \underline{\theta}' \begin{pmatrix} 1 & 0 & 0 \\ C_{1,1} & 1 & 0 \\ C_{1,2} & C_{1,1} & 1 \\ 0 & C_{1,2} & C_{1,1} \\ 0 & 0 & C_{1,2} \end{pmatrix} \begin{pmatrix} 1 \\ A_2 \\ B_2 \end{pmatrix}, \tag{2.4}$$

where

$$\underline{\theta} = \begin{pmatrix} 1 \\ \theta \\ \theta^2 \\ \theta^3 \\ \theta^4 \end{pmatrix}.$$

Case 2): Let $n = i$. Then the matrix is as follows:

$$(C_{i,0} \ C_{i,1} \ C_{i,2} \ C_{i,3} \ \dots \ C_{i,2i}) \underline{\theta} = \underline{\theta}' \begin{pmatrix} 1 & 0 & 0 \\ C_{i-1,1} & 1 & 0 \\ C_{i-1,2} & C_{i-1,1} & 1 \\ C_{i-1,3} & C_{i-1,2} & C_{i-1,1} \\ \vdots & \vdots & \vdots \\ C_{i-1,2(i-1)} & C_{i-1,2(i-1)-1} & C_{i-1,2(i-1)-2} \\ 0 & C_{i-1,2(i-1)} & C_{i-1,2(i-1)-1} \\ 0 & 0 & C_{i-1,2(i-1)} \end{pmatrix} \begin{pmatrix} 1 \\ A_i \\ B_i \end{pmatrix}, \quad (2.5)$$

where

$$\underline{\theta} = \begin{pmatrix} 1 \\ \theta \\ \theta^2 \\ \vdots \\ \theta^{2i} \end{pmatrix}.$$

Hence for $n \geq 2$, we have

i) If $k = 1$, then

$$\begin{cases} C_{n,0} = 1, \\ C_{n,1} = C_{n-1,1} + A_n, \\ C_{n-1,2n-1} = C_{n-1,2n} = 0. \end{cases} \quad (2.6)$$

ii) If $2 \leq k \leq 2n$, then

$$C_{n,k} = C_{n-1,k} + C_{n-1,k-1}A_n + C_{n-1,k-2}B_n. \quad (2.7)$$

Therefore,

$$\begin{aligned} P &= \prod_{i=1}^n (1 + A_i\theta + B_i\theta^2) \\ &= C_{n,0} + C_{n,1}\theta + C_{n,2}\theta^2 + \cdots + C_{n,k}\theta^k + \cdots + C_{n,2n}\theta^{2n}, \end{aligned} \quad (2.8)$$

where C 's are given in (2.6) and (2.7).

Hence (2.1) will become

$$f(\underline{x}, \underline{y}, \theta) = \sum_{j=0}^{2n} C_{n,j} \theta^j e^{-T_x - T_y - T_{xy}\theta}, \quad (2.9)$$

where

$$T_x = \sum_{i=1}^n x_i, \quad T_y = \sum_{i=1}^n y_i \quad \text{and} \quad T_{xy} = \sum_{i=1}^n x_i y_i.$$

3. BAYES ESTIMATOR OF θ

Let the prior distribution of θ be $g(\theta)$ as

$$g(\theta) = \frac{\theta^{a-1}(1-\theta)^{b-1}}{\beta(a,b)}, \quad 0 < \theta < 1, \quad a, b \geq 0, \quad (3.1)$$

where $\beta(a,b)$ is the beta function.

