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SOME NEW METHODS TO REDUCE THE NUMBER OF BLOCKS FOR NEIGHBOUR DESIGNS

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ABSTRACT

Neighbour balanced designs satisfy fairly restrictive combinatorial constraints and therefore, mostly such designs require large number of blocks. In the literature, partially neighbour balanced designs and generalized neighbour designs have been suggested to avoid a large number of blocks. But in this study, some new methods are proposed to overcome this problem. The reduction in number of blocks is made by using some extra treatment(s) whose response will not be measured. Plans for saving experimental material by using one or two extra treatments are also presented for v (number of treatments) ≤ 50.

1. INTRODUCTION

Neighbour balanced designs satisfy fairly restrictive combinatorial constraints and therefore, mostly such designs require large number of blocks. In many field experiments such as agriculture, it is impossible to have as much replication as is needed for neighbour designs. Wilkinson *et al.* (1983) defined a design to be partially neighbour balanced if each experimental treatment has every other treatment as a neighbour, on either side, at most once. In situations where resources are limited partially neighbour balanced designs are preferred. Misra *et al.* (1991) constructed generalized neighbour designs for odd v (number of treatments). Chaure and Misra (1996) constructed generalized neighbour designs for (i) $v = 4t+1$, $k = 3$ in $b = t(4t+1)$ blocks where t is a natural number, (ii) $GN_3 -$designs (the designs where some pairs of treatments appear once, some appear twice while all others appear three times as nearest neighbour) for $v = 4t$, $k = 2t$ in $b = 2(4t-1)$ blocks where $t > 2$, (iii) $GN_2 -$designs (the designs where some pairs of treatments appear once as nearest neighbour while others appear twice) for $v = 4t-1$, $k = 2n+1$, where 'n' is a positive integer and (iii) $GN_2 -$designs for $v = 4t-1$, $k = 2n$. Nutan (2007) constructed families of proper generalized neighbour designs. Kedia and Misra (2008) constructed some series of generalized neighbour designs which are obtained by developing the initial blocks, using Rees' principle. They constructed a series of $GN_2 -$designs for (i) $v = 3t+1$, $k = 4$, (ii) $v = 5t$, $k = 4$, (iii) $v = 6t+1$, $k = 4$, (iv) $v = 7t+1$, $k = 6$. They also constructed a series of $GN_3 -$designs for (i) $v = 5t + 1$, $k = 4$ and (ii) $v = 6t + 1$, $k = 6$. Ahmed *et al.* (2009) presented some series of economical GN_2 −designs.

Organization of the paper is as follows. In Section 2, neighbour designs are presented for $k = 2$ and v even. Neighbour designs with extra treatment are discussed in Section 3 while designs with two extra treatments are discussed in Section 4.

2. NEIGHBOR DESIGNS FOR k = 2 AND v EVEN

If the unused units are not wasted, using block size 2 for even v is the most economical. In general, for block size 2, the minimum number of blocks required for neighbour designs is $v(v-1)/2$. For $v = 2m$, $m = 2,3,\ldots$ the neighbour designs with $r = v-1$ and $\lambda' = 2$, can be generated through the $(m-1)$ initial blocks $(0,1)$, $(0,2)$, $\ldots,(0,m-1)$ with augmented blocks $(0,m)$, $(1,m+1),\ldots,(m-1,2m-1)$. Remaining $(v-1)$ blocks are obtained cyclically mod v from each initial block.

Example 2.1: Neighbour design for $v = 10$ and $k = 2$ can be generated through four initial blocks $(0,1)$, $(0,2)$, $(0,3)$, $(0,4)$ with 5 augmented blocks $(0,5)$, $(1,6)$, $(2,7)$, $(3,8)$, $(4,9)$. Nine blocks can be generated through each initial block cyclically mod 10.

3. NEIGHBOR DESIGNS WITH EXTRA TREATMENT

A new methodology is proposed to reduce the number of blocks, named as neighbour designs with extra treatment(s). In this methodology, a neighbour design for $v-1$ treatments is obtained through a design constructed for v treatments and then v-th treatment is excluded from the analysis.

