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PROFIT ANALYSIS OF JOB PROCESSING SYSTEM OF A MINI-RICE MILL

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ABSTRACT

Job processing systems in any industry play a significant role in optimizing cost and thus increasing the profit. This analysis for a mini-rice mill defers from what can be suitable for a major rice-mill particularly in terms of the number of functioning units. In view of a host of mini-rice mills engaged in the processing of paddy in the state of Chhattisgarh, this research paper has been worked out. The study is based on a mini-rice mill having multiple functioning units by taking failure rates as exponential and arbitrary repair rates. In this analysis various economic related measures such as mean time to system failure, Point wise Availability, Busy period of the repairman and expected profit are also evaluated to facilitate the research outcomes. A particular case has also been discussed along with the graphical representation.

1. INTRODUCTION

Reliability technology has been playing an important role in the modern age world of industrial growth. The reliability analysis deals with the proper functioning of the equipment and the system making more effective use of resources at command which in turn ensures the enhanced productivity and better quality of the end product. It also paves the way for optimization with regard to various inputs needed viz. money, manpower and material so that the whole process may run into a cost effective mode. The applications of reliability measures for example consideration of preventive maintenance of the complete system after a regular period of time, making the provision of standby units etc. have brought about greater returns by increasing the production with same input cost and consequently giving out more returns in terms of profit.

In any production system, the study of job processing phenomenon is essential from reliability point of view. The job processing system has been widely studied by many engineers, system analysts and scientists to meet out quality output standards. To achieve these objectives it is indispensable to design and develop systems that are free from any fault because even a single fault in the operation of the system not only adversely affects the quality and standards of the finished product but also obstructs the proper functioning of the related system. In this background, among others, Singh and Pathak (2001, 2002) studied personal computer system having a protective unit to keep the system free from any fault. Osaki and Kinugasa (1982) studied a hybrid redundant system from hardware redundancy point of view. Kochar (1983) developed a systematic system of investment decision on additional equipment to form a standby system in a production system.

In the present paper, a mini rice mill having four units namely, Rubber roll (R), Separator (S), Polisher (P) and the Conveyor belt (C) is considered. Paddy is used as raw material in the system where as rubber roll is used to remove husk.

2. SYSTEM DESCRIPTION AND ASSUMPTIONS

1. System consists of four units namely Rubber roll R, Separator S, Polisher P and a Conveyor belt C.
2. Only one job is taken for processing at one time till it passes through all the units.
3. It is assumed that a unit can fail while working with load or without load.
4. If a unit fails, then the job slated for this particular unit will wait for processing on it.
5. There are two types of repairs viz. major and minor. Major repair requires expert repairman whereas in case of minor repair, repair is possible even when the system is still working.
6. After repair a unit works as a new.
7. Arrival and processing time of the job are taken to be negative exponential.

   Failure time distribution of all the units are negative exponential while the repair time are taken to be arbitrary.

3. NOTATIONS AND STATES OF THE SYSTEM

p_{ij} : Transition probabilities from state S_i to S_j

μ_i : Mean Sojourn time in the state S_i

λ : Job arrival rate

η_1 : Job processing rate of Rubber roll (R)

η_2 : Job processing rate of Separator (S)

η : Job processing rate of Polisher (P)

α / α_1 : Major/Minor failure rate of Rubber roll (R)

β / β_1 : Major/Minor failure rate of Separator (S)

γ / γ_1 : Major/Minor failure rate of Polisher (P)

δ / δ_1 : Major/Minor failure rate of Conveyor belt (C)

$g_i(t)$: pdf of time to major repair of Rubber roll / Separator / Polisher / Conveyor Belt $i=1,2,3,4$

$g'(t)$: pdf of time to minor repair of Rubber roll / Separator / Polisher / Conveyor belt $i=1,2,3,4$

$R_{wol} / R_{wl} / R_g / R_r$: Rubber roll working without load / working with load / good and non-operative / under repair.

$S_{wol} / S_{wl} / S_g / S_r$: Separator working without load / working with load / good and non-operative / under repair.

$P_{wol} / P_{wl} / P_g / P_r$: Polisher working without load / working with load / good and non-operative / under repair.

