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THE STOCHASTIC MODELING AND ANALYSIS OF MILK POWDER MAKING SYSTEM IN A DAIRY PLANT

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ABSTRACT

This paper is concerned with modeling and analysis of a milk powder making system in a dairy plant consisting of a number of subsystems of varying nature. Taking constant failure rate and general repair time distribution for each subsystem, the system is analyzed by using regenerative point technique. A common cause failure is also considered in system modeling. The expressions for several system characteristics such as reliability, $MTSF$, steady state availability, busy period and expected profit, which are useful to system managers, engineers, training supervisors and reliability analysts, are obtained. The $MTSF$ and profit function have also been studied through graphs in respect of various parameters in a particular case when repair time distributions are taken as Erlangian.

1. INTRODUCTION

Milk powder making system in a dairy plant is of complex type, repairable engineering system, involving high risk which consists of 4 subsystems namely Milk condense unit (H), Air heaters (A_1, A_2), Dryer (D), Exhauster (E) working in a series network. Milk condense unit has one operative and one standby unit. Although, a lot of work have been done in the field of reliability but most of it is concerned with hypothetical models. Very few authors Sharma and Panigrahi (1990), Kumar (1991), Singh *et al.* (1993), Kumar *et al.* (1996), Singh and Nair (1997), Arora and Kumar (2000), Singh and Singh (2002) have studied the industrial system models with real existing situation.

For the purpose of analyzing real existing (industrial) system, a model of milk powder making system in a dairy plant is developed for its stochastic analysis. Such type of system is running near Meerut managed by Parag Industry. The working of different subsystems of milk powder making system is described as follows:

i) Milk condense unit (H) **-** The milk is first stored into the condense unit and then it is condensed by boiling through heat steam which is provided by steam chamber. The steam chamber has two similar steam making units, in which one is operative and the other is in standby. Heat steam provided by one unit is sufficient for the successful functioning of the milk condense unit.

ii) Air heaters (A_1, A_2) **–** To convert condense milk into powder the hot air is needed which is provided by air heaters. The number of air heaters depends upon plant capacity which functions independently. Here considered plant needs two air heaters, which provide the prefixed temperature hot air. The temperature is prefixed according to the type of milk used such as

a) For sweet condense milk– temp. $165^o C - 170^o C$

b) For SMP (separata) milk – temp. $185^o C - 190^o C$

c) For ordinary condense milk – temp. $170^o C - 175^o C$.

The functioning of both air heaters is necessary to run the plant. Here produced fixed temperature hot air is supplied to the dryer.

iii) Dryer (D) – This is the most important unit of the system. In dryer the condense milk is sprayed by automizer and prefixed temperature hot air runs around the dryer. The hot air dries the milk and converts it into powder. Dryer also has some automatic hammers which knock the dryer's output unit in the form of powder easily. Further, this powder is cooled up by an automatic cooler.

iv) Exhauster (E) **–** The milk powder obtained from the dryer passes through the exhauster which separates the low and good quality milk powder with the help of some special type of fens. The low quality powder (light powder) is again mixed with the milk for reprocessing to obtain the milk powder of good quality. If exhauster is failed, the system produces low quality powder which is not required. In this situation, the processing is stopped and the system enters into the failed state.

2. ASSUMPTIONS

i) Failures and repairs are stochastically independent.

ii) A single repair facility is available to repair a failed subsystem and the priority is being given to air heaters, dryer and exhauster over the milk condense subsystem (see states S_6 to S_9 of transition diagram, fig. 1).

iii) A repaired unit is as good as new and is immediately reconnected to the system

iv) To put the standby unit into operation a switching and sensing device is used which is instantaneous and perfect whenever required.

v) A large percentage of failures in system occur due to common cause failure. Common cause failure is defined as any instance where multiple units or components fail due to single cause. A common cause failure may occur due to voltage fluctuations, temperature, fire, operational and maintenance error, design deficiency etc.

vi) All the failure time distributions are taken as negative exponential whereas repair time distributions are taken as general.

vii) System/unit fails either due to its normal failure or due to common cause failure.

viii) If during the repair of the condense unit, air heaters, dryer or exhauster fails, then repair of condense unit is stopped and the later failed unit is taken for repair. The repair of condense unit is of pre-emptive repeat type i.e. the time already spent in the repair of condense unit goes waste.

