Aligarh Journal of Statistics
Vol. 28 (2008), 27-36

A BIVARIATE MODEL TO STUDY THE STOCHASTIC BEHAVIOUR OF BREASTFEEDING AND POST PARTUM AMENORRHEA

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ABSTRACT

A bivariate model is developed and characterized to study the inter-relationship between breastfeeding and post partum amenorrhea.

1. INTRODUCTION

The term post partum amenorrhea denotes the period of temporary sterility which immediately follows the termination of pregnancy and during which conception does not take place. The mean duration of post partum amenorrhea is a good indicator of the mean duration of ovulation. It is very difficult to ascertain the time after birth when the couple should begin to practice contraception. Generally, the resumption of the first menstruation is treated as the termination of PPA. The period of amenorrhea after birth is often called as lactational amenorrhea due to the fact that it is commonly known that majority of the women breastfeed their infants and breastfeeding is one of the principal determinants of the amenorrhea. Therefore, an assessment of the probable effects of breastfeeding on the period of amenorrhea becomes desirable. Amenorrhea is affected not only by the duration of breastfeeding but also by the type and intensity of breastfeeding. [Huffman *et al.* (1987). Malkani and Mirchandani (1960). Srinivasan *et al.* (1989)]. It is easy to ascertain the lactational amenorrhea period than the breastfeeding because the problems of related to the type and intensity of breastfeeding. Several authors make attempt to develop models on PPA in the past. Talwar (1965) used the asymmetrical triangular distribution for the post partum amenorrhea deriving the distribution of closed birth intervals.

Then discrete triangular distribution is used for deriving the probability distribution of PPA, Yadav (1966) treated the PPA duration to be distributed as a Chi-square while deriving distribution of the number of births as against constant PPA period. Biswas (1973) used a mixture of Gamma density, $f(x)$, with two parameters, say θ_1 and θ_2 to describe the variations in PPA duration, which is given by

$$f(x) = \pi \frac{e^x x^{\theta_1 - 1}}{\theta_1 !} + (1-\pi) \frac{e^x x^{\theta_2 - 1}}{\theta_2 !}, \quad 0 < x < \infty, \ \theta_1 \neq \theta_2 .$$

Singh and Bhaduri (1971) gave a more general distribution of PPA. They used two Type III distributions to derive the pattern of PPA. Using the same concept

Saxena and Pathak (1977) have used a mixture of two truncated Chi-square distribution with the assumption that there is minimum period of one month PPA that every woman has. Their proposed distribution is

$$f(x) = \pi f_1(x,n) + (1-\pi) f_2(x, n),$$

where

$$f_i(x,n) = \frac{x e^{-x/2} 2^{1/(2n_i-1)}}{c(n_i) 2^{1/(2n_i-1)} (n_i/2)!}, \quad 1 < x < \infty,$$

$$c(n_i) = 2^{1/(2n_i-1)} (n_i/2)! \int_1^{\infty} e^{-x/2} 2^{1/(2n_i-1)} dx$$

and

$$n_i = n_1, \qquad n_2 > 0.$$

But these mixture models are not found suitable to estimate the effect of breastfeeding on the PPA duration. Lesthaeghe and Page (1980) have discussed a more comprehensive system of distribution of PPA. A more general treatment of the behaviour of PPA period in relation to lactation can be found in Ginsberg (1973) where PPA is treated as a double stochastic process.

In this paper, the problem of estimating the residual Post Partum Amenorrhea (PPA) after the discontinuity of lactation is viewed in the same manner as a problem of life-testing when two components are in parallel configuration in the system and failure of one results in decline in the survival rate of the second. In order to study the inter-relationship between breast feeding and post partum amenorrhea, a bivariate model is developed.

2. RELATION BETWEEN *PPA* AND BREASTFEEDING

The study of the mechanism underlying the variation in the lactation amenorrhea acquires much more importance for the evaluation of the post-partum conception programmed as well as ascertainment of natural fertility of women. In general, breastfeeding delays the onset of menstruation; hence the study of impact of breastfeeding on PPA is quite desirable. The study of such impact can be done through the empirical models as well as through the stochastic models.