$C_{wol} / C_{wl} / C_g / C_r$: Conveyor Belt working without load / working with load / good and non-operative / under repair.

$R_g, (J_{wpR}) / R_r, (J_{wpR})$: Rubber roll under good and non-operative / repair (Job waiting for processing at R)

$S_g, (J_{wpS}) / S_r, (J_{wpS})$: Separator under good and non-operative / repair (Job waiting for processing at S)

$P_g, (J_{wpP}) / P_r, (J_{wpP})$: Polisher under good and non-operative / repair (Job waiting for processing at P)

Considering these symbols the system may be in one of the following states.

Up State : $\{S_0, S_1, S_6, S_{11}\}$.

Down State: $\{S_2 - S_5, S_7 - S_{10}, S_{12} - S_{19}\}$.

4. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Simple probabilistic considerations yield the following expressions for non-zero transition probabilities:

$$p_{01} = \frac{\lambda}{\lambda + x_1}, \quad p_{02} = \frac{\alpha_1}{\lambda + x_1}, \quad p_{03} = \frac{\beta_1}{\lambda + x_1}, \quad p_{04} = \frac{\delta_1}{\lambda + x_1},$$

$$p_{05} = \frac{\gamma_1}{\lambda + x_1}, \quad p_{16} = \frac{\eta_1}{\eta_1 + x_2}, \quad p_{17} = \frac{\beta_1}{\eta_1 + x_2}, \quad p_{18} = \frac{\gamma_1}{\eta_1 + x_2},$$

$$p_{19} = \frac{\alpha}{\eta_1 + x_2}, \quad p_{1,10} = \frac{\delta}{\eta_1 + x_2}, \quad p_{6,11} = \frac{\eta_2}{\eta_2 + x_3}, \quad p_{6,12} = \frac{\alpha_1}{\eta_2 + x_3},$$

$$p_{6,13} = \frac{\gamma_1}{\eta_2 + x_3}, \quad p_{6,14} = \frac{\beta}{\eta_2 + x_3}, \quad p_{6,15} = \frac{\delta}{\eta_2 + x_3},$$

$$p_{11,0} = \frac{\eta}{\eta + x_4}, \quad p_{11,16} = \frac{\gamma}{\eta + x_4}, \quad p_{11,17} = \frac{\alpha_1}{\eta + x_4},$$

$$p_{11,18} = \frac{\beta_1}{\eta + x_4}, \quad p_{11,19} = \frac{\delta}{\eta + x_4},$$

$$p_{20} = p_{30} = p_{40} = p_{50} = p_{71} = p_{81} = p_{91} = p_{10,1} = p_{12,6} = p_{13,6} = p_{14,6}$$

$$= p_{15,6} = p_{16,11} = p_{17,11} = p_{18,11} = p_{19,11} = 1,$$

where

$$x_1 = \alpha_1 + \beta_1 + \delta_1 + \gamma_1, x_2 = \alpha + \beta_1 + \delta + \gamma_1, x_3$$

$$= \alpha_1 + \beta + \delta + \gamma_1, x_4 = \alpha_1 + \beta_1 + \delta + \gamma.$$

The mean sojourn times in the states $S_i \quad (i = 0, 1, \cdots, 16)$ are given by

$$\mu_0 = \frac{1}{\lambda + x_1}, \quad \mu_1 = \frac{1}{\eta_1 + x_2}, \quad \mu_6 = \frac{1}{\eta_2 + x_3}, \quad \mu_{11} = \frac{1}{\eta + x_4},$$

$$\mu_2 = \mu_{12} = \mu_{17} = \int_0^t \overline{G_1'(t)}\, dt, \quad \mu_3 = \mu_7 = \mu_{18} = \int_0^t \overline{G_2'(t)}\, dt,$$

$$\mu_5 = \mu_8 = \mu_{13} = \int_0^t \overline{G_3'(t)}\, dt, \quad \mu_{10} = \mu_{15} = \mu_{19} = \int_0^t \overline{G_4(t)}\, dt,$$