Case 1): Assume that $a = b = 1$ and $g(\theta) = 1$ for $0 < \theta < 1$. Then

$$\begin{aligned} h(\underline{x}, \underline{y}) &= \int_0^1 f(\underline{x}, \underline{y} | \theta) g(\theta) d\theta \\ &= \int_0^1 \sum_{j=0}^{2n} C_{n,j} \theta^j e^{-T_x - T_y} e^{-T_{xy}\theta} d\theta \\ &= \sum_{j=0}^{2n} C_{n,j} e^{-T_x - T_y} \int_0^1 \theta^j e^{-T_{xy}\theta} d\theta. \end{aligned} \quad (3.2)$$

We know that

$$\int_0^1 \theta^j e^{-T_{xy}\theta} d\theta = \frac{\Gamma(j+1)}{T_{xy}^{j+1}} \left[1 - \sum_{r=0}^j e^{-T_{xy}} \frac{T_{xy}^r}{r!} \right].$$

Therefore,

$$h(\underline{x}, \underline{y}) = \sum_{j=0}^{2n} C_{n,j} e^{-T_x - T_y} \frac{\Gamma(j+1)}{T_{xy}^{j+1}} \left[1 - \sum_{r=0}^j e^{-T_{xy}} \frac{T_{xy}^r}{r!} \right] \tag{3.3}$$

and by using some elementary algebra

$$h(\theta | x, y) = \frac{\sum_{j=0}^{2n} C_{n,j} \theta^j e^{-T_{xy}\theta}}{\sum_{j=0}^{2n} C_{n,j} \frac{\Gamma(j+1)}{T_{xy}^{j+1}} \left[1 - e^{-T_{xy}} \sum_{r=0}^j \frac{T_{xy}^r}{r!} \right]}. \tag{3.4}$$

Thus Bayes estimate is given as

$$\hat{\theta}_B = \frac{\sum_{j=0}^{2n} C_{n,j} \frac{\Gamma(j+2)}{T_{xy}^{j+2}} \left[1 - e^{-T_{xy}} \sum_{r=0}^{j+1} \frac{T_{xy}^r}{r!} \right]}{\sum_{j=0}^{2n} C_{n,j} \frac{\Gamma(j+1)}{T_{xy}^{j+1}} \left[1 - e^{-T_{xy}} \sum_{r=0}^j \frac{T_{xy}^r}{r!} \right]}. \tag{3.5}$$

Case 2): Let $a > 1$ and $b = 1$. By the same way as it is done in the case 1), we get the Bayes estimate

$$\hat{\theta}_B = \frac{\sum_{j=0}^{2n} C_{n,j} \frac{\Gamma(a+j+2)}{T_{xy}^{a+j+2}} \left[1 - e^{-T_{xy}} \sum_{r=0}^{a+j+1} \frac{T_{xy}^r}{r!} \right]}{\sum_{j=0}^{2n} C_{n,j} \frac{\Gamma(a+j+1)}{T_{xy}^{a+j+1}} \left[1 - e^{-T_{xy}} \sum_{r=0}^{a+j} \frac{T_{xy}^r}{r!} \right]}. \tag{3.6}$$

Case 3): Assume that $a \geq 1$ and $b > 1$. Then

$$\begin{aligned} h(\underline{x}, \underline{y}) &= \int_0^1 \sum_{j=0}^{2n} C_{n,j} \theta^j e^{-T_x - T_y} e^{-T_{xy}\theta} \frac{\theta^{a-1} (1-\theta)^{b-1}}{\beta(a,b)} d\theta \\ &= \sum_{j=0}^{2n} C_{n,j} e^{-T_x - T_y} \int_0^1 \frac{\theta^{a+j-1} (1-\theta)^{b-1} e^{-T_{xy}\theta}}{\beta(a,b)} d\theta. \end{aligned} \tag{3.7}$$

Consider

$$\begin{aligned}
 I &= \int_0^1 \frac{\theta^{a+j-1} (1-\theta)^{b-1} e^{-T_{xy}\theta}}{\beta(a,b)} d\theta \\
 &= \int_0^1 \sum_{i=0}^{\infty} (-1)^i \frac{\theta^i T_{xy}^i}{i!} \frac{\theta^{a+j-1} (1-\theta)^{b-1}}{\beta(a,b)} d\theta \\
 &= \sum_{i=0}^{\infty} (-1)^i \frac{T_{xy}^i}{i!} \int_0^1 \frac{\theta^{a+i+j-1} (1-\theta)^{b-1}}{\beta(a,b)} d\theta.
 \end{aligned}$$