A number of studies inferred that the PPA can be extended up to an extent by prolonging the breast-feeding or lactation. The problem of estimating the residual PPA after the discontinuity of lactation is of prime importance. This problem can be viewed in the same manner as a problem of life testing when two components are in parallel configuration in a system and the failure of one result in decline in the survival rate of the second. For this situation Ferund (1961) proposed the point distribution.

Suppose that an instrument has two components A and B with life times X and Y having the exponential distribution (when both components are in operation) with parameters α and β respectively. The X and Y are dependent in a manner that a failure of either component changes the parameter of the life distribution of the other component. Thus when A fails, the parameter for Y becomes β' when B fails, the parameter for X becomes α'. There is no other dependence. Thus the joint density function of X and Y is

$$f(x, y) = \begin{cases} \alpha\beta' \exp[-\beta' y - (\alpha + \beta - \beta')x], & 0 < x < y \\ \alpha\beta' \exp[-\alpha' x - (\alpha + \beta - \alpha')y], & 0 < y < x \end{cases} \tag{2.1}$$

Suchindaran and Bhattacharya (1974) have implemented the above model to study *PPA* and breastfeeding where X and Y denote the length of post-partum amenorrhea and breastfeeding respectively. And also α_1 and β_1 are the probabilities of *PPA* and breastfeeding respectively to end before time t and α_1 and β_1 are the changed probabilities in case of one of them earlier than another.

Let (x_i, y_i), $i = 1, 2, \cdots, n$ be n pairs of independent random variables from Ferund's exponential bivariate distribution (which may be consider as a particular case of Marshall and Olkin's weibull bivariate distribution) with the distribution function.

$$F(x, y) = \begin{cases} \dfrac{\alpha_1}{\alpha_1 + \beta_1}[1 - \exp\{-(\alpha_1 + \beta_1)x\}] - \dfrac{\alpha_1 \exp\{-\beta_2 y\}}{\alpha_1 + \beta_1 - \beta_2} \\ \qquad \times [1 - \exp\{-(\alpha_1 + \beta_1 - \beta_2)x\}] \\ \dfrac{\alpha_1}{\alpha_1 + \beta_1}[1 - \exp\{-(\alpha_1 + \beta_1)y\}] - \dfrac{\beta_1 \exp\{-\alpha_2 x\}}{\alpha_1 + \beta_1 - \alpha_2} \\ \qquad \times [1 - \exp\{-(\alpha_1 + \beta_1 - \alpha_2)y\}] \end{cases}$$

$$0 < \alpha_1 < \alpha_2 \text{ and } 0 < \alpha_1 < \alpha_2 \tag{2.2}$$

Then the corresponding probability density function can be obtained as

$$f(x, y) = \frac{d}{dy}\left[\frac{d}{dx}\{F(x, y)\}\right]$$

i) For $0 < x < y$

$$\frac{d}{dx}\left[\frac{d}{dy}\left\{\frac{\alpha_1}{\alpha_1+\beta_1}[1-\exp\{-(\alpha_1+\beta_1)x\}]-\frac{\alpha_1\exp\{\beta_2 y\}}{\alpha_1+\beta_1-\beta_2}\right.\right.$$

$$\left.\left.\times[1-\exp\{-(\alpha_1+\beta_1-\beta_2)x\}]\right\}\right]$$

$$=\alpha_1\beta_2\exp\{-\beta_2 y-(\alpha_1+\beta_1-\beta_2)x\}.$$

ii) For $0 < y < x$

$$\frac{d}{dy}\left[\frac{d}{dx}\left\{\frac{\beta_1}{\alpha_1+\beta_1}[1-\exp\{-(\alpha_1+\beta_1)y\}]-\frac{\beta_1\exp\{-\alpha_2 x\}}{\alpha_1+\beta_1-\alpha_2}\right.\right.$$

$$\left.\left.\times[1-\exp\{-(\alpha_1+\beta_1-\alpha_2)y\}]\right\}\right]$$

$$=\alpha_2\beta_1\exp\{-\alpha_2 x-(\alpha_1+\beta_1-\alpha_2)y\}.$$

Which finally yields

$$f(x,y)=\begin{cases}\alpha_1\beta_2\exp\{-\beta_2 y-(\alpha_1+\beta_1-\beta_2)x\}], & 0<x<y\\ \alpha_1\beta_2\exp\{-\alpha_2 x-(\alpha_1+\beta_1-\beta_2)x\}], & 0<y<x\end{cases}.$$