$$\mu_4 = \int_0^t \overline{G_4'(t)}\, dt, \quad \mu_9 = \int_0^t \overline{G_1(t)}\, dt, \quad \mu_{14} = \int_0^t \overline{G_2(t)}\, dt,$$

$$\mu_{16} = \int_0^t \overline{G_3(t)}\, dt.$$

5. MEAN TIME TO SYSTEM FAILURE

Let T_i be the random variable depicting time to system failure when the system starts from the state $S_i \in E$ and $\pi_i(t) = \Pr[T_i \le t]$. To calculate the function $\pi_i(t)$, we consider the possible transitions from the state S_0. From S_0 the system may transit to any one state $S_i \quad (i = 0, 1, \cdots, 16)$. Suppose the system enters state S_1 during $(u, u + du)$ and then starting from S_1, it fails before the expiry of the further time $(t - u)$ and the probability of this contingency is

$$\int_0^t \pi_1(t-u)\, dQ_{01}(u) = Q_{01}(t) \boxed{s} \pi_1(t).$$

The other possibility is that starting from S_0, the system directly transits to failed state in time $(u,\, u+du)$. Thus, we finally have

$$\pi_0(t) = Q_{01}(t) \boxed{s} \pi_1(t) + Q_{02}(t) + Q_{03}(t) + Q_{04}(t) + Q_{05}(t) \tag{5.1}$$

$$\pi_1(t) = Q_{16}(t) \boxed{s} \pi_6(t) + Q_{17}(t) + Q_{18}(t) + Q_{19}(t) + Q_{1,10}(t) \tag{5.2}$$

$$\pi_6(t) = Q_{6,11}(t) \boxed{s} \pi_{11}(t) + Q_{6,12}(t) + Q_{6,13}(t) + Q_{6,14}(t) + Q_{6,15}(t) \tag{5.3}$$

$$\pi_{11}(t) = Q_{11,0}(t) \boxed{s} \pi_0(t) + Q_{11,16}(t) + Q_{11,17}(t) + Q_{11,17}(t) + Q_{11,18}(t) + Q_{11,19}(t) \tag{5.4}$$

Taking Lalace-Stieltjes transform of the equations (5.1), (5.2), (5.3) and (5.4) and solving for $\tilde{\pi}_0(s)$ using this, Mean time to system failure can be obtained as:

$$MTSF = E(T) = \frac{\mu_0 + \mu_1 p_{01} + \mu_6\, p_{01}\, p_{16} + \mu_{11} p_{01}\, p_{16}\, p_{6,11}}{1 - p_{01}\, p_{16}\, p_{6,11}\, p_{11,0}}\,. \tag{5.5}$$

6. AVAILABILITY ANALYSIS

Let M_i be the probability that the system initially in regenerative state S_i, is up at time t without passing to any other non-regenerative state or returning to itself through one or more non-regenerative state.

i) Probability that the system initially in S_0 is up at epoch t without transiting to any other regenerative state is $M_0(t)$.

ii) Probability that the system transits to $S_i \in E$ during $(u,\, u+du)$ and then starting from S_i it is up at epoch t and the probability of this contingency is

$$\int_0^t q_{0i}(u)\, A_i(t-u)\, du = q_{0i}(t) \boxed{c} A_i(t)\,.$$

By probabilistic arguments, we have

$$M_0(t) = e^{-(\lambda + X_1)t}, \quad M_1(t) = e^{-(\eta_1 + X_2)t}, \quad M_6(t) = e^{-(\eta_2 + X_3)t},$$

$$M_{11}(t) = e^{-(\eta\ + X_4)t}\,.$$

Recursive relations giving point wise availabilities are given as follows:

$$A_0(t) = M_0(t) + q_{01}(t) \boxed{c} A_1(t) + q_{02}(t) \boxed{c} A_2(t) + q_{03}(t) \boxed{c} A_3(t) + q_{04}(t) \boxed{c} A_4(t) + q_{05}(t) \boxed{c} A_5(t) \tag{6.1}$$

$$A_1(t) = M_1(t) + q_{16}(t) \boxed{c} A_6(t) + q_{17}(t) \boxed{c} A_7(t)$$
$$+ q_{18}(t) \boxed{c} A_8(t) + q_{19}(t) \boxed{c} A_9(t) + q_{1,10}(t) \boxed{c} A_{10}(t) \tag{6.2}$$