3. NOTATIONS AND STATES OF THE SYSTEM

E : Set of regenerative states $\cong \{S_0 \text{ to } S_9, S_{11}\}$

$\overline{E}$: Set of non-generative states $\cong \{S_{10}\}$

E_0 : Initial state of the system i.e. the state at time $t = 0$

$q_{ij}(.), Q_{ij}(.)$: pdf and cdf of one step or direct transition time from state $S_i \in E$ to $S_j \in E$

p_{ij} : Steady state transition probability from state S_i to S_j, such that

$$p_{ij} = Q_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(u)\,du$$

$q_{ij}^{(l)}$, $Q_{ij}^{(l)}$: pdf and cdf of transition from state $S_i \in E$ to $S_j \in E$ via state $S_1 \in \overline{E}$

$p_{ij}^{(1)}$: Steady state transition probability from state $S_i \in E$ to $S_j \in E$ via state $S_1 \in \overline{E}$, such that

$$p_{ij}^{(1)} = Q_{ij}^{(1)}(\infty) = \int_0^\infty q_{ij}^{(1)}(u)\,du$$

$Z_i(t)$: Probability that the system sojourns in state S_i upto time t

ψ_i : Mean sojourn in state S_i

α_i : Constant failure rates of the units $H / A_1 / A_2 / D / E$ respectively, for $i = 1, 2, 3, 4, 5$

α_c : Common cause failure rate of the system when it is in either state S_0 or S_1

$g_i(.), G_i(.)$: pdf and cdf of repair time of the units $H / A_1 / A_2 / D / E$ respectively, for $i = 1, 2, 3, 4, 5$

$g_c(.), G_c(.)$: pdf and cdf of repair time of the system in failed state S_{11} due to common cause failure

$*, \sim, s$: Symbols and dummy variable used for Laplace and Laplace-Stieltjes transforms e.g.

$$q_{ij}^*(s) = \int_0^\infty e^{-st} q_{ij}(t)\, dt$$

$$\tilde{q}_{ij}(s) = \int_0^\infty e^{-st} dQ_{ij}(t)$$

Symbols used for states of the system

$H_g / H_o / H_s / H_r / H_{wr}$: Unit H is good / operative / standby / under repair / waiting for repair

$A_{1g} / A_{1o} / A_{1r}$: Unit A_1 is good / operative / under repair

$A_{2g} / A_{2o} / A_{2r}$: Unit A_2 is good / operative / under repair

$D_g / D_o / D_r$: Unit D is good / operative / under repair

$E_g / E_o / E_r$: Unit E is good / operative / under repair

Using these symbols the various states of the system model are shown in transition diagram, where the states S_0 and S_1 are up states and the states S_2 to S_{11} are failed states.

4. TRANSITION PROBABILITIES AND SOJOURN TIMES

$$p_{01} = \alpha_1 \int_0^\infty \exp\left\{-\left(\sum_{i=1}^{5} \alpha_i + \alpha_c\right)\right\} du = \alpha_1 / \left(\sum_{i=1}^{5} \alpha_i + \alpha_c\right)$$

$$p_{02} = \alpha_4 / \left(\sum_{i=1}^{5} \alpha_i + \alpha_c\right),\ p_{03} = \alpha_5 / \left(\sum_{i=1}^{5} \alpha_i + \alpha_c\right),\ p_{04} = \alpha_3 / \left(\sum_{i=1}^{5} \alpha_i + \alpha_c\right),$$

