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CONSTRUCTION OF TEST-CONTROL TREATMENT BLOCK DESIGNS WHEN $k > v$

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ABSTRACT

Ting and Notz (1988) have only considered the construction of block designs for comparing test treatments with a control when the block size $k > v$, and have also given a catalogue of $A-$optimal designs for such comparisons. In this paper, we have constructed such a type of designs using a new method called cyclic shifts or set(s) of shifts. The important feature of this method is that the properties of a design can be obtained from the off-diagonal elements of the concurrence matrix without constructing the actual blocks of the design. These newly proposed designs possess the property of $A-$optimality for some specific values of v, b and block size k $(> v)$.

1. INTRODUCTION

In problems such as screening experiments or in the beginning of a long-term experimental investigation, it is desirable to determine the relative performance of new test treatments with respect to the control or standard treatment (Hedayat *et al.* 1988). Experiments to compare certain test treatments with a control treatment were first considered by Hoblyn, *et al.* (1954). Cox (1958) suggested augmenting an incomplete block design in test treatments with one or more replications of the control in each block to obtain a good design. Pearce (1960) developed a systematic approach for designing such type of comparative experiments and Pearce (1983) made two suggestions for such experiments; one is supplementation and the other is reinforcement (following Das, 1958). Pesek (1974) compared a balanced incomplete block design (*BIBD*) with an augmented *BIBD* suggested by Cox (1958) for estimating control-test treatment contrasts and noticed that the latter design was more efficient. Bechhofer and Tamhane (1981) developed the theory of incomplete block designs for comparing several treatments with a control. They did not consider the $A-$ or $MV-$optimality of a design but obtained optimal simultaneous confidence intervals. Their developments led to the concept of Balanced Treatment Incomplete Block (*BTIB*) designs; Notz and Tamhane (1983) studied their construction. Constantine (1983) showed that a *BIBD* in test treatments augmented by a replication of control in each block is $A-$optimal in the class of designs with exactly one replication of the control in each block. Jacroux

(1984) showed that Constantine's (1983) conclusion remains valid even when the $BIBD$ s are replicated by some divisible designs.

Majumdar and Notz (1983) gave a method of obtaining $A-$ and $MV-$optimal designs among all designs for block designs. Hedayat and Majumdar (1984) gave an algorithm and a catalogue of $A-$ and $MV-$optimal designs. Ture (1982, 1985) also studied $A-$optimal designs and suggested their construction. He constructed A-optimal designs when the control treatment replication size, r_0, is a multiple of b for fixed v and k. Hedayat and Majumdar (1985) gave families of $A-$ and $MV-$optimal designs. Notz (1985) proposed optimal row-columns designs for comparing test treatments with a control. Majumdar (1986) and Hedayat, Jacroux and Majumdar (1988) considered the problem of finding optimal designs for comparing the test treatments with two or more controls.

Jacroux (1987a, 1987b, 1988) gave new methods for obtaining $MV-$optimal design, and also gave catalogues for such designs. Jacroux (1986) also studied optimal two-column designs for comparing treatments with a control by utilizing techniques of Hall (1935) and Agrawal (1966). Hedayat and Majumdar (1988) studied designs simultaneously optimal under both block designs and row-column designs. Jacroux (1989) generalized the Hedayat and Majumdar's (1984) algorithm for finding $A-$optimal designs. Cheng, *et al.* (1988) introduced new families of $A-$ and $MV-$optimal block designs. Stufken (1986, 1987, 1988) also studied $A-$ and $MV-$optimal block designs. Mandal, *et al.* (2000) considered distance optimality criterion introduced by Sinha (1970) for comparing a test treatment with control treatments. The matter of comparing test treatments with two or more controls has been discussed in detail by Majumdar (1986) and Hedayat, *et al.* (1988) and Majumdar (1996). Jacroux (2000, 2001, 2002) also constructed $A-$optimal designs for comparing a set of test treatments to a set of standard (control) treatments.

Suppose in an experiment we are interested in comparing several test treatments $1, 2, \cdots, v$ with a control treatment denoted by '0'. For example we have a balance incomplete block design with $v = 3$ treatments in $b = 3$ blocks each of size $k = 4$, where $k > v$.

$$\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{matrix}$$

The test treatments (1, 2, 3) are replicated $r_1 = 2$ times and are compared with a control treatment '0', which is replicated $r_0 = 6$ times. Each treatment pair excluding the control treatment '0' appears together within blocks $\lambda_1 = 1$ time and the control treatment '0' appears with each test treatment within blocks $\lambda_0 = 4$ times.

