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DESIGNS WITH UNEQUAL BLOCK SIZES FOR COMPARING TEST TREATMENTS WITH A CONTROL

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ABSTRACT

In this paper, we have constructed designs with unequal block sizes for comparing test treatments with a control by using the method of cyclic shifts. These designs are also known as Balanced Treatment Incomplete Block Designs $(BTIBDs)$. An important feature of this method of construction is that the properties of a design can easily be obtained from off-diagonal elements of the concurrence matrix without constructing the actual blocks of the design. One representative case for such *BTIBDs* with unequal block sizes $k_1 = 5$ and $k_2 = 4$ for $v = 7$ is presented.

1. INTRODUCTION

Designs with unequal block sizes are likely to occur in practical situations especially when the experiments are large and the blocking of the experimental units is natural (Angelis and Moyssiadis, 1991). The need for using different block sizes in biological experiments was first noted by Pearce (1964). Different methods for the construction of such designs are available in literature. Balanced block designs with unequal replications were considered by Corsten (1962) and John (1964). Some methods for constructing block designs with unequal block sizes were given by Kulshreshtha et al. (1972) and Kageyama (1976, 1981). Kageyama and Tusji (1980a, 1980b) gave some useful results on the characterization of balanced block designs, and Kageyama (1980) gave further results for resolvable balanced block designs.

As remarked by Kageyama (1976), with the advent of high speed computers block designs with unequal block sizes may be particularly useful in large experiments in industry and agriculture. Gupta (1989a) studied E-optimality of block design with varying replicates and unequal block sizes. Iqbal and Jones (1995) constructed variance-balanced designs using cyclic shifts with unequal block sizes. John *et al.* (1999) constructed resolvable designs with blocks of unequal sizes. Williams *et al.* (1999) gave some examples of block designs which are useful in plant and tree breeding trials, when blocks sizes are unequal.

It is well known that the comparison of test treatments with a control treatment is an integral part of genetic and biological research. This situation also occurs in agronomy, chemistry, pathology, physiology and physics, where a new treatment (material) is compared with the other(s). After the introduction of such designs by Hoblyn *et al.* (1954), many authors have extended work on *BTIBDs*. Bechhofer and Tamhane (1981), Majumdar (1996), Majumdar and Notz (1983), Cheng *et al.* (1988), Jacroux (1984, 1989), Hedayat and Majumdar (1984, 1985), Hedayat *et al.* (1988), Stufken (1987), Ting and Notz (1988), Gupta (1989b) and Sinha (1992) considered comparing test treatments with a set of standard (control) treatments for equal blocks sizes. In literature Jacroux (1992) considered the problem of comparing v test treatments with a control using block designs, where b_1 blocks are of size k_1 and b_2 are of size k_2 $(k_1 > k_2)$ and Angelis and Moyssiadis (1991) considered A-optimal designs with unequal sizes for comparing test treatments with a control. In this paper, we have considered the construction of designs for comparing test treatments with a control when block are of unequal sizes using the method of cyclic shifts.

Example 1: Consider the situation in a *BTIBD*, where $v=4$ treatments appear in $b_1 = 8$ and $b_2 = 2$ blocks, each of sizes $k_1 = 4$ and $k_2 = 2$ respectively. The test treatments are replicated $r_1 = 5$ times and are compared with control treatment, which is replicated $r_0 = 16$ times.

0	0	0	0	0	0	0	0	1	2
0	0	0	0	0	0	0	0	3	4
1	2	3	4	1	2	3	4		
2	3	4	1	2	3	4	1		

In the above design, one control treatment is compared with four test treatments (1, 2, 3, 4). We see that the control treatment 0 appears with all the other test treatments within blocks an equal number of times. If we consider the test treatments only, we see that the test treatment 1 appears with test treatments 2 and 4 two times in blocks of size 4 and once with treatment 3 in blocks of size 2. That is, the concurrences of treatment 1 with others treatments are (2, 0, 2) in blocks of size 4 and (0, 1, 0) in blocks of size 2. To obtain variance-balanced designs, we have to consider weighted concurrences (discussed in section 3). Then we will divide the respective concurrences by their block sizes to obtain the weighted concurrences, which are in this case equal to (1 /2). Therefore, by considering this property and that each test treatment appears with the control treatment an equal number of times within each block size, we can construct the required designs.