3. MARGINAL PROBABILITY DENSITY FUNCTIONS OF POST PARTUM AMENORRHEA AND BREASTFEEDING

Now, let (X_i, Y_i) $(i=1,2,\cdots,n)$ be n pairs of observations for PPA and breastfeeding for a cohort. In the present section the marginal probability density functions and distribution functions of $PPA\,(X)$ and breastfeeding (Y) are obtained.

The marginal probability density function of $PPA\,(X)$ can be obtained as

$$g(x)=\int_{-\infty}^{\infty} f(x,y)\,dy=\int_0^{\infty} f(x,y)\,dy$$

$$=\int_0^x \alpha_2\beta_1[\exp\{-\alpha_2 x-(\alpha_1+\beta_1-\alpha_2)y\}]\,dy$$

$$+\int_x^{\infty}\alpha_1\beta_2[\exp\{-\beta_2 y-(\alpha_1+\beta_1-\beta_2)x\}]\,dy$$

$$=\left[\frac{\alpha_2\beta_1\exp\{-(\alpha_1+\beta_1)x\}}{-(\alpha_1+\beta_1-\alpha_2)}-\frac{\alpha_2\beta_1\exp(-\alpha_2 x)}{-(\alpha_1+\beta_1-\alpha_2)}\right]+\left[\frac{\alpha_1\beta_2\exp(\alpha_1+\beta_1)x}{-\beta_2}\right]$$

On simplification one can get

$$g(x) = \frac{(\alpha_1 - \alpha_2)(\alpha_1 + \beta_2)\exp\{-(\alpha_1 + \beta_1)x\}}{(\alpha_1 + \beta_1 - \alpha_2)} + \frac{\alpha_2 \beta_1 \exp(-\alpha_2 x)}{(\alpha_1 + \beta_1 - \alpha_2)}, \quad x > 0 \tag{3.1}$$

and the marginal distribution function of $PPA\ (X)$ is

$$G(x) = \int_0^x g(x)\,dx = \int_0^x \frac{(\alpha_1 - \alpha_2)(\alpha_1 + \beta_1)\exp\{-(\alpha_1 + \beta_1)x\}}{(\alpha_1 + \beta_1 - \alpha_2)} + \frac{\alpha_2 \beta_1 \exp(-\alpha_2 x)}{(\alpha_1 + \beta_1 - \alpha_2)}\,dx$$

$$= \frac{(\alpha_1 - \alpha_2)[1 - \exp\{-(\alpha_1 + \beta_1)x\}]}{(\alpha_1 + \beta_1 - \alpha_2)} + \frac{\beta_1 [1 - \exp(-\alpha_2 x)]}{(\alpha_1 + \beta_1 - \alpha_2)}.$$

Thus

$$G(x) = 1 - \frac{(\alpha_1 - \alpha_2)\exp\{-(\alpha_1 + \beta_1)x\} + \beta_1 \exp(-\alpha_2 x)}{(\alpha_1 + \beta_1 - \alpha_2)}, \quad x > 0 \tag{3.2}$$

Similarly the marginal probability density function of breastfeeding (Y) will be

$$h(y) = \int_{-\infty}^{\infty} f(x, y)\,dy = \int_0^{\infty} f(x, y)\,dy$$

$$= \frac{(\beta_1 - \beta_2)(\alpha_1 + \beta_1)\exp\{-(\alpha_1 + \beta_1)y\}}{(\alpha_1 + \beta_1 - \beta_2)} + \frac{\alpha_1 \beta_2 \exp(-\beta_2 y)}{(\alpha_1 + \beta_1 - \beta_2)}, \quad y > 0 \tag{3.3}$$

and the marginal distribution function of breastfeeding (Y) is

$$H(y) = \int_0^y h(y)\,dy$$

$$= 1 - \frac{(\beta_1 - \beta_2)\exp\{-(\alpha_1 + \beta_1)y\} + \alpha_1 \exp(-\beta_2 y)}{(\alpha_1 + \beta_1 - \beta_2)}, \quad y > 0 \tag{3.4}$$