Only Ting and Notz (1988) considered $A-$optimal designs for such situations and gave a catalogue for comparing v $(k > v)$ test treatments with one control treatment. They considered designs for the following values of the parameters $2 \le b \le 50$, $v \le k \le 50$ and $2 \le v \le 20$.

v	k
2	4, 7, 10, 13, 14, 17, 20, 21, 22, 24, 26, 27, 28, 29.
3	5, 9, 10, 14, 16, 17, 19, 22, 24, 25, 27, 28, 30.
4	5, 6, 7, 9, 11, 12, 13, 15, 17, 18, 19, 21, 23, 24, 25, 27, 29, 30.
5	7, 9, 10, 14, 15, 16, 19, 22, 23, 26, 29.
6	7, 10, 11, 14, 17, 20, 21, 24, 25, 27, 28.
7	8, 10, 11, 15, 18, 19, 21, 22, 25, 26, 29.
8	11, 19, 20, 22, 23.
9	12, 24.
10	13, 16, 25, 26, 29.
11	14, 29.
12	14, 17.
13	17, 18.
14	18
15	19
16	20
17	21
18	22
19	22, 23.

In this paper, we have developed the above type of designs but restricted our consideration to only two values of k, i.e. $k = v + 1$ and $v + 2$, for $v \le 10$. We used the method of cyclic shifts for the construction of such designs.

The paper is organized as follows. The *BTIB* designs are briefly described in section 2. The cyclic shift method is elaborated in section 3. The methods for the construction of *BTIB* designs are elaborated in section 4 and the newly proposed *BTIB* designs for block size $\boldsymbol{k > v}$ have been developed in section 5. The last section concludes this paper with some final remarks.

2. THE BALANCE TEST-TREATMENTS INCOMPLETE BLOCK ($BTIB$) DESIGNS

Suppose v test treatments and a control is to be compared in b blocks of size k each. The test treatments will be labeled as $1,2,\cdots,v$ and the control as '0'. Then the model for the response Y_{ijq} by applying $i-$th treatment to the $q-$th unit in $j-$th block is

$$Y_{ijq} = \mu + \tau_i + \beta_j + \varepsilon_{ijq}, \quad i = 0,1,\cdots,v, \quad j = 1,2,\cdots,b$$

$$q = 1,2,\cdots,n_{ij} \quad (n_{ij} = 0,1,2,\cdots),$$

where n_{ij} denotes the number of experimental units in block j assigned to treatment i. There is no observation Y_{ijq} if $n_{ij} = 0$. The unknown constants μ, τ_i and β_j represent the general mean, the effect of treatment i, the effect of block j and ε_{ijq} is a random variable having mean zero and variance σ^2.

Let $D(v,b,k)$ be the set of all possible designs and let $(\hat{\tau}_0 - \hat{\tau}_i)$; $i = 0,1,\cdots,v$ be the best linear unbiased estimator of $(\tau_0 - \tau_i)$. Our objective here is to allocate the treatments $0,1,2,\cdots,v$ to the blocks in a way that allows the best possible inference on the vector of control-test treatment contrasts $(\tau_0 - \tau_1, \tau_0 - \tau_2, \cdots, \tau_0 - \tau_v)$ using the criteria of $A-$optimality. A design is called $A-$optimal if it minimizes

$$\sum_{i=1}^{v} \text{var}(\hat{\tau}_0 - \hat{\tau}_i).$$

Bechhofer and Tamhane (1981) defined a class of designs, known as $BTIB$ designs and discussed some optimal properties of these designs for setting simultaneous confidence bounds for the set of control-test treatment contrasts. A $BTIB$ designs is an incomplete block design in which each test treatment appears in the same block with the control the same number of times $(= \lambda_0)$ and any pair of test treatment appears together in the same block the same number of times $(= \lambda_1)$. Formally, we may define a $BTIB$ design by the relation

$$\sum_{j=1}^{b} n_{oj}\, n_{ij} = \lambda_0 \quad \text{for} \quad i = 1,2,\cdots,v$$

and

$$\sum_{j=1}^{b} n_{ij}\, n_{i^*j} = \lambda_1 \qquad \text{for} \quad i \neq i^*, \quad i = 1,2,\cdots,v.$$

Note that a *BIBD* is a *BTIB* design with $n_{ij} = 0,1$ and $\lambda_0 = \lambda_1$.