$$A_6(t) = M_6(t) + q_{6,11}(t) \boxed{c} A_{11}(t) + q_{6,12}(t) \boxed{c} A_{12}(t)$$
$$+ q_{6,13}(t) \boxed{c} A_{13}(t) + q_{6,14}(t) \boxed{c} A_{14}(t) + q_{6,15}(t) \boxed{c} A_{15}(t) \tag{6.3}$$

$$A_{11}(t) = M_{11}(t) + q_{11,0}(t) \boxed{c} A_0(t) + q_{11,16}(t) \boxed{c} A_{16}(t)$$
$$+ q_{11,17}(t) \boxed{c} A_{17}(t) + q_{11,18}(t) \boxed{c} A_{18}(t) + q_{11,19}(t) \boxed{c} A_{19}(t) \tag{6.4}$$

$$A_i(t) = q_{i0}(t) \boxed{c} A_0(t), \quad i = 2,3,4,5 \tag{6.5}$$

$$A_j(t) = q_{j1}(t) \boxed{c} A_1(t), \quad j = 7,8,9,10 \tag{6.6}$$

$$A_k(t) = q_{k6}(t) \boxed{c} A_6(t), \quad k = 12,13,14,15 \tag{6.7}$$

$$A_h(t) = q_{h,11}(t) \boxed{c} A_{11}(t), \quad h = 16,17,18,19. \tag{6.8}$$

Taking Lalace-transform of the equations (6.1), (6.2), (6.3), (6.4), (6.5), (6.6), (6.7) and (6.8) and solving for $A_0^*(s)$, the steady-state availability is given by

$$A_0(\infty) = \lim_{S \to 0} A_0^*(s) = \frac{N_2(0)}{D_2'(0)},$$

where

$$N_2(0) = p_{16} p_{6,11} p_{11,0}\, \mu_0 + p_{01} p_{6,11} p_{11,0} \mu_1$$
$$+ p_{01}\, p_{16}\, p_{11,0} \mu_6 + p_{01}\, p_{16}\, p_{6,11} \mu_{11}$$

$$D_2'(0) = \mu_0 p_{16}\, p_{6,11}\, p_{11,0} + \mu_1 p_{01}\, p_{6,11}\, p_{11,0} + \mu_6 p_{16}\, p_{01}\, p_{11,0}$$
$$+ \mu_{11} p_{16} p_{6,11} p_{01} + (\mu_2 p_{02} + \mu_3 p_{03} + \mu_4 p_{04} + \mu_5 p_{05})$$
$$p_{16}\, p_{6,11} p_{11,0} + (\mu_7 p_{17} + \mu_8 p_{18} + \mu_9 p_{19} + \mu_{10} p_{1,10}) p_{01} p_{6,11} p_{11,0}$$
$$+ (\mu_{12} p_{6,12} + \mu_{13} p_{6,13} + \mu_{14} p_{6,14} + \mu_{15} p_{6,15}) p_{16} p_{01} p_{11,0}$$
$$+ (\mu_{16} p_{11,16} + \mu_{17} p_{11,17} + \mu_{18} p_{11,18} + \mu_{19} p_{11,19}) p_{16} p_{6,11} p_{01}.$$

7. BUSY PERIOD ANALYSIS

Let $W_i(t)$ denote the probability that the repairman is busy in regenerative state S_i and remains busy at epoch t without transiting to any other regenerative state. By probabilistic argument, we have

$$W_i(t) = \overline{G_1'(t)}, \quad (i = 2,12,17), \quad W_j(t) = \overline{G_2'(t)}, \ (j = 3,7,18),$$

$$W_k(t) = \overline{G_3'(t)}, \quad (k = 5,8,13), \quad W_h(t) = \overline{G_4(t)}, \ (h = 10,15,19),$$

$$W_4(t) = \overline{G_4'(t)}, \ W_9(t) = \overline{G_1(t)}, \ W_{14}(t) = \overline{G_2(t)}, \ W_{16}(t) = \overline{G_3(t)},$$