Example 3.1: A neighbour design for $v = 8$, $k = 3$ required 56 blocks while a neighbour design for $v = 9$, $k = 3$ can be constructed in $b = 12$. Considering $9-$th treatment (labelled as 8) as extra treatment, the required design is: $(0,2,3)$, $(1,3,4)$, $(2,4,5)$, $(3,5,6)$, $(4,6,7)$, $(5,7,0)$, $(6,0,1)$, $(7,1,2)$, (0,4,**8**), (1, 5, **8**), (2,6,**8**), (3,7,**8**). Proposed design with extra treatment saves 78.57 % experimental material. For analysis, it will be a neighbour design for $v = 8$ with $k_1 = 3$ and $k_2 = 2$.

Theorem 3.1: If v is an even number, $v > k$, d_1 is common divisor of $v(v-1)$ and k, d_2 is common divisor of $v(v+1)/2$ and k then neighbour designs with extra treatment will reduce the experimental material at least $[1-\{d_1(v+1)/2d_2(v-1)\}]100$ percent if: (i) $v > (2d_2 + d_1)/(2d_2 - d_1)$ and (ii) $2d_2 > d_1$.

Proof: If v is an even number, $v > k$, d_1 is common divisor of $v(v-1)$ and k, d_2 is common divisor of $v(v+1)$ and k then neighbour design requires at least $b = v(v+1)/d_1$ and our proposed design requires $b = v(v+1)/2d_2$. Hence, our proposed design reduces at least $[v(v-1)/d_1 - v(v-1)/2d_2]$ blocks which saves at least $[1-\{d_1(v+1)/2d_2(v-1)\}]\,100$ percent materials.

Corollary 3.1: If v is even and k is relatively prime to $v(v+1)$ and $v(v+1)/2$ then neighbour designs with extra treatment will reduce the experimental material at least 35.7 %.

Proof: If v is even and k is relatively prime to $v(v-1)$ and $v(v+1)/2$, neighbour design requires at least $v(v-1)$ blocks and our proposed design requires at least $v(v+1)/2$ blocks.

Saved material $= v(v-1) - v(v+1)/2$

$$= [100\{v(v-1) - v(v+1)/2\}/v(v-1)]\%$$

$$= 50[1-(2/(v-1))]\%.$$

The smallest v which satisfied the above conditions is 8. For $v = 8$, neighbour designs with extra treatment will reduce the experimental material 35.7 %.

Corollary 3.2: If v is even and k is relatively prime to $v(v-1)$, d is common divisor of k and $v(v+1)$, where $d > 1$, neighbour designs with extra treatment will reduce experimental material at least 50%.

Proof: If $v(>2)$ is even and k is relatively prime to $v(v-1)/2$ then neighbour design required at least $v(v-1)$ blocks. If d is common divisor of k and $v(v+1)$ then $v(v+1)/2$ blocks are required for $v+1$.

Saved material $= v(v-1) - v(v+1)/2d$

$$= [100\{v(v-1) - v(v+1)/2d\}/v(v-1)]\%.$$

Smallest value of d is 2 for which minimum saved material

$$= [25(3v-5)/(v-1)]\%.$$

Since $[(3v-5)/(v-1)] > 2$, for $v > 3$, hence proved that neighbour designs with extra treatment will reduce experimental material at least 50%.

Theorem 3.2: If v is an odd number, $v > k$, d_1 is common divisor of $v(v-1)/2$ and k, d_2 is common divisor of $v(v+1)$ and k then neighbour designs with extra treatment will reduce at least

$100[1-\{2d_1(v+1)d_2(v-1)\}]\%$ experimental material if: (i) $v>(d_2+2d_1)/(d_2-2d_1)$ and (ii) $d_2>2d_1$.

Proof: If v is an odd number, $v>k$, d_1 is common divisor of $v(v-1)/2$ and k, d_2 is common divisor of $v(v+1)$ and k then neighbour design requires at least $b=v(v-1)/2d_1$ and our proposed design requires at least $b=v(v+1)/d_2$. So our proposed design reduces at least $[v(v-1)/2d_1-v(v+1)/d_2]$ blocks which saves atleast $100[1-\{2d_1(v+1)/d_2(v-1)\}]\%$ material.

Theorem 3.3: For complete block neighbour designs when v is even and $v>3$ then our proposed neighbour design with extra treatment will reduce $[50(v-3)/(v-1)]\%$ experimental material.