Cheng, *et al.* (1988), Hedayat and Majumdar (1984), and Hedayat, *et al.* (1988) defined two kinds of *BTIB* designs, rectangular ($R-$) type and step ($S-$) type. A *BTIB* (v,b,k,t,s) is called $R-$type design when $s = 0$, and $S-$type design when $s > 0$, where s is the number of replicates of the control treatment in addition to the obligatory bt replicates of the control treatment. Here each block has t replicates of the control. The layouts of these *BTIB* designs are pictured in Figure 1 and Figure 2, with columns as blocks.

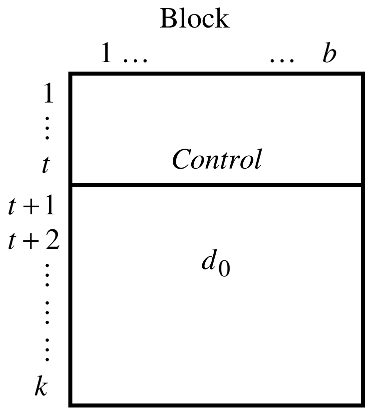

Figure 1. $R-$type *BTIB* design

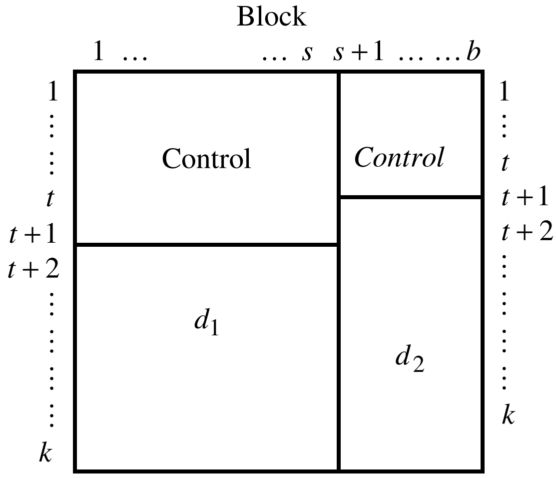

Figure 2. $S-$type *BTIB* design

In $R-$type *BTIB*, the remaining part of the design, d_0, is a binary design in b blocks of size $(k-t)$ in the test treatments and is a *BIBD*. In $S-$type design the parts d_1 and d_2 are the components of the design, which involves the test treatments only. The s blocks in d_1 are having $(k-t-1)$ units each while the $(b-s)$ blocks in d_2 are having $(k-t)$ units each.

3. METHOD OF CYCLIC SHIFTS

The method of cyclic shifts is a particular way of constructing test treatments versus control block designs. Here the v treatments are labeled as $0,1,\cdots,v-1$ and we consider the equireplicate binary design for v treatments in $b = v$ blocks of size k. The method of construction is to allocate to the first plot in the $i-$th block the treatment i; $i = 0,1,\cdots,v-1$. We denote this using the vector $\underline{u}_1 = [0,1,\cdots,v-1]'$, which holds the treatments allocated to the first plot in each of the blocks $1,2,\cdots,v$ respectively. To obtain the treatment allocation of the remaining plots in each block, we cyclically shift the treatments allocated to the first plot. In order to define a cyclic shift, let $\underline{u}_i$ denote the $1 \times v$ vector, which defines the allocation of treatments to the $i-$th plot in each block. That is, the $j-$th element of $\underline{u}_i$ is the treatment allocated to plot i of block j. A cyclic shift of size q_i, when applied to plot i, is such that $\underline{u}_{i+1} = [\underline{u}_i + q_i \underline{1}]'$, where addition is mod v, $\underline{1}$ is a $1 \times v$ vector of ones, $1 \le i \le k-1$ and $1 \le q_i \le v-1$. Assuming that we always start with $\underline{u}_1$ as defined above, a design is completely defined by a set of $k-1$ shifts, $\underline{Q}$, say, where $\underline{Q} = [q_1, q_2, \cdots, q_{k-1}]'$. To avoid a treatment occurring more than once in a block, one must ensure that sum of any two successive shifts, the sum of any three successive shifts, $\cdots$, the sum of any $k-1$ successive shifts is not equal to zero mod v. Subject to this constraint, $\underline{Q}$ may consist of any combination of shifts including repeats. Also the shifts need only range from 1 to $[v/2]$ inclusive, where $[v/2]$ is the greatest integer less than or equal to $[v/2]$. This is because a shift of size q is equivalent to one of size $[v-q]$ mod v.