The mean of the random variables X and Y having the probability density function as defined in (2.3) and (3.3) respectively will be

$$E(X) = \int_0^{\infty} x\, g(x)\,dx$$

$$= \frac{1}{(\alpha_1 + \beta_1 - \alpha_2)}\left[\frac{(\alpha_1 - \alpha_2)}{(\alpha_1 + \beta_1)} + \frac{\beta_1}{\alpha_2}\right] \tag{3.5}$$

and

$$E(X^2) = \int_0^\infty x^2 \, g(x)\, dx$$

$$= \frac{1}{(\alpha_1 + \beta_1 - \alpha_2)} \left[\frac{(\alpha_1 - \alpha_2)}{(\alpha_1 + \beta_1)^2} + \frac{\beta_1}{\alpha_2^2} \right]. \qquad (3.6)$$

Now

$$E(Y) = \int_0^\infty y\, h(y)\, dy$$

$$= \frac{1}{(\alpha_1 + \beta_1 - \beta_2)} \left[\frac{(\beta_1 - \beta_2)}{(\alpha_1 + \beta_1)} + \frac{\beta_1}{\beta_2} \right] \qquad (3.7)$$

and

$$E(Y^2) = \int_0^\infty y^2 \, h(y)\, dy$$

$$= \frac{1}{(\alpha_1 + \beta_1 - \beta_2)} \left[\frac{(\beta_1 - \beta_2)}{(\alpha_1 + \beta_1)^2} + \frac{\beta_1}{\beta_2^2} \right] \qquad (3.8)$$

4. CONDITIONAL PROBABILITY DENSITY FUNCTIONS OF POST-PARTUM AMENORRHEA AND BREASTFEEDING

Now, in the present section the conditional probability density functions of Y for given X and of X for given Y are obtained.

The conditional probability density function of X for given Y can be obtained as follows

$$g(x \mid y) = \frac{f(x, y)}{h(y)}$$

$$= \begin{cases} \dfrac{\alpha_1 \beta_2 (\alpha_1 + \beta_1 - \beta_2) \exp\{-\beta_2 y - (\alpha_1 + \beta_1 - \beta_2) x\}}{(\beta_1 - \beta_2)(\alpha_1 + \beta_1) \exp\{-(\alpha_1 + \beta_1) y\} + \alpha_1 \beta_2 \exp(-\beta_2 y)}, & 0 < x < y \\[2ex] \dfrac{\alpha_2 \beta_1 (\alpha_1 + \beta_1 - \beta_2) \exp\{-\alpha_2 x - (\alpha_1 + \beta_1 - \alpha_2) y\}}{(\beta_1 - \beta_2)(\alpha_1 + \beta_1) \exp\{-(\alpha_1 + \beta_1) y\} + \alpha_1 \beta_2 \exp(-\beta_2 y)}, & 0 < y < x \end{cases}$$

(4.1)

Similarly, the conditional probability density function of Y for given X can be obtained as follows:

$$h(y \mid x) = \frac{f(x, y)}{g(x)}$$

$$= \begin{cases} \dfrac{\alpha_2\beta_1(\alpha_1+\beta_1-\alpha_2)[\exp\{-\alpha_2 x-(\alpha_1+\beta_1-\alpha_2)y\}]}{(\alpha_1-\alpha_2)(\alpha_1+\beta_1)\exp\{-(\alpha_1+\beta_1)x\}+\alpha_2\beta_1\exp(-\alpha_2 x)}, & 0<y<x \\ \\ \dfrac{\alpha_1\beta_2(\alpha_1+\beta_1-\alpha_2)[\exp\{-\beta_2 y-(\alpha_1+\beta_1-\beta_2)x\}]}{(\alpha_1-\alpha_2)(\alpha_1+\beta_1)\exp\{-(\alpha_1+\beta_1)x\}+\alpha_2\beta_1\exp(-\alpha_2 x)}, & 0<x<y \end{cases}$$

(4.2)