Proof: We have always a complete block neighbour design for odd v in $(v-1)/2$ blocks. For even v at least $(v-1)$ blocks are required for complete block neighbour design, it means for v even, neighbour design with extra treatment will require $b=v/2$ blocks.

$$\text{Saved material } =[100\{v(v-1)-v(v+1)/2\}/v(v-1)]\%$$

$$=50(v-3)/(v-1))]\% \text{ for } v>3.$$

Plans to construct neighbour designs using extra treatment are presented in Appendix A which saves at least 70% experimental material for $8\le v\le 50$ and $3\le k\le 20$.

4. NEIGHBOUR DESIGNS WITH TWO EXTRA TREATMENTS

Here, a neighbour design for $v-2$ treatments is obtained through a design constructed for v treatments and then $v-$th and $(v-1)-$th treatments are excluded from the analysis.

Example 4.1: A neighbour design for $v=23$, $k=3$ can be constructed in $b=253$, while such design for $v=25$, $k=3$ can be constructed in 100 blocks. Our proposed design saves 60.5% experimental material by considering $24-$th and $25-$th treatments as extra treatments. For analysis, it will be a neighbour design for $v=23$ with $k_1=3$ and $k_2=2$.

Theorem 4.4: If v is even and k is relatively prime to $v(v-1)$ and $v(v+1)/2$ but not to $(v+2)(v+1)$ then neighbour designs with two extra treatments will reduce $[1-(v+2)(v+1)/dv(v-1)100$percent experimental material.

Proof: If v is even and k is relatively prime to $v(v-1)$ and $v(v+1)/2$ then at least $v(v-1)$ blocks are required for neighbour designs. Let d , $(d>2)$ be common divisor of k and $(v+2)(v+1)$ then $(v+2)(v+1)/d$ blocks are required. Hence, our proposed design with two extra treatments will reduce $[v(v-1)-(v+2)(v+1)/d]$ blocks.

Corollary 4.1: If $v=4t$, $k=m+1$, $m=2t+1$, where $t>1$ then a neighbour design can always be constructed through our proposed design with two extra treatments, which saves $[1-2(v+1)/v(v-1)]100\%$ material.

Proof: If $v=4t$, $k=m+1$, $m=2t+1$, where $s>1$ then $v+2=2k$. In this situation neighbour design for v treatments will require $v(v-1)$ blocks while for $v+2$, it will take $2(v+1)$ blocks.

Plans to construct neighbour designs using two extra treatments are presented in Appendix *B* which saves at least 70% experimental material for $15\leq v\leq 50$ and $4\leq k\leq 20$.

APPENDIX A

Plans using extra treatment for $8 \le v \le 50$ and $3 \le k \le 20$.

Original designs				Designs with extra treatment					
v	k	b	Units	v	k	b	Units	Saved Units	Reduction %
8	3	56	168	9	3	12	36	132	78.6
14	3	182	546	15	3	35	105	441	80.8
20	3	380	1140	21	3	70	210	930	81.6
26	3	650	1950	27	3	117	351	1599	82.0
32	3	992	2976	33	3	176	528	2448	82.3
38	3	1406	4218	39	3	247	741	3477	82.4
44	3	1892	5676	45	3	330	990	4686	82.6
14	5	182	910	15	5	21	105	805	88.46
24	5	552	2760	25	5	60	300	2460	89.1
34	5	1122	5610	35	5	119	595	5015	89.4
44	5	1892	9460	45	5	198	990	8470	89.5
8	6	28	168	9	6	6	36	132	78.6
20	6	190	1140	21	6	35	210	930	81.6
32	6	496	2976	33	6	88	528	2448	82.3
38	6	703	4248	39	6	147	882	3366	79.2
44	6	946	5676	45	6	165	990	4686	82.6
20	7	380	2660	21	7	30	210	2450	92.1
34	7	1122	7854	35	7	85	595	7259	92.4
41	7	820	5740	42	7	246	1722	4018	70.0
48	7	2256	15792	49	7	168	1176	14616	92.6
15	8	105	840	16	8	30	240	600	71.4
23	8	253	2024	24	8	69	552	1472	72.7
47	8	1081	8648	48	8	282	2256	6392	73.9
14	9	182	1638	15	9	35	315	1328	80.8
17	9	136	1224	18	9	34	306	918	75.0
20	9	380	3420	21	9	70	630	2790	81.6
26	9	650	5850	27	9	39	351	5499	94.0
32	9	992	8928	33	9	176	1584	7344	82.3
35	9	595	5355	36	9	140	1260	4095	76.5
38	9	1406	12654	39	9	247	2223	10431	82.4
44	9	1892	17028	45	9	110	990	16038	94.2
50	9	2450	22050	51	9	425	3825	18225	82.7
14	10	91	910	15	10	21	210	700	76.9
19	10	171	1710	20	10	38	380	1330	77.8
24	10	276	2760	25	10	30	300	2460	89.1

