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RECURRENCE RELATIONS FOR SINGLE AND PRODUCT MOMENTS OF GENERALIZED ORDER STATISTICS FROM DOUBLY TRUNCATED WEIBULL DISTRIBUTION

R.U. Khan, Zaki Anwar and Haseeb Athar

ABSTRACT

Some recurrence relations for moments of generalized order statistics are obtained by Kamps (1995), Cramer and Kamps (2000), Kamps and Cramer (2001) Pawlas and Szynal (2001a, b), Athar and Islam (2004) among others. We have established recurrence relations for single and product moments of generalized order statistics (gos) from doubly truncated Weibull distribution which includes relations for order statistics, $k-$th record values, sequential order statistics and order statistics with non- integral sample size.

1. INTRODUCTION

Kamps (1995) introduced the concept of the generalized order statistics (gos) as follows:

Let $X_1, X_2, \cdots$ be a sequence of independent and identically distributed (iid) random variables (rv) with absolutely continuous distribution function (df) $F(x)$ and probability density function (pdf) $f(x)$, $x \in (\alpha, \beta)$. Let $n \in N$, $n \ge 2$, $k > 0$, $\tilde{m} = (m_1, m_2, \cdots, m_{n-1}) \in \Re^{n-1}$, $M_r = \sum_{j=r}^{n-1} m_j$, such that $\gamma_r = k + (n-r) + M_r > 0$ for all $r \in \{1, \cdots, n-1\}$. Then $X(r, n, \tilde{m}, k)$, $r = 1, 2, \cdots, n$ are called gos if their joint pdf is given by

$$k \left(\prod_{j=1}^{n-1} \gamma_j \right) \left[\prod_{i=1}^{n-1} [1 - F(x_i)]^{m_i} f(x_i) \right] [1 - F(x_n)]^{k-1} f(x_n) \tag{1.1}$$

on the cone $F^{-1}(0) \le x_1 \le \cdots \le x_n \le F^{-1}(1)$.

Here we assume two cases

Case I: $m_1 = m_2 = \cdots = m_{n-1} = m$.

Case II: $\gamma_i \ne \gamma_j$, $i, j = 1, \cdots, n-1$.

For **case I**, gos will be denoted as $X(r,n,m,k)$ with its pdf (Kamps, 1995)

$$f_r(x) = \frac{C_{r-1}}{(r-1)!}[\overline{F}(x)]^{\gamma_r - 1} f(x)\, g_m^{r-1}(F(x)) \tag{1.2}$$

and the joint pdf of $X(r,n,m,k)$ and $X(s,n,m,k)$, $1 \le r < s \le n$, is

$$f_{rs}(x,y) = \frac{C_{s-1}}{(r-1)!(s-r-1)!}[\overline{F}(x)]^m\, g_m^{r-1}(F(x))$$

$$[h_m(F(y)) - h_m(F(x))]^{s-r-1}[\overline{F}(y)]^{\gamma_s - 1} f(x) f(y),$$

$$\alpha \le x < y \le \beta, \tag{1.3}$$

where

$$\overline{F}(x) = 1 - F(x) \tag{1.4}$$

$$\gamma_i = k + (n-i)(m+1) \tag{1.5}$$

$$C_{r-1} = \prod_{i=1}^{r} \gamma_i \tag{1.6}$$

$$h_m(x) = \begin{cases} -\dfrac{1}{m+1}(1-x)^{m+1}, & m \ne -1 \\ \log\left(\dfrac{1}{1-x}\right), & m = -1 \end{cases} \tag{1.7}$$

and

$$g_m(x) = h_m(x) - h_m(0),\ x \in [0,1) \tag{1.8}$$

For **case II**, the pdf of $X(r,n,\tilde{m},k)$ is (Kamps and Cramer, 2001)

$$f_{X(r,n,\tilde{m},k)}(x) = C_{r-1} f(x) \sum_{i=1}^{r} a_i(r)[\overline{F}(x)]^{\gamma_i - 1} \tag{1.9}$$

and the joint pdf of $X(r,n,\tilde{m},k)$ and $X(s,n,\tilde{m},k)$, $1 \le r < s \le n$, is

$$f_{X(r,n,\tilde{m},k),\, X(s,n,\tilde{m},k)}(x,y) = C_{s-1} \sum_{i=r+1}^{s} a_i^{(r)}(s)$$

$$\left[\frac{\overline{F}(y)}{\overline{F}(x)}\right]^{\gamma_i} \left[\sum_{i=1}^{r} a_i(r)[\overline{F}(x)]^{\gamma_i}\right] \frac{f(x)}{\overline{F}(x)} \frac{f(y)}{\overline{F}(y)},$$

$$\alpha \le x < y \le \beta, \tag{1.10}$$

where

$$a_i(r)=\prod_{\substack{j=1\\ j\neq i}}^{r}\frac{1}{(\gamma_j-\gamma_i)}\,,\quad \gamma_j\neq\gamma_i\,,\qquad 1\le i\le r\le n \tag{1.11}$$

$$a_i^{(r)}(s)=\prod_{\substack{j=r+1\\ j\neq i}}^{s}\frac{1}{(\gamma_j-\gamma_i)}\,,\quad \gamma_j\neq\gamma_i\,,\qquad r+1\le i\le s\le n\,. \tag{1.12}$$

Now since $\lim_{m\to-1} h_m(x)=\log\left(\frac{1}{1-x}\right)$, therefore, we will consider only the case

$h_m(x)=-\frac{1}{m+1}(1-x)^{m+1}$ for all m, unless needed otherwise.