5. CONDITIONAL EXPECTATION AND CONDITIONAL VARIANCE OF THE POST-PARTUM AMENORRHEA AND BREASTFEEDING

In the present section, the conditional expectation and conditional variance of X for given Y and of Y for given X are obtained. The conditional expectation of X for given Y can be obtained as follows:

$$E(X\mid Y)=\frac{(\alpha_1+\beta_1-\beta_2)}{(\beta_1-\beta_2)(\alpha_1+\beta_1)\exp\{-(\alpha_1+\beta_1)y\}+\alpha_1\beta_2\exp(-\beta_2 y)}$$

$$\left\{\frac{\alpha_2\beta_1\exp[-(\alpha_1+\beta_1-\alpha_2)y]\exp(-\alpha_2 x)}{(-\alpha_2)}\left(x-\frac{1}{\alpha_2}\right)\right.$$

$$\left.+\alpha_1\beta_2\exp(-\beta_2 y)\frac{\exp[-(\alpha_1+\beta_1-\beta_2)x]}{(\alpha_1+\beta_1-\beta_2)}\left(x+\frac{1}{(\alpha_1+\beta_1-\beta_2)}\right)\right\}$$

and

$$E(X^2\mid Y)=\frac{(\alpha_1+\beta_1-\beta_2)}{(\beta_1-\beta_2)(\alpha_1+\beta_1)\exp[-(\alpha_1+\beta_1)y]+\alpha_1\beta_2\exp(-\beta_2 y)}$$

$$\left[\alpha_2\beta_1\exp[-(\alpha_1+\beta_1-\alpha_2)y]\exp(-\alpha_2 x)\left\{\frac{x^2}{(-\alpha_2)}-\frac{2}{\alpha_2^2}\left(x+\frac{1}{\alpha_2}\right)\right\}\right.$$

$$+\frac{\alpha_1\beta_2\exp(-\beta_2 y)\exp[-(\alpha_1+\beta_1-\beta_2)x]}{(\alpha_1+\beta_1-\beta_2)}$$

$$\left.\times\left\{x^2+\frac{2}{(\alpha_1+\beta_1-\beta_2)}\left(x+\frac{1}{(\alpha_1+\beta_1-\beta_2)}\right)\right\}\right].$$

Now conditional variance of the X for given Y can be obtained as

$$V(X\mid Y)=\frac{(\alpha_1+\beta_1-\beta_2)}{(\beta_1-\beta_2)(\alpha_1+\beta_1)\exp[-(\alpha_1+\beta_1)y]+\alpha_1\beta_2\exp(-\beta_2 y)}$$

$$\left[\alpha_2\beta_1\exp[-(\alpha_1+\beta_1-\alpha_2)y]\exp(-\alpha_2 x)\left\{\frac{x^2}{(-\alpha_2)}-\frac{2}{\alpha_2^2}\left(x+\frac{1}{\alpha_2}\right)\right\}\right.$$

$$+\frac{\alpha_1\beta_2\exp(-\beta_2 y)\exp[-(\alpha_1+\beta_1-\beta_2)x]}{(\alpha_1+\beta_1-\beta_2)}$$

$$\left.\times\left\{x^2+\frac{2}{(\alpha_1+\beta_1-\beta_2)}\left(x+\frac{1}{(\alpha_1+\beta_1-\beta_2)}\right)\right\}\right]$$

$$-\left[\frac{(\alpha_1+\beta_1-\beta_2)}{(\beta_1-\beta_2)(\alpha_1+\beta_1)\exp\{-(\alpha_1+\beta_1)y\}+\alpha_1\beta_2\exp(-\beta_2 y)}\right.$$

$$\left\{\frac{\alpha_2\beta_1\exp[-(\alpha_1+\beta_1-\alpha_2)y]\exp(-\alpha_2 x)}{(-\alpha_2)}\left(x-\frac{1}{\alpha_2}\right)\right.$$

$$\left.\left.+\alpha_1\beta_2\exp(-\beta_2 y)\frac{\exp[-(\alpha_1+\beta_1-\beta_2)x]}{(\alpha_1+\beta_1-\beta_2)}\left(x+\frac{1}{(\alpha_1+\beta_1-\beta_2)}\right)\right\}\right]^2.$$