$$W_{19}(t) = \overline{G_4(t)}.$$

Let $B_i(t)$ be the probability that the repairman is busy at epoch t starting from the state S_i. By probabilistic arguments as in the previous section, we have

$$B_0(t) = \sum_{i=1-5} q_{0i}(t) \boxed{c} B_i(t), \quad B_1(t) = \sum_{i=6-10} q_{1i}(t) \boxed{c} B_i(t)$$

$$B_6(t) = \sum_{i=11-15} q_{6,i}(t) \boxed{c} B_i(t), \quad B_{11}(t) = \sum_{i=0,16-19} q_{11,i}(t) \boxed{c} B_i(t)$$

$$B_j(t) = W_j(t) + q_{j0}(t) \boxed{c} B_0(t), \quad (j = 2,3,4,5),$$

$$B_j(t) = W_j(t) + q_{j1}(t) \boxed{c} B_1(t), \quad (j = 7,8,9,10),$$

$$B_j(t) = W_j(t) + q_{j6}(t) \boxed{c} B_6(t), \quad (j = 12,13,14,15),$$

$$B_j(t) = W_j(t) + q_{j,11}(t) \boxed{c} B_{11}(t), \quad (j = 16,17,18,19)$$

Taking Laplace-transform of the above equations and solving for $B_0^*(s)$, in the long run , the fraction of time for which the repairman is busy with repair of the failed unit, is given by

$$B_0(\infty) = \lim_{t\to\infty} B_0(t) = \lim_{S\to 0} B_0^*(s) = \frac{N_3(0)}{D_2'(0)}.$$

$$\begin{aligned}
N_3(0) = {} & (\mu_2 p_{02} + \mu_3 p_{03} + \mu_4 p_{04} + \mu_5 p_{05}) p_{16} p_{6,11} p_{11,0} \\
& + (\mu_7 p_{17} + \mu_8 p_{18} + \mu_9 p_{19} + \mu_{10} p_{1,10}) p_{01} p_{6,11} p_{11,0} \\
& + (\mu_{12} p_{6,12} + \mu_{13} p_{6,13} + \mu_{14} p_{6,14} + \mu_{15} p_{6,15}) p_{16} p_{01} p_{11,0} \\
& + (\mu_{16} p_{11,16} + \mu_{17} p_{11,17} + \mu_{18} p_{11,18} + \mu_{19} p_{11,19}) p_{16} p_{6,11} p_{01}.
\end{aligned}$$

8. PARTICULAR CASE

When all the repair time distribution are exponential i.e.

$$g_i(t) = r_i \exp(-r_i t) \text{ and } g_i'(t) = r_i' \exp(-r_i' t) \quad (i = 1,2,3,4),$$

then the steady state equation become:

$$MTSF = K_0 / K_1.$$

$$\text{Availability} = K_{01} / K_2.$$

$$\text{Busy Period} = K_{02} / K_2.$$

$$K_0 = \frac{1}{L_{01}L_{12}L_{23}L_{04}}[\lambda\eta_1\eta_2 + \lambda\eta_1 L_{04} + \lambda L_{23}L_{04} + L_{12}L_{23}L_{04}],$$

$$K_1 = 1 - \frac{\lambda\eta\eta_1\eta_2}{L_{01}L_{12}L_{23}L_{04}},$$

$$K_{01} = \frac{\eta\eta_1\eta_2 + \lambda\eta\eta_2 + \lambda\eta\eta_1 + \lambda\eta_1\eta_2}{L_{01}L_{12}L_{23}L_{04}},$$

$$K_2 = \frac{(1+F_1)\eta\eta_1\eta_2 + (1+F_2)\lambda\eta\eta_2 + (1+F_3)\lambda\eta\eta_1 + (1+F_4)\lambda\eta_1\eta_2}{L_{01}L_{12}L_{23}L_{04}},$$

$$K_{02} = \frac{F_1\eta\eta_1\eta_2 + F_2\lambda\eta\eta_2 + F_3\lambda\eta\eta_1 + F_4\lambda\eta_1\eta_2}{L_{01}L_{12}L_{23}L_{04}},$$