The organization of this paper is as follows. In Section 2, we have explored the relation between the concurrence matrix and cyclic shifts. The method of cyclic shifts used to construct unequal block sized *BTIBDs* is given in Iqbal and Tahir (2008, 2009). In Section 3, we have presented a representative case for the construction of designs for comparing test treatments with a control when block are of unequal sizes. In Section 4, we gave some concluding remarks.

2. CYCLIC SHIFTS AND PROPERTIES OF BLOCK DESIGNS

The standard setting for a block design is an integer triple setting with parameters (*v*, *b*, *k*) specifying that it is an arrangement of *v* treatments in *b* blocks, each of size *k*, where $k < v$. The conditions for a BIBD are that (i) each treatment has *r* replications, (ii) no treatment appears more than once in any block, and (iii) all unordered pair of treatments appears exactly in λ blocks, where $\lambda = b(k-1)/(r-1)$ is often referred to as the concurrence parameter of a *BIBD*.

Let $D(v,b,k)$ denote the class of all block designs for the setting (v,b,k) which are available in an experiment whose contrasts are estimable. Let $\mathrm{N} = (n_{ij})$; $i = 1,2,\ldots,v$ and $j = 1,2,\ldots,b$ be the $(v \times b)$ treatment-block incidence matrix associated with any design $d \in D(v,b,k)$ whose elements n_{ij} signify the number of units in block *j* allocated to treatment *i*. The matrix, $\mathbf{NN'}$, is referred to as concurrence matrix of design *d*, and its entries, the concurrences parameters are denoted by λ_{ij}. For any equi-replicated block design, $\mathbf{NN'}$, the treatment concurrence with diagonal elements are equal to *r* and the off-diagonal elements are equal to the number of times any pair of treatment occur together within blocks. In a balanced design, the off-diagonal entries of $\mathbf{NN'}$ are all equal to a constant, λ, say, that is, the common replication for a *BIBD* is *r*, and the common pairwise treatment concurrence is λ. The Bose information matrix for estimating treatment contrasts using design *d* is

$$\mathbf{C} = \mathbf{R} - \mathbf{N}\mathbf{K}^{-1}\mathbf{N}',$$

where

$$\mathbf{R} = \begin{pmatrix} r_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & r_v \end{pmatrix} \text{ and } \mathbf{K} = \begin{pmatrix} k_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & k_b \end{pmatrix}.$$

$\mathbf{C}$ is also known as **C**-matrix of the design, which determines the statistical properties of a BIBD. $\mathbf{C}$ is symmetric and non-negative definite, with rank $v-1$.

We will now describe the *method of cyclic shifts* for constructing a design which has $\mathbf{NN'}$ matrix of any pre-specified form. Suppose that we want to construct a design which has a particular pattern of $\mathbf{NN'}$, the entries of $\mathbf{NN'}$ are built up as follows.

Since $\mathbf{NN}'$ is a $v \times v$ matrix, having *v* diagonal elements equal to *r* and next to the diagonal there are $v-1$ off-diagonal elements. Next to these, we have further $v-2$ off-diagonal elements, and so on.

We want to systematically convert some or all of the off-diagonal zeros into positive integers. For $v=b$ and $k=2$, we can convert exactly *v* zeros into 1's. Suppose we convert the $v-1$ zeros next to the main diagonal into 1's and for the *v*th element we convert the last zero of the first row into 1. Then for $r=2$, $k=2$, the concurrence matrix will be

$$\mathbf{NN}' = \begin{pmatrix} 2 & 1 & 0 & 0 & \cdots & 1 \\ 1 & 2 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 2 & 1 & \cdots & 0 \\ 0 & 0 & 1 & 2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & 0 & \cdots & 2 \end{pmatrix}$$

The corresponding design is

$$\begin{bmatrix} 0 & 1 & 2 & 3 & \cdots & v-1 \\ 1 & 2 & 3 & 4 & \cdots & 0 \end{bmatrix},$$

where the columns represent the blocks.