Then

$$I = \sum_{i=0}^{\infty} (-1)^i \frac{T_{xy}^i}{i!} \frac{\beta(a+i+j,b)}{\beta(a,b)}. \quad (3.8)$$

Therefore,

$$h(\underline{x}, \underline{y}) = \sum_{j=0}^{2n} C_{n,j} e^{-T_x - T_y} \sum_{i=0}^{\infty} (-1)^i \frac{T_{xy}^i}{i!} \frac{\beta(a+i+j,b)}{\beta(a,b)} \quad (3.9)$$

and posterior probability density function of θ is

$$\begin{aligned}
 h(\theta | \underline{x}, \underline{y}) &= \frac{\sum_{j=0}^{2n} C_{n,j} \theta^{a+j-1} (1-\theta)^{b-1} e^{-T_{xy}\theta}}{\sum_{j=0}^{2n} C_{n,j} \sum_{i=0}^{\infty} (-1)^i \frac{T_{xy}^i}{i!} \beta(a+i+j,b)}.
 \end{aligned} \quad (3.10)$$

Hence Bayes estimate for $a \geq 1$ and $b > 1$ is given as

$$\hat{\theta}_B = \frac{\sum_{j=0}^{2n} \sum_{i=0}^{\infty} C_{n,j} (-1)^i \frac{T_{xy}^i}{i!} \beta(a+i+j+1,b)}{\sum_{j=0}^{2n} \sum_{i=0}^{\infty} C_{n,j} (-1)^i \frac{T_{xy}^i}{i!} \beta(a+i+j,b)}. \quad (3.11)$$

Consider the confluent hypergeometric function (see Abramowitz and Stegun (1972)) as

$${}_1F_1(c; d; x) = \sum_{r=0}^{\infty} \frac{\Gamma(c+r)}{\Gamma(c)} \frac{\Gamma(d)}{\Gamma(d+r)} \frac{x^r}{r!}, \quad (3.12)$$

and

$${}_1F_1(c; d; x) = e^x {}_1F_1(d-c; d; -x). \quad (3.13)$$

By using the above two equations we get

$$\hat{\theta}_B = \frac{\sum_{j=0}^{2n} C_{n,j} \frac{\Gamma(a+j+1)}{\Gamma(a+b+j+1)} {}_1F_1(a+j+1; a+b+j+1; -T_{xy})}{\sum_{j=0}^{2n} C_{n,j} \frac{\Gamma(a+j)}{\Gamma(a+b+j)} {}_1F_1(a+j; a+b+j; -T_{xy})}, \quad (3.14)$$

or

$$\hat{\theta}_B = \frac{\sum_{j=0}^{2n} C_{n,j} \frac{\Gamma(a+j+1)}{\Gamma(a+b+j+1)} {}_1F_1(b; a+b+j+1; T_{xy})}{\sum_{j=0}^{2n} C_{n,j} \frac{\Gamma(a+j)}{\Gamma(a+b+j)} {}_1F_1(b; a+b+j; T_{xy})}. \quad (3.15)$$

4. JOINT DISTRIBUTION OF $(X_1, Y_1, X_2, Y_2, \dots, X_n, Y_n)$ FOR GUMBEL'S TYPE II DISTRIBUTION

$$\begin{aligned} f(\underline{x}, \underline{y}, \theta) &= \prod_{i=1}^n e^{-x_i - y_i} [1 + \alpha(2e^{-x_i} - 1)(2e^{-y_i} - 1)] \\ &= \prod_{i=1}^n [1 + \alpha(2e^{-x_i} - 1)(2e^{-y_i} - 1)] e^{-\sum_{i=1}^n x_i - \sum_{i=1}^n y_i} \\ &= Q e^{-\sum_{i=1}^n x_i - \sum_{i=1}^n y_i}, \end{aligned} \quad (4.1)$$