To illustrate the above method of construction, let us consider the construction of a design for $v=6$ and $k=4$. The set of shifts are defined by $\underline{Q} = [q_1, q_2, q_3]'$, where $q_i = 1,2,3,4,5$; $i = 1,2,3$. Suppose that $\underline{Q} = [1,2,5]$, then $\underline{u}_1 = [0,1,2,3,4,5]'$, $\underline{u}_2 = [1,2,3,4,5,0]'$, $\underline{u}_3 = [3,4,5,0,1,2]'$ and $\underline{u}_4 = [2,3,4,5,0,1]'$. Then the complete design will be

0	1	2	3	4	5
1	2	3	4	5	0
3	4	5	0	1	2
2	3	4	5	0	1

The properties of a design depend on the number of concurrences between the pairs of treatments. A concurrence between two treatments occurs when both treatments are in the same block. Because of the cyclic nature of the construction, the number of concurrences between any treatment and the remainder can be obtained from the number of concurrences between treatment 0 and the remainder. Also the number of concurrences between 0 and the remainder can easily be obtained from $\underline{Q}$ (the set of shifts used to construct the design).

If shifts q_1 and q_2, for example, are applied successively to treatment 0, the result is a concurrence between treatment 0 and treatment q_1 and q_2, and a concurrence between treatment 0 and treatment $q_1 + q_2$. If a third shift, q_3 say, is applied after q_1 and q_2, then the following treatments will also concur with treatment 0: q_3, $q_2 + q_3$ and $q_1 + q_2 + q_3$. This adding of shifts to get the treatments, which concur with 0 works for the general case and so enables the number of concurrences of a design to be obtained directly from the shifts, which defines it. In general, if shifts $q_1, q_2, \cdots, q_{i-1}$ have been applied successively to treatment 0, then the additional concurrences, which results when shift q_i is applied are between treatment 0 and treatments q_i, $q_i + q_{i-1}, \cdots, q_2 + q_3 + \cdots + q_i, \cdots, q_1 + q_2 + \cdots + q_i$ when addition is mod v. It can also be noted that any shift of size q that results in concurrence between treatment 0 and treatments q also results in a concurrence between treatment 0 and treatment $(v-q)$ mod v.

In the above design, $q_1 = 1$, $q_2 = 2$ and $q_3 = 5$ were used. For this one obtains $q_1 + q_2 = 3$, $q_2 + q_3 = 7 = 1$(mod 6) and $q_1 + q_2 + q_3 = 8 = 2$(mod 6). In the above design treatment 1 appears twice (i.e. $q_1 = 2$ and $q_2 + q_3 = 1$) and since 1 is symmetric to 5 and 5 appears once (i.e. $q_3 = 5$), therefore the concurrence between treatment 0 and treatment 1 is 3 and between treatment 0 and treatment 5 is also 3. Similarly treatment 2 appears twice (i.e. $q_2 = 1$ and $q_1 + q_2 + q_3 = 2$) and since 2 is symmetric to 4, therefore, the concurrence between treatment 0 and treatment 2 is 2 and also between treatment 0 and treatment 4 is 2. And treatment 3 appears once (i.e. $q_1 + q_2 = 3$). Since treatment 3 is symmetric to itself, therefore the concurrence between treatment 0 and treatment 3 is 2. Therefore the concurrences between treatment 0 and treatments 1, 2, 3, 4, 5 are 3, 2, 2, 2, and 3 respectively. The concurrences between treatment 1 and treatments 2, 3, 4, 5 follow the same pattern, i.e. the concurrences are 3, 2, 2, 2. Similarly the concurrences between treatment 2 and treatments 3, 4, 5 are 3, 2, 2 and so on.

By using certain combinations of shifts we can construct designs that are made up of complete replicates of smaller designs. When v and k are not relatively prime, then partial sets of v/d blocks can also be obtained, where 'd' is any common divisor of v and k. The shifts producing such partial sets of blocks can be obtained as follows. The smallest integer 'a' is found where $(a \times v)1/k = n$ and 'n' is an integer. Then the set of shifts used to construct the design is such that the sum of every 'a' successive shifts is equal to n. The design will contain v/n blocks. Designs which are constructed using such shifts are referred to as fractional designs.

For even v and k, the fractional designs can be constructed by setting the middle shift $(q_{k/2})$ equal to $v/2$ and ensuring that shifts q_1 and q_{k-i}; $i = 1, 2, \cdots, (k/2-1)$ are complement of each other. For example, a fractional design for $v = 6$ and $k = 4$ by using the set of cyclic shifts $[2,3,4]1/2$ is given by

0	1	2	3	4	5
2	3	4	5	0	1
5	0	1	2	3	4
3	4	5	0	1	2

In order to construct a design with more than v blocks, we combine the blocks obtained from more than one sets of shifts. As an illustration, given below is a design for $v=6$ treatments in 15 blocks of size 4 which has been constructed by combining together the blocks which are obtained from the three sets of shifts [1,1,2], [1,1,3] and $[2,3,4]1/2$.