v	k	b	Units
34	10	561	5610
39	10	741	7410
44	10	946	9460
21	11	210	2310
32	11	992	10912
43	11	903	9933
23	12	253	3036
29	12	203	2436
35	12	595	7140
47	12	1081	12972
25	13	300	3900
38	13	1406	18278
51	13	1275	16575
20	14	190	2660
27	14	351	4914
34	14	561	7854
44	14	946	13244
48	14	1128	15792
20	15	75	1140
24	15	184	2760
29	15	406	6090
32	15	992	14880
38	15	1406	21090
44	15	1892	28380
23	16	253	4048
31	16	465	7440
39	16	741	11856
47	16	1081	17296
33	17	528	8976
50	17	2450	41650
26	18	325	5850
32	18	496	8928
35	18	595	10710
44	18	946	17028
37	19	666	12654
24	20	138	2760
34	20	561	11220
39	20	741	14820
44	20	473	9460

v	k	b	Units	Saved Units	Reduction %
35	10	119	1190	4420	78.8
40	10	156	1560	5850	78.9
45	10	99	990	8470	89.5
22	11	42	462	1848	80.0
33	11	48	528	10384	95.16
44	11	172	1892	8041	81.0
24	12	46	552	2484	81.8
30	12	145	1740	696	82.6
36	12	105	1260	5880	82.4
48	12	188	2256	10716	82.6
26	13	50	650	3250	83.3
39	13	57	741	17537	95.9
52	13	204	2652	13923	84.0
21	14	15	210	2450	92.1
28	14	54	756	4158	84.6
35	14	85	1190	6664	84.8
45	14	495	6930	6314	91.1
49	14	84	1008	14784	93.6
21	15	14	210	930	81.6
25	15	20	300	2460	89.1
30	15	58	870	5220	85.7
33	15	176	2640	12240	82.3
39	15	247	3705	17385	82.4
45	15	66	990	27390	96.5
24	16	69	1104	2944	72.7
32	16	62	992	6448	86.7
40	16	195	3120	8736	73.7
48	16	141	2256	15040	87.0
34	17	66	1122	7854	87.5
51	17	75	1275	40375	96.9
27	18	39	702	5148	88.0
33	18	88	1584	7344	82.3
36	18	70	1260	9450	88.2
45	18	55	990	16038	94.2
38	19	74	1406	11248	88.9
25	20	15	300	2460	89.1
35	20	119	2380	8840	78.8
40	20	78	1560	13260	89.5
45	20	99	1980	7480	79.1

APPENDIX B

Plans using two extra treatments for $15 \le v \le 50$ and $4 \le k \le 20$.

Original designs

v	k	b	Units
15	4	105	420
23	4	253	1012
31	4	465	1860
39	4	741	2964
47	4	1081	4324
13	5	78	390
18	5	306	1530
19	5	171	855
23	5	253	1265
28	5	756	3780
33	5	528	2640
38	5	1406	7030
39	5	741	3705
43	5	903	4515
48	5	2256	11280
49	5	1176	5880
11	6	55	330
23	6	253	1518
35	6	595	3570
47	6	1081	6486
12	7	132	924
13	7	78	546
19	7	171	1197
26	7	650	4550
27	7	351	2457
40	7	1560	10920
41	7	820	5740
47	7	1081	7567
15	8	105	840
22	8	231	1848
31	8	465	3720
38	8	703	5624
46	8	1035	8280
47	8	1081	8648
17	9	136	1224
23	9	506	4554
29	9	812	7308
35	9	595	5355
18	10	153	1530
19	10	171	1710
23	10	253	2530
28	10	378	3780
38	10	703	7030
39	10	741	7410
43	10	903	9030