$$F_1 = \frac{\alpha_1}{r_1'} + \frac{\beta_1}{r_2'} + \frac{\gamma_1}{r_3'} + \frac{\delta_1}{r_4'}, \quad F_2 = \frac{\alpha}{r_1} + \frac{\beta_1}{r_2'} + \frac{\gamma_1}{r_3'} + \frac{\delta}{r_4},$$

$$F_3 = \frac{\alpha_1}{r_1'} + \frac{\beta}{r_2} + \frac{\gamma_1}{r_3'} + \frac{\delta}{r_4}, \quad F_4 = \frac{\alpha_1}{r_1'} + \frac{\beta_1}{r_2'} + \frac{\gamma}{r_3} + \frac{\delta}{r_4}, \quad L_{01} = \lambda + X_1,$$

$$L_{12} = \eta_1 + X_2, \quad L_{23} = \eta_2 + X_3, \quad L_{04} = \eta + X_4, \quad \mu_0 = \frac{1}{\lambda + X_1},$$

$$\mu_1 = \frac{1}{\eta_1 + X_2}, \quad \mu_6 = \frac{1}{\eta_2 + X_3}, \quad \mu_{11} = \frac{1}{\eta + X_4}, \quad \mu_2 = \mu_{12} = \mu_{17} = \frac{1}{r_1'},$$

$$\mu_3 = \mu_7 = \mu_{18} = \frac{1}{r_2'}, \quad \mu_5 = \mu_8 = \mu_{13} = \frac{1}{r_3'}, \quad \mu_{10} = \mu_{15} = \mu_{19} = \frac{1}{r_4},$$

$$\mu_4 = \frac{1}{r_4'}, \quad \mu_9 = \frac{1}{r_1}, \quad \mu_{14} = \frac{1}{r_2}, \quad \mu_{16} = \frac{1}{r_3}.$$

9. PROFIT ANALYSIS

The profit analysis of the system can be carried out by considering the expected busy period of the repairman in the repair of the failed units in $(0,t]$.

Therefore,

$G(t) =$ Expected revenue earned by the system in $(0,t]$ – Expected repair cost of the repairman in $(0,t]$.

$$= C_1\mu_{up}(t) - C_2\mu_b(t),$$

where

$$\mu_{up}(t) = \int_0^\infty A_0(t)\,dt, \qquad \mu_b(t) = \int_0^\infty B_0(t)\,dt.$$

The expected profit per unit of time in steady state is given by

$$G = \lim_{t\to\infty} \frac{G(t)}{t} = \lim_{s\to 0} s^2 G^*(s) = C_1\mu_{up}(t) - C_2\mu_b(t),$$

where

C_1 and C_2 are the revenue per unit up time and repair cost.

REFERENCES

Singh S.K. and Pathak V.K. (2006): Analysis of the feeding system of rice mill. *J. Ravishankar Univ.*, **19**, 73-85.

Singh S.K. and Pathak V.K. (2002): Stochastic analysis of a system having three types of dissimilar units. *J. Nat. Acad. Math.*, **16**, 85-96.

Singh S.K. and Pathak V.K. (2001): Stochastic analysis of a system having one blast furnace and two rolling machines. *J. Ultra Scientist Phys. Sci.*, **13,** 388-395.

Singh S.K. (1992): Profit evaluation of a two unit cold standby system with a proviso of rest. *Internat J. Manag. Syst.,* **8**, 277-288.

Dhillon B.S. and Natesan J. (1987): Probabilistic analysis of a pulverlizer system with common cause failures. *Microelectron Reliab.*, **22**, 1121-1133.

Natesan and Jardine A.K.S. (1984): Stochastic analysis of outdoor power system in fluctuating environment. *Microelectron Reliab.*, **26**,1045-1055.

Kochar Indrapal Singh (1983): Reliability analysis and investment in electric motors for irrigation. *Microelectron Reliab.*, **23**, 173-174.

Osaki, S. Kinugasa, M. (1982): Performance-related reliability evaluation of a three-unit hybrid redundant system. *Internat. J. Systems Sci.*, **13**, 1-19.

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