0	1	2	3	4	5	0	1	2	3	4	5	0	1	2
1	2	3	4	5	0	1	2	3	4	5	0	2	3	4
2	3	4	5	0	1	2	3	4	5	0	1	5	0	1
4	5	0	1	2	3	5	0	1	2	3	4	3	4	5

The above design has been constructed by using shifts $[1,1,2]+[1,1,3]$ $+[2,3,4]1/2$, where the "+" signs indicate that the blocks constructed from the separate sets of shifts must be combined together.

4. CONSTRUCTION OF TEST-CONTROL TREATMENT BLOCK DESIGNS

Method 1: If D_1 is a $BTIB$ design that contains the control t times in each block $(t=1,2,\cdots)$, then the design D_2 in the test treatments obtained by deleting the control from each block of D_1 satisfies the definition of a $BIBD$. Thus one easy way of constructing a $BTIB$ design is to start with a BIB design D_2 in the test treatments and to augment each block of D_2 with the control t times for some value of $t=1,2,\cdots$. $BTIB$ designs that are constructed using this augmentation process are called augmented BIB designs $(ABIBD's)$ as defined by Majumdar and Notz (1983).

In our method of construction, we first construct a design in which each treatment pair appears together within blocks an equal number of times. The block sizes may or may not be equal. If the block sizes are equal, we have an $R-$type design, and then each block of the design can be augmented by t replicates of the control treatments, where $t\geq 1$. If the block sizes are not equal, then the design is an $S-$type. In this case, each block is augmented by a possibly different value of $t\geq 0$, so that the blocks of the design also become equal. As we have our own catalogues of $BIBDs$ and methods of constructing such designs, we therefore have a large number of designs for different values of v, k and b to choose from. Therefore, we can construct designs for many different values of r_0 and r_1.

Method 2: Bechhofer and Tamhane (1981) defined another method of design construction, which is as follows. Starting with a *BIBD* containing $t > v$ treatments in b blocks, one can replace the treatments $v+1, v+2, \cdots, t$ by control '0' to obtain a new *BTIB* design with possibly an additional block or blocks, each one of the latter containing only one test treatment or only the control treatment. After deleting all of these one-treatment blocks and identifying the support of the resulting *BTIB* design, we obtain the derived generator design (s). A generator design is defined by Bechhofer and Tamhane (1981) as *BTIB* design, which is such that no proper subset of its blocks forms a *BTIB* design, and no block of which contains only one of the $(v+1)$ treatments. Bechhofer and Tamhane (1981) pointed out for the design (3.7a) in their method II; that every *BIBD* involving t treatments yields a *BTIB* design with $v = t-1$ test treatments.

In our second method of construction, we construct a design for $(v+1)$ test treatments. If the design is a *BIB*, then each treatment appears together an equal number of times, and if we consider the 'zero' treatment as the control, then we have a *BTIB* design for v test treatments and one control. In this case $r_{01} = r_1$, (where r_{01} is number of replicates of the control treatment before augmentation). Here we can also augment each block by a control treatment t times. Then we have $r_0 = r_{01} + bt$. In this case we can also take designs with different values of k, and then augment the blocks with a control treatment to make all the blocks of equal size.

We can also obtain a *BTIB* design by combining Method 1 and Method 2.

Example 1: Let $v=3$, $k=4$, $b=12$, $t=1$, $s=9$, $r_0=21$, $r_1=9$ and the sets of shifts [11] C +[1(3)] $2C$, where [11] means the set of shifts [1,1], [1(3)] means shift [1] is applied three times in the same row, C means augment each block of this part of the design with a control treatment once and $2C$ means augment each block of this part of design with the control treatment two times, i.e. each block of this part of the design contains two replicates of the control treatment. The design is given below:

Design 1

0	0	0	0	0	0	0	0	0	0	0	0
1	2	3	0	0	0	0	0	0	0	0	0
2	3	1	1	2	3	1	2	3	1	2	3
3	1	2	2	3	1	2	3	1	2	3	1

Example 2: Let $v=3$, $k=5$, $b=15$, $t=0$, $s=36$, $r_0=36$, $r_1=13$ and sets of shifts are [1111] + [1(4)] $3C$.