where

$$Q = \prod_{i=1}^n [1 + \alpha(2e^{-x_i} - 1)(2e^{-y_i} - 1)].$$

Let $D_i = (2e^{-x_i} - 1)(2e^{-y_i} - 1)$. Then

$$Q = \prod_{i=1}^n (1 + \alpha D_i). \quad (4.2)$$

Case 1): Assume that $n = 2$ and $E_{1,1} = D_1$

$$\begin{aligned} Q &= (1 + D_1\alpha)(1 + D_2\alpha) = \prod_{i=1}^2 (1 + D_i\alpha) \\ &= (1 + E_{1,1}\alpha)(1 + D_2\alpha) \\ &= 1 + (E_{1,1} + D_2)\alpha + E_{1,1}D_2\alpha^2 \\ &= 1 + E_{2,1}\alpha + E_{2,2}\alpha^2 \\ &= E_{2,0} + E_{2,1}\alpha + E_{2,2}\alpha^2. \end{aligned} \quad (4.3)$$

Hence,

$$E_{2,0} = 1, \quad E_{2,1} = E_{1,1} + D_2 \quad \text{and} \quad E_{2,2} = E_{1,1}D_2.$$

This can be written as

$$(E_{2,0} \quad E_{2,1} \quad E_{2,2}) \underline{\alpha} = \underline{\alpha}' \begin{pmatrix} 1 & 0 \\ E_{1,1} & 1 \\ 0 & E_{1,1} \end{pmatrix} \begin{pmatrix} 1 \\ D_2 \end{pmatrix}, \quad (4.4)$$

where

$$\underline{\alpha} = \begin{pmatrix} 1 \\ \alpha \\ \alpha^2 \end{pmatrix}.$$

Case 2): Assume that $n = i$. Then

$$Q = \prod_{j=1}^i (1 + \alpha D_j). \quad (4.5)$$

So the matrix is as follows:

$$(E_{i,0} \ E_{i,1} \ E_{i,2} \ E_{i,3} \ \cdots \ E_{i,i}) \underline{\alpha} = \underline{\alpha}' \begin{pmatrix} 1 & 0 \\ E_{i-1,1} & 1 \\ E_{i-1,2} & E_{i-1,1} \\ E_{i-1,3} & E_{i-1,2} \\ \vdots & \vdots \\ E_{i-1,i-2} & E_{i-1,i-3} \\ E_{i-1,i-1} & E_{i-1,i-2} \\ 0 & E_{i-1,i-1} \end{pmatrix} \begin{pmatrix} 1 \\ D_i \end{pmatrix}, \quad (4.6)$$

where

$$\underline{\alpha} = \begin{pmatrix} 1 \\ \alpha \\ \alpha^2 \\ \vdots \\ \alpha^i \end{pmatrix}.$$

Hence for $n > 2$

i) If $k = 1$, then

$$\begin{cases} E_{n,0} = 1, \\ E_{n,1} = E_{n-1,1} + D_n, \\ E_{n-1,1} = 0. \end{cases} \quad (4.7)$$

ii) If $2 \leq k \leq 2n$, then

$$E_{n,k} = E_{n-1,k} + E_{n-1,k-1} D_n. \quad (4.8)$$

Therefore,

$$\begin{aligned} Q &= \prod_{i=1}^n (1 + \alpha D_i) \\ &= E_{n,0} + E_{n,1} \alpha + E_{n,2} \alpha^2 + \cdots + E_{n,n} \alpha^n \\ &= \sum_{j=0}^n E_{n,j} \alpha^j, \end{aligned} \quad (4.9)$$

where E^i 's are given in (4.7) and (4.8).

Hence (4.1) will become

$$f(\underline{x}, \underline{y}, \alpha) = \sum_{j=0}^n E_{n,j} \alpha^j e^{-T_x - T_y}, \quad (4.10)$$

where

$$T_x = \sum_{i=1}^n x_i \quad \text{and} \quad T_y = \sum_{i=1}^n y_i.$$

5. BAYES ESTIMATOR OF α

Let the prior distribution of α be $g(\alpha)$, where

$$g(\alpha) = \frac{1}{2}, \quad -1 \leq \alpha \leq 1. \quad (5.1)$$

The joint distribution of $(\underline{X}, \underline{Y})$ is as follows:

$$\begin{aligned} h(\underline{x}, \underline{y}) &= \int_{-1}^1 \sum_{j=0}^n E_{n,j} \frac{\alpha^j}{2} e^{-T_x - T_y} d\alpha \\ &= \sum_{j=0}^n E_{n,j} e^{-T_x - T_y} \frac{1}{2} \int_{-1}^1 \alpha^j d\alpha \\ &= \sum_{j=0}^n E_{n,j} e^{-T_x - T_y} \left[\frac{1 + (-1)^{j+1}}{2(j+1)} \right]. \end{aligned} \quad (5.2)$$