$$p_{05} = \alpha_2 / \left(\sum_{i=1}^{5} \alpha_i + \alpha_c\right),\ p_{0,11} = \alpha_c / \left(\sum_{i=1}^{5} \alpha_i + \alpha_c\right),\ p_{10} = \tilde{G}_1 / \left(\sum_{i=1}^{5} \alpha_i + \alpha_c\right),$$

$$p_{16} = \left\{\alpha_4 / \left(\sum_{i=1}^{5} \alpha_i + \alpha_c\right)\right\}\left\{1 - \tilde{G}_1\left(\sum_{i=1}^{5} \alpha_i + \alpha_c\right)\right\},$$

$$p_{17} = \left\{\alpha_5 / \left(\sum_{i=1}^{5} \alpha_i + \alpha_c\right)\right\}\left\{1 - \tilde{G}_1\left(\sum_{i=1}^{5} \alpha_i + \alpha_c\right)\right\},$$

$$p_{18} = \left\{\alpha_2 / \left(\sum_{i=1}^{5}\alpha_i + \alpha_c\right)\right\}\left\{1 - \tilde{G}_1\left(\sum_{i=1}^{5}\alpha_i + \alpha_c\right)\right\}$$

$$p_{19} = \left\{\alpha_3 / \left(\sum_{i=1}^{5}\alpha_i + \alpha_c\right)\right\}\left\{1 - \tilde{G}_1\left(\sum_{i=1}^{5}\alpha_i + \alpha_c\right)\right\},$$

$$p_{1,11} = \left\{\alpha_c / \left(\sum_{i=1}^{5}\alpha_i + \alpha_c\right)\right\}\left\{1 - \tilde{G}_1\left(\sum_{i=1}^{5}\alpha_i + \alpha_c\right)\right\},$$

$$p_{11}^{(10)} = \int_0^\infty \alpha_1 \exp\left\{-\left(\sum_{i=1}^{5}\alpha_i + \alpha_1\right)u\right\}\tilde{G}_1(u)\,du\int_u^\infty \frac{1}{G_1(u)}\,dG_1(v)$$

$$= \left\{\alpha_1 / \left(\sum_{i=1}^{5}\alpha_i + \alpha_1\right)\right\}\int_0^\infty\left[1 - \exp\left\{-\left(\sum_{i=1}^{5}\alpha_i + \alpha_1\right)v\right\}\right]dG_1(v)$$

$$= \left\{\alpha_1 / \left(\sum_{i=1}^{5}\alpha_i + \alpha_c\right)\right\}\left\{1 - \tilde{G}_1\left(\sum_{i=1}^{5}\alpha_i + \alpha_c\right)\right\}$$

$$p_{20} = p_{30} = p_{40} = p_{50} = 1$$

$$p_{61} = p_{71} = p_{81} = p_{91} = 1$$

It is obvious that,

$$p_{01} + p_{02} + p_{03} + p_{04} + p_{05} + p_{c,11} = 1$$

$$p_{10} + p_{16} + p_{17} + p_{18} + p_{19} + p_{1,1}^{(10)} + p_{1,11} = 1$$

Let X_i denote the sojourn time in state S_i, then the mean sojourn time in state S_i is given by

$$\Psi_i = \int_0^\infty \Pr[X > t]\,dt$$

so that,

$$\Psi_0 = \int_0^\infty \exp\left\{-\left(\sum_{i=1}^{5}\alpha_i + \alpha_c\right)t\right\}dt = 1/\left(\sum_{i=1}^{5}\alpha_i + \alpha_c\right),$$

$$\psi_1 = \left\{1 - \bar{G}_1\left(\sum_{i=1}^{5}\alpha_1 + \alpha_c\right)\right\}/\left(\sum_{\mathrm{i}=1}^{5}\alpha_{\mathrm{i}} + \alpha_c\right),$$