Design 2

1	2	3	0	0	0	0	0	0	0	0	0	0	0	0
2	3	1	0	0	0	0	0	0	0	0	0	0	0	0
3	1	2	0	0	0	0	0	0	0	0	0	0	0	0
1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
2	3	1	2	3	1	2	3	1	2	3	1	2	3	1

Example 3: Let $v=3$, $k=5$, $b=15$, $t=1$, $s=33$, $r_0=48$, $r_1=9$ and sets of shifts $[1]3C+\{[1(2){+}2]\,(t+1)\,\}3C$. In this case we have constructed the design by using the sets of shifts $[1]$, and $[1(2)+2]$ for $v+1$, i.e. for $0,1,2,3$ treatments (denoted in the sets of shifts by adding $t+1$ at the end) and then augmented the full design by the control treatment thrice. The final design is given below:

Design 3

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	2	3	0	1	2	3	0	1	2	3	0	1	2	3
2	3	4	1	2	3	4	1	2	3	4	2	3	4	0

5. NEW *BTIB* DESIGNS WHEN $k > v$

Ting and Notz (1988) only considered $A-$optimal *BTIB* designs for $k>v$ situation and also provided a catalogue for comparing v test treatments with one control. They considered designs for the following values of the parameters $2\le b\le 50$, $v\le k\le 50$ and $2\le v\le 20$. So, the designs obtained by our methods described in the preceding section are listed in Table 1, which follows.

In Table 1, we have only given one representative case for $v=4$ and $k=5$. It can be seen from Table 1 that we have found many new designs, which were not obtained earlier. For example Ting and Notz (1988) gave the designs for $v=4$, $k=5$ and $b=6, 12,\cdots$, whereas we have developed designs for $v=4$, $k=5$ and $b=4, 5,\cdots,20$. They gave only one design for the above values of v, b and k for $r_0=10$ and $r_1=5$, whereas in addition to this design we have proposed design for $r_0=18$ and $r_1=3$, and $r_0=2$ and $r_1=7$. Our designs have been developed using newly proposed method based on cyclic shifts. We have given sets of shifts that must be used to construct such designs for $v=4$ and $k=5$ in Table 1. The new designs can be considered for the following sets of v, k and different values of b, r_0 and r_1.

Table 1: Suggested designs for one representative case $v = 4$ and $k = 5$.

b	t	s	r_0	r_1	Sets of Shifts
4	2	0	8	3	$[11]2C$
5	1	4	9	4	$[11(1/4)]C + [11]2C$
	0	1	1	6	$[1111] + [111(1/4)]C$
6	3	0	18	3	$[1 + 2(1/2)]3C$
	1	4	10	5	$[111(2/4)]C + [11]2C$
	0	2	2	7	$[1111] + [111(2/4)]C$
7	1	12	19	4	$[111(1/4)]C + [1 + 2(1/2)]3C$
	1	4	11	6	$[111(3/4)]C + [11]2C$
	0	3	3	8	$[1111] + [111(3/4)]C$
8	2	0	16	6	$[11(2)]2C$
	0	4	4	9	$[1111] + [111]C$
	1	12	20	5	$[111(2/4)]C + [1 + 2(1/2)]3C$
	1	4	12	7	$[111]C + [11]2C$
9	1	4	13	8	$[111(5/4)]C + [11]2C$
	1	12	21	6	$[111(3/4)]C + [1 + 2(1/2)]3C$
	1	8	17	7	$[111(1/4)]C + [11(2)]2C$
	0	5	5	10	$[1111] + [111(5/4)]C$
	0	1	1	11	$[1111(2)] + [111(1/4)]C$
10	1	4	14	9	$[111(6/4)]C + [11]2C$
	1	12	22	7	$[111]C + [1 + 2(1/2)]3C$
	1	8	18	8	$[111(2/4)]C + [11(2)]2C$
	0	6	6	11	$[1111] + [111(6/4)]C$
	0	2	2	12	$[1111(2)] + [111(2/4)]C$