The conditional expectation and conditional variance of the Y for given X can be obtained as

$$E(Y\mid X)=\frac{(\alpha_1+\beta_1-\alpha_2)}{(\alpha_1-\alpha_2)(\alpha_1+\beta_1)\exp\{-(\alpha_1+\beta_1)x\}+\alpha_2\beta_1\exp(-\alpha_2 x)}$$

$$\left\{\frac{\alpha_1\beta_2\exp[-(\alpha_1+\beta_1-\beta_2)x]\exp(-\beta_2 y)}{(-\beta_2)}\left(y-\frac{1}{\beta_2}\right)\right.$$

$$\left.+\frac{\alpha_2\beta_1\exp(-\alpha_2 x)\exp[-(\alpha_1+\beta_1-\alpha_2)y]}{(\alpha_1+\beta_1-\alpha_2)}\left(y+\frac{1}{(\alpha_1+\beta_1-\alpha_2)}\right)\right\}$$

and

$$E(Y^2 \mid X) = \frac{(\alpha_1+\beta_1-\alpha_2)}{(\alpha_1-\alpha_2)(\alpha_1+\beta_1)\exp[-(\alpha_1+\beta_1)x]+\alpha_2\beta_1\exp(-\alpha_2 x)}$$

$$\left[\alpha_1\beta_2\exp[-(\alpha_1+\beta_1-\beta_2)x]\exp(-\beta_2 y)\left\{\frac{y^2}{(-\beta_2)}-\frac{2}{\beta_2^2}\left(y+\frac{1}{\beta_2}\right)\right\}\right.$$

$$+\frac{\alpha_2\beta_1\exp(-\alpha_2 x)\exp[-(\alpha_1+\beta_1-\alpha_2)y]}{(\alpha_1+\beta_1-\alpha_2)}$$

$$\left.\times\left\{y^2+\frac{2}{(\alpha_1+\beta_1-\alpha_2)}\left(y+\frac{1}{(\alpha_1+\beta_1-\alpha_2)}\right)\right\}\right].$$

Now conditional variance of the Y for given X can be obtained as

$$V(Y \mid X) = \frac{(\alpha_1+\beta_1-\alpha_2)}{(\alpha_1-\alpha_2)(\alpha_1+\beta_1)\exp[-(\alpha_1+\beta_1)x]+\alpha_2\beta_1\exp(-\alpha_2 x)}$$

$$\left[\alpha_1\beta_2\exp[-(\alpha_1+\beta_1-\beta_2)x]\exp(-\beta_2 y)\left\{\frac{y^2}{(-\beta_2)}-\frac{2}{\beta_2^2}\left(y+\frac{1}{\beta_2}\right)\right\}\right.$$

$$+\frac{\alpha_2\beta_1\exp(-\alpha_2 x)\exp[-(\alpha_1+\beta_1-\alpha_2)y]}{(\alpha_1+\beta_1-\alpha_2)}$$

$$\left.\times\left\{y^2+\frac{2}{(\alpha_1+\beta_1-\alpha_2)}\left(y+\frac{1}{(\alpha_1+\beta_1-\alpha_2)}\right)\right\}\right]$$

$$-\left[\frac{(\alpha_1+\beta_1-\alpha_2)}{(\alpha_1-\alpha_2)(\alpha_1+\beta_1)\exp\{-(\alpha_1+\beta_1)x\}+\alpha_2\beta_1\exp(-\alpha_2 x)}\right.$$

$$\left\{\frac{\alpha_1\beta_2\exp[-(\alpha_1+\beta_1-\beta_2)x]\exp(-\beta_2 y)}{(-\beta_2)}\left(y-\frac{1}{\beta_2}\right)\right.$$

$$\left.\left.+\alpha_2\beta_1\exp(-\alpha_2 x)\frac{\exp[-(\alpha_1+\beta_1-\alpha_2)y]}{(\alpha_1+\beta_1-\alpha_2)}\left(y+\frac{1}{(\alpha_1+\beta_1-\alpha_2)}\right)\right\}\right]^2.$$

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Received : 01-07-2005

Revised : 17-08-2007

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