Now, instead of considering the elements next to the main diagonal, let us convert the elements in the next-but-one position to the main diagonal into 1's. Since, we have converted $v-2$ zeros into 1's, so we also have to convert the second last element of the first row and the last element of the second row into 1's. That is

$$\mathbf{NN}' = \begin{pmatrix} 2 & 0 & 1 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 2 & 0 & 1 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 2 & 0 & 1 & \cdots & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 2 \end{pmatrix}.$$

The corresponding design is

$$\begin{bmatrix} 0 & 1 & 2 & \cdots & v-2 & v-1 \\ 2 & 3 & 4 & \cdots & 0 & 1 \end{bmatrix}.$$

The conversion of the elements next to the main diagonal into 1's is named as 'using shift 1' and the conversion of the elements in the next-but-one position is named as 'using shift 2' and so on. In general, for any value of k and $v=b$, we have to convert $kv(k-1)/2$ zeros into ones. So, when we convert the elements in the ith diagonal position next to the main diagonal, we call it 'using cyclic shift i' or 'using shift i', $i=1,2,\ldots,v-1$. However, we do not need to construct the $\mathbf{NN'}$ matrix directly because we can find its off-diagonal elements immediately from the shifts.

For further understanding of this method, we again explain the method of cyclic shifts as follows. Let us start with a set of $v=b$ blocks, each containing one plot. The treatments on these plots are respectively, $0,1,\ldots,v-1$. By using a shift q, say, we mean adding a constant q (mod v) to each of the treatments. This then gives a design with $k=2$. In the previous two designs, we have used shifts $q_1=1$ and $q_2=2$ respectively.

For a design with $b=2v$ blocks, we can use the shifts, say q_1 and q_2, each separately to two sets of v plots. If $q_1=1$ and $q_2=2$, then

$$\mathbf{NN'} = \begin{pmatrix} 4 & 1 & 1 & 0 & 0 & \cdots & 0 & 1 & 1 \\ 1 & 4 & 1 & 1 & 0 & \cdots & 0 & 0 & 1 \\ 1 & 1 & 4 & 1 & 1 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 1 & 4 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 1 & 1 & 4 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 4 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 4 & 1 \\ 1 & 1 & 0 & 0 & 0 & \cdots & 1 & 1 & 4 \end{pmatrix}$$

and the corresponding design will be

$$\begin{bmatrix} 0 & 1 & 2 & \cdots & \mathrm{v}-1 & 0 & 1 & 2 & \cdots & \mathrm{v}-2 & \mathrm{v}-1 \\ 1 & 2 & 3 & \cdots & 0 & 2 & 3 & 4 & \cdots & 0 & 1 \end{bmatrix}.$$

For a design with unequal block sizes k_1 and k_2, (where $k_1 > k_2$), we can use the shifts, say q_1 and q_2, each separately to two sets of v plots. Suppose, if $q_1=1,2$ and $q_2=4$, then

$$\mathbf{NN}' = \begin{pmatrix} 5 & 1 & 1 & 0 & 1 & 0 & \cdots & 0 & 1 & 0 & 1 & 1 \\ & 5 & 1 & 1 & 0 & 1 & \cdots & 0 & 0 & 1 & 0 & 1 \\ & & 5 & 1 & 1 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 \\ & & & 5 & 1 & 1 & \cdots & 1 & 0 & 0 & 0 & 1 \\ & & & & 5 & 1 & \cdots & 0 & 1 & 0 & 0 & 0 \\ & & & & & 5 & \cdots & 1 & 0 & 1 & 0 & 0 \\ & & & & & & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & & & & & & & 5 & 1 & 1 & 0 & 1 \\ & & & & & & & & 5 & 1 & 1 & 0 \\ & & & & & & & & & 5 & 1 & 1 \\ & & & & & & & & & & 5 & 1 \\ & & & & & & & & & & & 5 \end{pmatrix}$$