$$\Psi_2 = \int_0^\infty \overline{G}_4(t)\,dt = m_1 \text{(say)}, \qquad \Psi_3 = \int_0^\infty \overline{G}_5(t)\,dt = m_2,$$

$$\Psi_4 = \int_0^\infty \overline{G}_3(t)\,dt = m_3, \; \Psi_5 = \int_0^\infty \overline{G}_2(t)\,dt = m_4, \; \Psi_6 = \int_0^\infty \overline{G}_4(t)\,dt = m_4,$$

$$\Psi_7 = \int_0^\infty \overline{G}_5(t)\,dt = m_5, \; \Psi_8 = \int_0^\infty \overline{G}_2(t)\,dt = m_2, \; \Psi_9 = \int_0^\infty \overline{G}_3(t)\,dt = m_3,$$

$$\Psi_{10} = \int_0^\infty \overline{G}_1(t)\,dt = m_1, \quad \Psi_c = \int_0^\infty \overline{G}_c(t)\,dt = m_c.$$

5. RELIABILITY AND *MTSF*

Let the random variable T_i be the time to system failure when the system starts its operation from state $S_i \in E$ then the reliability of the system is given by

$$R_i(t) = \Pr[T_i > t].$$

To determine $R_i(t)$, we assume the failed states $(S_3$ to $S_{11})$ of the system as absorbing. Using simple probabilistic arguments, one can easily develop the recurrence relations among $R_i(t)$, $i = 0,1$. Taking Laplace transforms of the relations and simplifying the resulting set of algebraic equations for $R_0^*(s)$, we get

$$R_0^*(s) = \frac{Z_0^*(s) + q_{10}^*(s)\, Z_1^*(s)}{1 - q_{01}^*(\mathrm{s})\, \mathrm{q}_{10}^*(\mathrm{s})}, \tag{5.1}$$

where

$$Z_0^*(s) \text{ and } Z_1^*(s) \text{ are } LT \text{ of } Z_0(t) = \exp\left\{ -\left(\sum_{i=1}^{5} \alpha_i + \alpha_c \right) t \right\}$$

and

$$Z_1(t) = \exp\left\{ -\left(\sum_{i=1}^{5} \alpha_i + \alpha_c \right) t \right\} \overline{G}_1(t).$$

Taking inverse Laplace transform of (5.1) we can get the reliability of the system. The $MTSF$ is given by

$$E(T_0) = \lim_{s \to 0} R_0^*(s) = \frac{\Psi_0 + p_{01}\Psi_1}{1 - p_{01}\, p_{10}}. \tag{5.2}$$

6. AVAILABILITY ANALYSIS

Let us define $A_i(t)$ as the probability that the system is up at epoch t, when it is initially started from state $S_i \in E$. Using the definition of $A_i(t)$ and probabilistic concept, the recurrence relations among $A_i(t)$ $(i = 1, 2, \cdots, 9, c)$ can easily be developed. Using the technique of LT, the value of $A_0(t)$ in terms of its LT is as follows:

$$A_0^*(s) = N_2(s) / D_2(s), \tag{6.1}$$

where

$$N_2(s) = Z_0^*(1 - q_{16}^* q_{61}^* - q_{17}^* q_{71}^* - q_{18}^* q_{81}^* - q_{19}^* q_{91}^* - q_{11}^{(10)*}) + q_{01}^* Z_1^*$$

$$D_2(s) = (1 - q_{02}^* q_{20}^* - q_{03}^* q_{30}^* - q_{04}^* q_{40}^* - q_{05}^* q_{50}^* - q_{0c}^* q_{c0}^*)$$
$$(1 - q_{16}^* q_{61}^* - q_{17}^* q_{71}^* - q_{18}^* q_{81}^* - q_{19}^* q_{91}^* - q_{11}^{(10)*}$$
$$- q_{01}^* q_{10}^* - q_{01}^* q_{1c}^* q_{c0}^*) \tag{6.2}$$