A random variable X is said to have Weibull distribution if the *pdf* of X is of the form

$$f_1(x)=p\,x^{p-1}\,e^{-x^p}\,,\ x>0\,,\ p>0 \tag{1.13}$$

and the corresponding *df* is

$$F_1(x)=1-e^{-x^p}\,,\ x>0\,,\ p>0\,. \tag{1.14}$$

Now if for given P_1 and Q_1

$$\int_0^{Q_1} f_1(x)dx=Q \quad\text{and}\quad \int_0^{P_1} f_1(x)dx=P$$

then the truncated *pdf* is given by

$$f(x)=\frac{p\,x^{p-1}\,e^{-x^p}}{P-Q}\,,\ -\log(1-Q)\le x^p\le-\log(1-P)\,,\ p>0 \tag{1.15}$$

and the corresponding truncated *df* $F(x)$ is

$$\overline{F}(x)=-P_2+\frac{1}{p}x^{1-p}\,f(x)\,, \tag{1.16}$$

where

$$Q_1^p=-\log(1-Q)\,,\ P_1^p=-\log(1-P)\,,\ Q_2=\frac{1-Q}{P-Q}\ \text{and}\ P_2=\frac{1-P}{P-Q}\,.$$

Here in this paper, we have obtained recurrence relations for single and product moments of generalized order statistics from doubly truncated Weibull distribution and its various deductions and particular cases are discussed.

2. RECURRENCE RELATIONS FOR SINGLE MOMENTS

Case I: $m_i = m_j = m$, $i, j = 1, 2, \cdots, n-1$.

Lemma 2.1: For $2 \le r \le n$, $n \ge 2$ and $k = 1, 2, \cdots$.

i) $E[X^j(r,n,m,k)] - E[X^j(r-1,n,m,k)]$

$$= \frac{C_{r-2}}{(r-1)!} j \int_{Q_1}^{P_1} x^{j-1} [\bar{F}(x)]^{\gamma_r} g_m^{r-1}(F(x))\, dx. \tag{2.1}$$

ii) $E[X^j(r-1,n,m,k)] - E[X^j(r-1,n-1,m,k)]$

$$= -\frac{(m+1) C_{r-2}}{\gamma_1 (r-2)!} j \int_{Q_1}^{P_1} x^{j-1} [\bar{F}(x)]^{\gamma_r} g_m^{r-1}(F(x))\, dx. \tag{2.2}$$

iii) $E[X^j(r,n,m,k)] - E[X^j(r-1,n-1,m,k)]$

$$= \frac{C_{r-1}}{\gamma_1 (r-1)!} j \int_{Q_1}^{P_1} x^{j-1} [\bar{F}(x)]^{\gamma_r} g_m^{r-1}(F(x))\, dx. \tag{2.3}$$

Proof: Results can be established in view of Athar and Islam (2004).

Theorem 2.1: For the given Weibull distribution and $n \in N$, $m \in \Re$, $2 \le r \le n$.

$$E[X^j(r,n,m,k)] - E[X^j(r-1,n-1,m,k)]$$

$$= -P_2 K \{E[X^j(r,n-1,m,k+m)] - E[X^j(r-1,n-1,m,k+m)]\}$$

$$+ \frac{j}{p\gamma_1} E[X^{j-p}(r,n,m,k)], \tag{2.4}$$

where

$$K = \frac{C_{r-2}^{(n-1)}}{C_{r-2}^{(n-1,k+m)}} = \prod_{i=1}^{r-1} \left(\frac{\gamma_i^{(n-1)}}{\gamma_i^{(n-1)} + m} \right), \quad \gamma_i^{(n-1)} = k + (n-1-i)(m+1).$$