and the corresponding design will be

$$\begin{bmatrix} 0 & 1 & 2 & \cdots & v-1 & 0 & 1 & 2 & 3 & \cdots & v-1 \\ 1 & 2 & 3 & \cdots & 0 & 4 & 5 & 6 & 7 & \cdots & 3 \\ 3 & 4 & 5 & \cdots & 2 & & & & & & \end{bmatrix}.$$

From the above examples, it is clear that by using certain shifts individually or in combination, one or more sets of b blocks can be constructed. Details of the method of cyclic shifts are given in Iqbal and Tahir (2008).

3. CONSTRUCTION OF BTIBDS WITH UNEQUAL BLOCK SIZES

In this section, we will consider the construction of designs for comparing test treatments with a control, when the blocks are of unequal sizes. Gupta and Jones (1983) gave some methods for constructing equireplicate balanced block designs. Jones *et al.* (1987) extended the method given by Gupta and Jones (1983) by using triangular PBIB designs. In the method described by Gupta and Jones (1983), we take two block designs, say D_1 and D_2. Design D_1 consists of b_1 blocks of size k_1 and design D_2 of b_2 blocks of size k_2. The treatments replication in each of these designs is r_1 and r_2, respectively. Let λ_{11}, λ_{12}, …, λ_{1n} be the number of concurrences between treatment 0 and treatments 1,2,…, n respectively in D_1, where $n = v/2$ if v is even and $n = (v-1)/2$, if v is odd. Similarly, λ_{21}, λ_{22}, …, λ_{2n} are the concurrences between treatment 0 and treatments 1, 2, …, n in D_2. Obviously a balanced design in $(c_1 b_1 + c_2 b_2)$ blocks of sizes k_1 and k_2 can be obtained by adding together c_1 copies of the blocks in D_1 and c_2 copies of blocks in D_2, if $c_1 \lambda_{11}/k_1 + c_2 \lambda_{21}/k_2 = \lambda$ for i=1, 2, …, *n* and λ is some positive integer. The replication in the new design will be $c_1 r_1 + c_2 r_2$.

Using this procedure, we can construct many new designs for different values of (v, k_1, k_2) for comparing test treatments with one control when blocks are of unequal sizes. For illustration one representative case for $v = 7$, $k_1 = 5$ and $k_2 = 4$ is presented in Table 3.1. In these designs C means augment each block of this part of the design with a control treatment once while [1(2)] means add shift [1] to the design after repeating it twice with *v* treatments. For example, the BTIB design discussed in example 1 (section 1) can easily be constructed by using the following sets of shifts [1(2)] 2C+ [2($\frac{1}{2}$)].

Table 3.1: Suggested designs for one representative case $v = 7$, $k_1 = 5$ and $k_2 = 4$.

b_1	b_2	r_0	r_1	Sets of Shifts
7	7	7	8	[122]C+[112]
7	14	7	12	[112]C+[112(2)]
7	21	7	16	[112]C+[112(3)]
⋮	⋮	⋮	⋮	⋮
14	7	14	12	[112(2)]C+[112]
14	14	14	16	[112(2)]C+[112(2)]
14	21	14	20	[112(2)]C+[112(3)]
⋮	⋮	⋮	⋮	⋮
21	7	21	16	[112(3)]C+[112]
21	14	21	20	[112(3)]C+[112(2)]
21	21	21	24	[112(3)]C+[112(3)]
⋮	⋮	⋮	⋮	⋮
7	14	14	11	[112]C+[12]C+[112]
7	21	14	15	[112]C+[12]C+[112(2)]
7	28	14	19	[112]C+[12]C+[112(3)]
⋮	⋮	⋮	⋮	⋮
7	21	21	14	[112]C+[12(2]C+[112]
7	28	21	18	[112]C+[12(2)]C+[112)(2)]
7	35	21	22	[112]C+[12(2)]C+[112(3)]
⋮	⋮	⋮	⋮	⋮
7	28	28	17	[112]C+[12(3)]C+[112]
7	35	28	21	[112]C+[12(3)]C+[112(2)]
7	42	28	25	[112]C+[12(3)]C+[112(3)]
⋮	⋮	⋮	⋮	⋮
14	14	21	15	[112(2)]C+[12]C+[112]
14	21	21	19	[112(2)]C+[12]C+[112(2)]
14	28	21	23	[112(2)]C+[12]C+[112(3)]