Designs with two extra treatments

v	k	b	Units	Saved Units	Reduction %
17	4	34	136	316	75.2
25	4	75	300	712	70.4
33	4	132	528	1332	71.6
41	4	205	820	2144	72.3
49	4	294	1176	3148	72.8
15	5	21	105	285	73.1
20	5	76	380	1150	75.2
21	5	42	210	645	75.4
25	5	60	300	965	76.3
30	5	174	870	2910	77.0
35	5	119	595	2045	77.5
40	5	312	1560	5470	77.8
41	5	164	820	2885	77.9
45	5	198	990	3625	78.1
50	5	490	2450	8830	78.3
51	5	255	1275	4605	78.3
13	6	13	78	252	76.4
25	6	50	300	1218	80.2
37	6	111	666	2904	81.3
49	6	196	1176	5310	81.9
14	7	26	182	742	80.3
15	7	15	105	441	80.8
21	7	30	210	987	82.5
28	7	108	756	3794	83.4
29	7	58	406	2051	83.5
42	7	246	1722	9198	84.2
43	7	129	903	4837	84.3
49	7	168	1176	6391	84.5
17	8	17	136	704	83.8
24	8	69	552	1296	70.1
33	8	66	528	3192	85.8
40	8	195	1560	4064	72.3
48	8	282	2256	6024	72.8
49	8	147	1176	7472	86.4
19	9	19	171	1053	86.0
25	9	100	900	3654	80.2
31	9	155	1395	5913	80.9
37	9	74	666	4689	87.6
20	10	38	380	1150	75.2
21	10	21	210	1500	87.7
25	10	30	300	2230	88.1
30	10	87	870	2910	77.0
40	10	156	1560	5470	77.8
41	10	82	820	6590	88.9
45	10	99	990	8040	89.0

v	*k*	*b*	Units
48	10	1128	11280
20	11	380	4180
21	11	210	2310
31	11	465	5115
42	11	1722	18942
43	11	903	9933
14	12	91	1092
23	12	253	3036
26	12	325	3900
38	12	703	8436
47	12	1081	12972
24	13	552	7176
25	13	300	3900
37	13	666	8658
50	13	2450	31850
19	14	171	2394
26	14	325	4550
27	14	351	4914
47	14	1081	15134
19	15	57	855
23	15	506	7590
28	15	252	3780
29	15	406	6090
38	15	1406	21090
43	15	301	4515
22	16	231	3696
30	16	435	6960
31	16	465	7440
38	16	703	11248
46	16	1035	16560
32	17	992	16864
33	17	528	8976
49	17	1176	19992
23	18	253	4554
35	18	595	10710
43	18	301	5418
47	18	1081	19458
36	19	1260	23940
37	19	666	12654
23	20	253	5060
34	20	561	11220
39	20	741	14820
43	20	903	18060
47	20	1081	21620

v	*k*	*b*	Units	Saved Units	Reduction %
50	10	245	2450	8830	78.3
22	11	42	462	3718	88.9
23	11	23	253	2057	89.0
33	11	48	528	4587	89.7
44	11	172	1892	17050	90.0
45	11	90	990	8943	90.0
16	12	20	240	852	78.0
25	12	25	300	2736	90.1
28	12	63	756	3144	80.6
40	12	130	1560	6876	81.5
49	12	98	1176	11796	90.9
26	13	50	650	6526	90.9
27	13	27	351	3549	91.0
39	13	57	741	7917	91.4
52	13	204	2652	29198	91.7
21	14	15	210	2184	91.2
28	14	54	756	3794	83.4
29	14	29	406	4508	91.7
49	14	84	1008	14126	93.3
21	15	14	210	645	75.4
25	15	20	300	7290	96.0
30	15	58	870	2910	77.0
31	15	31	465	5625	92.4
40	15	104	1560	19530	92.6
45	15	66	990	3525	78.1
24	16	69	1104	2592	70.1
32	16	62	992	5968	85.7
33	16	33	528	6912	92.9
40	16	195	3120	8128	72.3
48	16	141	2256	14304	86.4
34	17	66	1122	15742	93.3
35	17	35	595	8381	93.4
51	17	75	1275	18717	93.6
25	18	50	900	3654	80.2
37	18	37	666	10044	93.8
45	18	55	990	4428	81.7
49	18	196	3528	15930	81.9
38	19	74	1406	22534	94.1
39	19	39	741	11913	94.1
25	20	15	300	4760	94.1
36	20	63	1260	9960	88.8
41	20	41	820	14000	94.5
45	20	99	1980	16080	89.0
49	20	294	5880	15740	72.8

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