	2	6	26	6	$[11]2C+[1+2(1/2)]3C$
	3	4	34	4	$\{[1+2](t+1)\}3C$
	2	10	30	5	$\{[11](t+1)\}2C+\{[2](t+1)\}3C$
11	1	4	15	10	$[111(7/4)]C+[11]2C$
	1	12	23	8	$[111(5/4)]C+[1+2(1/2)]3C$
	1	8	19	9	$[111(3/4)]C+[11(2)]2C$
	0	7	7	12	$[1111]+[111(7/4)]C$
	0	3	3	13	$[1111(2)]+[111(3/4)]C$
	1	16	27	7	$[111(1/4)]C+[11]2C+[1+2(1/2)]3C$
	1	24	35	5	$[111(1/4)]C+\{[1+2](t+1)\}3C$
	1	20	31	6	$[111(1/4)]C+\{[11](t+1)\}2C+\{[2](t+1)\}3C$
12	1	4	16	11	$[111(2)]C+[11]2C$
	1	12	24	9	$[111(6/4)]C+[1+2(1/2)]3C$
	1	8	20	10	$[111]C+[11(2)]2C$
	0	8	8	13	$[1111]+[111(2)]C$
	0	4	4	14	$[1111(2)]+[111]C$
	1	16	28	8	$[111(2/4)]C+[11]2C+[1+2(1/2)]3C$
	3	0	36	6	$[1(2)+2]3C$
	1	20	32	7	$[111(2/4)]C+\{[11](t+1)\}2C+\{[2](t+1)\}3C$
13	1	4	17	12	$[111(9/4)]C+[11]2C$
	1	12	25	10	$[111(7/4)]C+[1+2(1/2)]3C$
	1	8	21	11	$[111(5/4)]C+[11(2)]2C$
	0	9	9	14	$[1111]+[111(9/4)]C$
	0	5	5	15	$[1111(2)]+[111(5/4)]C$
	0	1	1	16	$[1111(3)]+[111(1/4)]C$

	1	16	29	9	$[111(3/4)]C+[11(2)]2C+[1+2(1/2)]3C$
	1	24	37	7	$[111(1/4)]C+[1(2)+2]3C$
	1	20	33	8	$[111(3/4)]C+\{[11](t+1)\}2C+\{[2](t+1)\}3C$
14	1	4	18	13	$[111(10/4)]C+[11]2C$
	1	12	26	11	$[111(2)]C+[1+2(1/2)]3C$
	1	8	22	12	$[111(6/4)]C+[11(2]2C$
	0	10	10	15	$[1111]+[111(10/4)]C$
	0	6	6	16	$[1111(2)]+[111(6/4)]C$
	0	2	2	17	$[1111(3)]+[111(2/4)]C$
	1	16	30	10	$[111]C+[11]2C+[1+2(1/2)]3C$
	1	24	38	8	$[111(2/4)]C+[1(2)+2]3C$
	2	6	34	9	$[11(2)]2C+[1+2(1/2)]3C$
	2	14	42	7	$[111]2C+\{[1+2](t+1)\}3C$
15	1	4	19	14	$[111(11/4)]C+[11]2C$
	1	12	27	12	$[111(9/4)]C+[1+2(1/2)]3C$
	1	8	23	13	$[111(7/4)]C+[11(2]2C$
	0	11	11	16	$[1111]+[111(11/4)]C$
	0	7	7	17	$[1111(2)]+[111(7/4)]C$
	0	3	3	18	$[1111(3)]+[111(3/4)]C$
	1	16	31	11	$[111(5/4)]C+[11]2C+[1+2(1/2)]3C$
	1	24	39	9	$[111(3/4)]C+[1(2)+2]3C$
	1	20	35	10	$[111(1/4)]C+[11(2)]2C+[1+2(1/2)]3C$
	1	28	43	8	$[111]2C+\{[1+2](t+1)\}3C+[111(1/4)]C$
16	1	4	20	15	$[111(3)]C+[11]2C$
	1	12	28	13	$[111(10/4)]C+[1+2(1/2)]3C$

	1	8	24	14	$[111(2)]C+[11(2)]2C$
	0	12	12	17	$[1111]+[111(3)]C$
	0	8	8	18	$[1111(2)]+[111(2)]C$
	0	4	4	19	$[1111(3)]+[111]C$
	1	16	32	12	$[111(6/4)]C+[11]2C+[1+2(1/2)]3C$
	1	24	40	10	$[111]C+[1(2)+2]3C$
	1	20	36	11	$[111(2/4)]C+[11(2)]2C+[1+2(1/2)]3C$
	2	12	44	9	$[11]2C+[1(2)+2]3C$
	2	16	48	8	$[1+2(1/2)]3C+\{[1+2](t+1)\}3C$
	3	4	52	7	$[1+2(1/2)]3C+\{[11](t+1)\}2C+\{[2](t+1)\}3C$
17	1	4	21	16	$[111(13/4)]C+[11]2C$
	1	12	29	14	$[111(11/4)]C+[1+2(1/2)]3C$
	1	8	25	15	$[111(9/4)]C+[11(2)]2C$
	0	13	13	18	$[1111]+[111(13/4)]C$
	0	9	9	19	$[1111(2)]+[111(9/4)]C$
	0	5	5	20	$[1111(3)]+[111(5/4)]C$
	0	1	1	21	$[1111(4)]+[111(1/4)]C$
	1	16	33	13	$[111(7/4)]C+[11]2C+[1+2(1/2)]3C$
	1	24	41	11	$[111(5/4)]C+[1(2)+2]3C$
	1	20	37	12	$[111(3/4)]C+[11(2)]2C+[1+2(1/2)]3C$
	1	28	45	10	$[111(1/4)]C+[11]2C+[1(2)+2]3C$
	1	32	49	9	$[1+2(1/2)]3C+\{[1+2](t+1)\}3C+[111(1/4)]C$
	1	36	53	8	$[1+2(1/2)]3C+\{[11](t+1)\}2C+\{[2](t+1)\}3C+[111(1/4)]C$
18	1	4	22	17	$[111(14/4)]C+[11]2C$
	1	12	30	15	$[111(12/4)]C+[1+2(1/2)]3C$