For brevity the argument $'s'$ has been omitted from $q_{ij}^*(s)$ and $Z_i^*(s)$

The steady state availability of the system is given by

$$A_0 = \lim_{t \to \infty} A_0(t) = \lim_{s \to 0} sA_0^*(s) = N_2(0) / D_2'(0), \tag{6.3}$$

where

$$N_2(0) = \Psi_0 (p_{10} + p_{1c}) + \Psi_1\, p_{01},$$

$$D_2'(0) = \Psi_0\, (p_{10} + p_{1c}) + \Psi_1\, p_{01} + (p_{10} + p_{1c})$$
$$(\Psi_2\, p_{02} + \Psi_3\, p_{03} + \Psi_4\, p_{04} + \Psi_5\, p_{05} + \Psi_6\, p_{0c})$$
$$+ p_{01} (\Psi_6\, p_{16} + \Psi_7\, p_{17} + \Psi_8\, p_{18} + \Psi_9\, p_{19} + \Psi_{10}\, p_{11}^{(10)}). \tag{6.4}$$

The expected up time of the system during $(0, t]$ is given by

$$\mu_{up} = \int_0^t A(t)\, du, \tag{6.5}$$

so that

$$\mu_{up}^*(s) = A_0^*(s) / s. \tag{6.6}$$

7. BUSY PERIOD ANALYSIS

We define $B_i(t)$ as the probability that repair facility is busy in the repair of the failed unit when system initially starts from state $S_i \in E$. Using simple probabilistic arguments, the value of $B_0(t)$ can easily be obtained in terms of its LT as follows:

$$B_0^*(s) = N_3(s)/D_2(s), \tag{7.1}$$

where

$$N_3(s) = (q_{02}^* Z_2^* + q_{03}^* Z_3^* + q_{04}^* Z_4^* + q_{05}^* Z_5^* + q_{0c}^* Z_c^*)$$

$$(1 - q_{16}^* q_{61}^* - q_{17}^* q_{71}^* - -q_{18}^* q_{81}^* - q_{19}^* q_{91}^* - q_{11}^{(10)})$$

$$+ q_{01}^* (Z_1^* + q_{16}^* Z_6^* + q_{17}^* Z_7^* + q_{18}^* Z_8^* + q_{19}^* Z_9^* + q_{1c}^* Z_c^*),$$

where

$Z_i^*(s)$ $(i = 4,5,6,7,8,9$ and $c)$ is the LT of $Z_i(t) = \overline{G_i}(t)$ and

$D_2(s)$ is same as given in availability analysis by expression (6.2).

In the long run, the probability that the repairman is busy, is given by

$$B_0 = \lim_{t\to\infty} B_0(t) = \lim_{s\to 0} s B_0^*(s) = N_3(0)/D_2'(0), \tag{7.2}$$

where

$$N_3(0) = (p_{02}\Psi_2 + p_{03}\Psi_3 + p_{04}\Psi_4 + p_{05}\Psi_5 + p_{0c}\Psi_c)$$

$$(p_{10} + p_{1c}) + p_{01}(\Psi_1 + p_{16}\Psi_6 + p_{17}\Psi_7 + p_{18}\Psi_8 + p_{19}\Psi_9 + p_{1c}\Psi_c)$$

and

$D_2'(0)$ may be seen from (6.4).

The expected busy period of the repairman during (0, t] is given by

$$\mu_b(t) = \int_0^t B_0(u)\,du \tag{7.3}$$

so that,

$$\mu_b^*(s) = B_0^*(s)/s. \tag{7.4}$$

8. COST BENEFIT ANALYSIS

Let

C_0 = revenue per unit up time by the system.

C_1 = cost per unit time for which the system is under repair.