So the posterior distribution of α is

$$h(\alpha | \underline{x}, \underline{y}) = \frac{\sum_{j=0}^n E_{n,j} \alpha^j}{\sum_{j=0}^n E_{n,j} [1 + (-1)^j] [0.5(j+1)^{-1}]} \quad (5.3)$$

and Bayes estimate of α is

$$\hat{\alpha}_B = \frac{\sum_{j=0}^n E_{n,j} [1 - (-1)^j] (j+2)^{-1}}{\sum_{j=0}^n E_{n,j} [1 + (-1)^j] (j+1)^{-1}}. \quad (5.4)$$

6. CONCLUSION

We therefore conclude that Bayesian methods and consequently Bayesian estimates are advantageous as the difficulty of exponential integrals has been overcome.

Also in order to get an idea of the Bias and the mean square error (*MSE*) of the Bayes estimator of parameters, we have generated a sample of size 5,10,...,50 from the Gumbel's type I and II distributions with $\theta = 0.3, 0.5, 0.7$ and $\alpha = 0.9, -0.6, -0.2, 0.2, 0.6, 0.9$, respectively. We have given Tables based on one thousand independent replication by *R* software.

Table 6.1: Bias and *MSE* of Bayes estimator of parameter of Gumbel's type-I distribution for $a = 1$ and $b = 1$

θ	0.3		0.5		0.7	
n	<i>Bias</i>	<i>MSE</i>	<i>Bias</i>	<i>MSE</i>	<i>Bias</i>	<i>MSE</i>
5	-0.17450	0.04190	-0.00598	0.00975	0.16956	0.03650
10	-0.15501	0.04267	-0.00700	0.01438	0.14935	0.03432
15	-0.13402	0.04122	-0.01302	0.01835	0.13266	0.03439
20	-0.12555	0.04006	-0.02109	0.02151	0.11009	0.03116
25	-0.11380	0.03840	-0.00390	0.02267	0.10296	0.02755
30	-0.09835	0.03682	-0.02129	0.02366	0.08804	0.02587
35	-0.08402	0.03102	-0.01112	0.02513	0.07900	0.02594
40	-0.09230	0.03618	-0.02627	0.02607	0.07251	0.02529
45	-0.08063	0.03104	-0.0276	0.02444	0.06008	0.02236
50	-0.07315	0.03071	-0.01795	0.02425	0.04793	0.02124

Tables 6.1, 6.2 and 6.3 show the Bias and *MSE* of Bayes estimator of parameter of Gumbel's type I distribution for $(a = 1, b = 1)$, $(a = 2, b = 1)$ and $(a = 3, b = 2)$, respectively. We have inserted the *Bias* and *MSE* of Bayes estimator of parameter of Gumbel's type II distribution in Tables 6.4 and 6.5 for $\alpha = -0.9, -0.6, -0.2, 0.2, 0.6, 0.9$, respectively.

Tables are shown that the *MSE* of the Bayes estimators are decreasing when n increases.

Table 6.2: Bias and *MSE* of Bayes estimator of parameter of Gumbel's type-I distribution for $a = 2$ and $b = 1$