	1	8	26	16	$[111(10/4)]C+[11(2)]2C$
	0	14	14	19	$[1111]+[111(14/4)]C$
	0	10	10	20	$[1111(2)]+[111(10/4)]C$
	0	6	6	21	$[1111(3)]+[111(6/4)]C$
	0	2	2	22	$[1111(4)]+[111(2/4)]C$
	1	16	34	14	$[111(2)]C+[11]2C+[1+2(1/2)]3C$
	1	24	42	12	$[111(6/4)]C+[1(2)+2]3C$
	1	20	38	13	$[111]C+[11(2)]2C+[1+2(1/2)]3C$
	1	28	46	11	$[111(2/4)]C+[11]2C+[1(2)+2]3C$
	1	32	50	10	$[1+2(1/2)]3C+\{[1+2](t+1)\}3C+[111(2/4)]C$
	3	0	54	9	$[1(3)+3(3/2)]3C$
19	1	4	23	18	$[111(15/4)]C+[11]2C$
	1	12	31	16	$[111(13/4)]C+[1+2(1/2)]3C$
	1	8	27	17	$[111(11/4)]C+[11(2)]2C$
	0	15	15	20	$[1111]+[111(15/4)]C$
	0	11	11	21	$[1111(2)]+[111(11/4)]C$
	0	7	7	22	$[1111(3)]+[111(7/4)]C$
	0	3	3	23	$[1111(4)]+[111(3/4)]C$
	1	16	35	15	$[111(9/4)]C+[11]2C+[1+2(1/2)]3C$
	1	24	43	13	$[111(7/4)]C+[1(2)+2]3C$
	1	20	39	14	$[111(5/4)]C+[11(2)]2C+[1+2(1/2)]3C$
	1	28	47	12	$[111(3/4)]C+[11]2C+[1(2)+2]3C$
	1	32	51	11	$[1+2(1/2)]3C+\{[1+2](t+1)\}3C+[111(3/4)]C$
	1	36	55	10	$[111(1/4)]C+[1(3)+2(3/2)]3C$
20	1	4	24	19	$[111(4)]C+[11]2C$

	1	12	32	17	$[111(14/4)]C + [1+2(1/2)]3C$
	1	8	28	18	$[1111(3)]C + [11(2)]2C$
	0	16	16	21	$[1111] + [111(4)]C$
	0	12	12	22	$[1111(2)] + [111(3)]C$
	0	8	8	23	$[1111(3)] + [111(2)]C$
	0	4	4	24	$[1111(4)] + [111]C$
	1	16	36	16	$[111(10/4)]C + [11]2C + [1+2(1/2)]3C$
	1	24	44	14	$[111(2)]C + [1(2)+2]3C$
	1	20	40	15	$[111(6/4)]C + [11(2)]2C + [1+2(1/2)]3C$
	1	28	48	13	$[111]C + [11]2C + [1(2)+2]3C$
	1	32	51	12	$[1+2(1/2)]3C + \{[1+2](t+1)\}3C + [111]C$
	1	36	56	11	$[111(2/4)]C + [1(3)+2(3/2)]3C$
	3	8	68	8	$\{[1(2)+2(2)](t+1)\}3C$
	2	24	64	9	$\{[11](t+1)\}2C + \{[1+2(2)](t+1)\}3C$
	2	20	60	10	$\{[11+12](t+1)\}2C + \{[1+2](t+1)\}3C$

6. SOME REMARKS

In this paper, we have proposed new methods for the construction of block designs for comparing test treatments with a control when block size $k > v$ by using the method of cyclic shifts. These new designs also provide a flexible family of *BTIB* designs. An important feature of these designs is that, for many cases the $A-$optimal design may also be present in this class. We may use a different rotation to construct the tables of these designs. In cases where the *BIB* design does not exist, we can fill the gap by using Regular Graph designs (See Iqbal and Jones, 1999) but here we have given only *BTIB* designs.

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