Then, the expected profit incurred by the system during $(0,1]$ is given by

$P(t) =$ Expected total revenue in $(0,1]$ – Expected total repair cost in $(0,1]$

$$= C_0 \mu_{up}(t) - C_1 \mu_b(t) \tag{8.1}$$

The expected profit per unit time in steady state is

$$P = \lim_{t\to\infty} P(t)/t$$

$$= C_0 \lim_{t\to\infty} \frac{\mu_{up}(t)}{t} - C_1 \lim_{t\to\infty} \frac{\mu_b(t)}{t}$$

$$= C_0 \lim_{s\to 0} s A_0^*(s) - C_1 \lim_{s\to\infty} s B_0^*(s)$$

$$= C_0 A_0 - C_1 B_0 \tag{8.2}$$

9. PARTICULAR CASE

When all repair time distributions are taken as r-phase Erlangian distribution i.e.

$$g_i(t) = \frac{(r\lambda_i)^r t^{r-1}}{(r-1)!} e^{-r\lambda_i t},$$

so that,

$$\overline{G}_i(t) = \sum_{j=0}^{r-1} \frac{e^{-r\lambda_i t} (r\lambda_i t)^j}{j!}, \quad \lambda_i > 0, \ r \text{ is an integer } \geq 1$$

for $i = 1,2,3,4,5$ and c.

Then in results (5.2), (6.3) and (7.2) we have the following changes in transition probabilities and sojourn times

$$p_{10} = 1 - Z, \quad p_{16} = \frac{\alpha_4}{Y}, \quad p_{17} = \frac{\alpha_5}{Y}, \quad p_{18} = \frac{\alpha_2}{Y}, \quad p_{19} = \frac{\alpha_3}{Y},$$

$$p_{11}^{(10)} = \frac{\alpha_1}{Y}, \quad p_{1c} = \frac{\alpha_c}{Y}, \quad \Psi_1 = \frac{1}{Y}, \quad \Psi_2 = \Psi_6 = \frac{1}{\lambda_4}, \quad \Psi_3 = \Psi_7 = \frac{1}{\lambda_5},$$

$$\Psi_5 = \Psi_8 = \frac{1}{\lambda_2}, \quad \Psi_4 = \Psi_9 = \frac{1}{\lambda_3}, \quad \Psi_c = \frac{1}{\lambda_c}, \quad \Psi_{10} = \frac{1}{\lambda_1},$$

where

$$Z = 1 - \left(\frac{r\lambda_1}{X + r\lambda_1}\right)^r, \quad Y = \frac{X}{Z} \text{ and } X = \left(\sum_{i=1}^{5} \alpha_i + \alpha_c\right).$$

Further, when $r = 1$, then all repair time distributions are converted into negative exponential distribution.

10. GRAPHICAL REPRESENTATION

For more concrete study of the system behaviour, we plot curves for *MTSF* and profit function *w.r.t.* failure rate of dryer (α_4).

Fig. 1 shows the variation in *MTSF* *w.r.t.* α_4 for different values of common cause failure rate $\alpha_c = 0.01, 0.02$ and 0.03 when other parameters are kept fixed as $r = 1$, $\alpha_1 = \alpha_2 = 0.001$, $\alpha_3 = 0.002$, $\alpha_5 = 0.004$. It is observed from graph that the *MTSF* rapidly decreases initially and tends to vanish as α_4 become large. Also, when we increase the value of α_c the *MTSF* decreases.

In fig. 2 the smooth and dotted curves represent the change in profit function respectively for $\alpha_2 = 0.003$ and 0.007 for varying values of α_4 and α_1 when the other parameters are kept fixed as $C_0 = 500$, $C_1 = 100$, $r = 1$, $\alpha_2 = \alpha_3 = 0.001$, $\alpha_5 = 0.004$, $\alpha_c = 0.001$, $\lambda_1 = \lambda_2 = \lambda_3 = 0.01$, $\lambda_4 = \lambda_c = 0.04$. From the graph we observe that both the profit curves decline rapidly initially and tend to vanish as α_4 becomes large. Also, the profit tends to decrease as we increase the values of failure rates α_1 and α_2.

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