θ	0.3		0.5		0.7	
N	<i>Bias</i>	<i>MSE</i>	<i>Bias</i>	<i>MSE</i>	<i>Bias</i>	<i>MSE</i>
5	-0.41492	0.17729	-0.23779	0.05975	-0.04722	0.00487
10	-0.38334	0.15664	-0.21881	0.05498	-0.04905	0.00680
15	-0.36848	0.14836	-0.21211	0.05247	-0.04074	0.00787
20	-0.3316	0.12579	-0.19148	0.04751	-0.04170	0.00825
25	-0.31514	0.11751	-0.18659	0.04610	-0.05074	0.01065
30	-0.28003	0.09782	-0.17731	0.04488	-0.04537	0.01099
35	-0.27451	0.09322	-0.17093	0.04314	-0.04393	0.01061
40	-0.25136	0.08271	-0.16237	0.04252	-0.04288	0.01196
45	-0.23468	0.07401	-0.15578	0.03879	-0.03998	0.01193
50	-0.23228	0.07328	-0.14550	0.03780	-0.04075	0.01214

Table 6.3: Bias and *MSE* of Bayes estimator of parameter of Gumbel's type- I distribution for $a = 3$ and $b = 2$

θ	0.3		0.5		0.7	
n	<i>Bias</i>	<i>MSE</i>	<i>Bias</i>	<i>MSE</i>	<i>Bias</i>	<i>MSE</i>
5	-0.46938	0.22440	-0.09516	0.01234	0.09119	0.01050
10	-0.43803	0.20061	-0.08752	0.01264	0.08559	0.01112
15	-0.41373	0.18084	-0.09162	0.01457	0.08079	0.01198
20	-0.38766	0.16310	-0.07364	0.01307	0.06944	0.01031
25	-0.36912	0.15059	-0.06875	0.01513	0.06633	0.01074
30	-0.34240	0.13340	-0.06901	0.01442	0.06665	0.01198
35	-0.32099	0.12053	-0.06263	0.01456	0.05151	0.01015
40	-0.30599	0.11204	-0.06786	0.01569	0.05547	0.01165
45	-0.28894	0.10036	-0.05606	0.01533	0.05875	0.01212
50	-0.26895	0.09078	-0.06301	0.01607	0.05180	0.01060

Table 6.4: Bias and *MSE* of Bayes estimator of parameter of Gumbel's type II distribution

α	-0.9		-0.6		-0.2	
n	<i>Bias</i>	<i>MSE</i>	<i>Bias</i>	<i>MSE</i>	<i>Bias</i>	<i>MSE</i>
5	0.63864	0.44908	-0.10459	0.05532	-0.02699	0.04467
10	-0.31011	0.15649	-0.26582	0.13489	-0.61354	0.43477
15	-0.03309	0.07903	-0.10339	0.08309	0.18417	0.09955
20	-0.22852	0.12486	-0.18782	0.11395	0.18734	0.10481
25	-0.09170	0.08819	-0.01283	0.08275	-0.07521	0.07771
30	0.70910	0.58203	-0.22949	0.14773	0.20624	0.12064
35	0.29207	0.16089	0.00041	0.09523	0.00200	0.08098
40	-0.09116	0.09320	-0.19369	0.12945	0.04559	0.08006
45	-0.12886	0.10451	-0.28151	0.18432	0.06159	0.09386
50	0.09362	0.10108	-0.30416	0.18846	0.11833	0.09710

Table 6.5: Bias and *MSE* of Bayes estimator of parameter of Gumbel's type II distribution

α	0.2		0.6		0.9	
n	<i>Bias</i>	<i>MSE</i>	<i>Bias</i>	<i>MSE</i>	<i>Bias</i>	<i>MSE</i>
5	0.00076	0.04389	0.13913	0.05798	-0.05058	0.04626
10	0.07437	0.06007	-0.15674	0.08817	0.13969	0.07197
15	0.26964	0.14314	0.11524	0.08851	0.19198	0.10968
20	0.14535	0.10619	0.03908	0.08608	0.24636	0.13957
25	-0.09948	0.09186	-0.23520	0.13678	0.23829	0.13282
30	0.31575	0.18728	0.24708	0.13991	0.05836	0.07198
35	0.04629	0.07587	-0.01303	0.09087	-0.07379	0.08963
40	0.09603	0.09330	-0.00898	0.07683	-0.14136	0.09500
45	0.03082	0.06653	0.08164	0.08379	0.13689	0.09629
50	-0.02609	0.08529	-0.08994	0.09883	0.10402	0.09487

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