⋮	⋮	⋮	⋮	⋮
14	21	28	18	[112(2)]C+[12(2)]C+[112]
14	28	28	22	[112(2)]C+[12(2)]C+[112(2)]
14	35	28	26	[112(2)]C+[12(2)]C+[112(3)]
⋮	⋮	⋮	⋮	⋮
14	28	35	21	[112(2)]C+[12(3)]C+[112]
14	35	35	25	[112(2)]C+[12(3)]C+[112(2)]
14	42	35	29	[112(2)]C+[12(3)]C+[112(3)]

21	14	28	19	[112(3)]C+[12]C+[112]
21	21	28	23	[112(3)]C+[12]C+[112(2)]
21	28	28	27	[112(3)]C+[12]C+[112(3)]
⋮	⋮	⋮	⋮	⋮
21	21	35	22	[112(3)]C+[12(2)]C+[112]
21	28	35	26	[112(3)]C+[12(2)]C+[112(2)]
21	35	35	30	[112(3)]C+[12(2)]C+[112(3)]
⋮	⋮	⋮	⋮	⋮
21	28	42	25	[112(3)]C+[12(3)]C+[112]
21	35	42	29	[112(3)]C+[12(3)]C+[112(2)]
21	42	42	33	[112(3)]C+[12(3)]C+[112(3)]
⋮	⋮	⋮	⋮	⋮
28	7	7	23	[1111+1112+1121]+[112]C+[112]
28	14	7	27	[1111+1112+1121]+[112]C+[112(2)]
28	21	7	31	[1111+1112+1121]+[112]C+[112(3)]
⋮	⋮	⋮	⋮	⋮
35	7	14	27	[1111+1112+1121]+[112(2)]C+[112]
35	14	14	31	[1111+1112+1121]+[112(2)]C+[112(2)]
35	21	14	35	[1111+1112+1121]+[112(2)]C+[112(3)]
⋮	⋮	⋮	⋮	⋮
42	7	21	31	[1111+1112+1121]+[112(3)]C+[112]
42	14	21	35	[1111+1112+1121]+[112(3)]C+[112(2)]
42	21	21	39	[1111+1112+1121]+[112(3)]C+[112(3)]
⋮	⋮	⋮	⋮	⋮

4. REMARKS

In general, the optimality property of block designs requires that the off-diagonal elements of the concurrence matrix should be as close as possible. That actually depends on the number of concurrences between the pair of treatments. Using the method of cyclic shifts, the number of concurrence between any pair of treatments can be obtained from the set of shifts used to construct the BTIBDs. It means that the concurrence matrix or the concurrences among off-diagonal elements play a vital role for a block design or BTIBD to be optimal or not. According to John (1987) "…the information matrix C and concurrence matrix NN' of a cyclic design are circulant..." Further, the circulant matrix can be specified by the elements in the first row, since the other rows are obtained from the first row by a cyclic rotation. This is the main reason for which we can quickly and easily obtain the properties of a design directly without constructing blocks of the design. It has been found on the basis of off-diagonal elements of concurrence matrix, and also due to circulant property that all these cyclic BTIBDs are A-optimal, since they are constructed through cyclic shifts.

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