Proof: From equations (1.16) and (2.3), we have

$$E[X^j(r,n,m,k)] - E[X^j(r-1,n-1,m,k)]$$

$$= \frac{C_{r-1}}{\gamma_1 (r-1)!} j \int_{Q_1}^{P_1} x^{j-1} [\bar{F}(x)]^{\gamma_r - 1} \left\{ -P_2 + \frac{1}{p} x^{1-p} f(x) \right\} g_m^{r-1}(F(x))\, dx$$

$$= -P_2 \frac{C_{r-1}}{\gamma_1 (r-1)!} j \int_{Q_1}^{P_1} x^{j-1} [\overline{F}(x)]^{\gamma_r - 1} g_m^{r-1}(F(x))\, dx$$

$$+ \frac{j}{p\,\gamma_1} \frac{C_{r-1}}{(r-1)!} \int_{Q_1}^{P_1} x^{j-p} [\overline{F}(x)]^{\gamma_r - 1} f(x)\, g_m^{r-1}(F(x))\, dx$$

$$= -P_2 \frac{C_{r-2}^{(n-1)}}{(r-1)!} j \int_{Q_1}^{P_1} x^{j-1} [\overline{F}(x)]^{\gamma_r^{(n-1,k+m)}} g_m^{r-1}(F(x))\, dx$$

$$+ \frac{j}{p\,\gamma_1} E[X^{j-p}(r,n,m,k)]$$

as $\gamma_r - 1 = \gamma_r^{(n-1,k+m)} = (k+m) + (n-1-r)(m+1)$, $C_{r-1} = \gamma_1 C_{r-2}^{(n-1)}$ and hence the required result.

If we put $p = 1$ in the above expression, we get corresponding result for the exponential distribution. For the non-truncated case one has to put $P = 1$, $Q = 0$.

Remark 2.1: Recurrence relation for single moments of order statistics $(m = 0,\ k = 1)$ is

$$E(X_{r:n}^{j}) - E(X_{r-1:n-1}^{j}) = -P_2 \{E(X_{r:n-1}^{j}) - E(X_{r-1:n-1}^{j})\} + \frac{j}{np} E(X_{r:n}^{j-p}) \tag{2.5}$$

or $$E(X_{r:n}^{j}) = Q_2\, E(X_{r-1:n-1}^{j}) - P_2\, E(X_{r:n-1}^{j}) + \frac{j}{np} E(X_{r:n}^{j-p}). \tag{2.6}$$

For $r = 1$

$$E(X_{1:n}^{j}) = Q_2\, Q_1^{j} - P_2\, E(X_{1:n-1}^{j}) + \frac{j}{np} E(X_{1:n}^{j-p}). \tag{2.7}$$

For $r = n$

$$E(X_{n:n}^{j}) = Q_2\, E(X_{n-1:n-1}^{j}) - P_2\, P_1^{j} + \frac{j}{np} E(X_{n:n}^{j-p}), \tag{2.8}$$

where by convention we use $X_{n:n-1} = P_1$ and $X_{0:n} = Q_1$

as obtained by Khan *et al*. (1983a).

Remark 2.2: For $k-$th record statistics $(m = -1)$ recurrence relation for single moments reduces as

$$E(X_r^j)^k - E(X_{r-1}^j)^k = -P_2\left(\frac{k}{k-1}\right)^{r-1}\{E(X_r^j)^{k-1} - E(X_{r-1}^j)^{k-1}\}$$
$$+\frac{j}{pk}E(X_r^{j-p})^k$$

as $K = \dfrac{C_{r-2}^{(n-1)}}{C_{r-2}^{(n-1,k+m)}} = \prod_{i=1}^{r-1}\left(\dfrac{k}{k-1}\right)$, $\quad \gamma_1 = k$, for $m = -1$.

Similarly, the recurrence relations for single moments of order statistics with non-integral sample size for $m = 0$, $k = \alpha - n + 1$, $\alpha \in \Re_+$ and for sequential order statistics for $m = \alpha - 1$, $k = \alpha$ may be obtained.

Theorem 2.2: For the given Weibull distribution and $n \in N$, $m \in \Re$, $2 \le r \le n$.

$$E[X^j(r,n,m,k)] - E[X^j(r-1,n-1,m,k)]$$
$$= \frac{(P-Q)}{p\gamma_1}K^* j E[\phi\{X(r,n,m,k+1)] \tag{2.9}$$
$$= \frac{j}{p\gamma_1}\{-(1-P)E[\phi(X(r,n,m,k)] + E[X^{j-p}(r,n,m,k)]\}, \tag{2.10}$$

where

$$\phi(x) = x^{j-p}e^{x^p}, \qquad K^* = \frac{C_{r-1}}{C_{r-1}^{(k+1)}} = \prod_{i=1}^{r}\left(\frac{\gamma_i}{\gamma_i+1}\right).$$

Proof: In view of equation (1.15), (2.3) becomes

$$E[X^j(r,n,m,k)] - E[X^j(r-1,n-1,m,k)]$$
$$= \frac{C_{r-1}}{\gamma_1(r-1)!} j\int_{Q_1}^{P_1} x^{j-1}[\bar F(x)]^{\gamma_r}\left\{\frac{(P-Q)f(x)}{p\,x^{p-1}e^{-x^p}}\right\} g_m^{r-1}(F(x))\,dx$$
$$= \frac{(P-Q)C_{r-1}}{p\gamma_1 C_{r-1}^{(k+1)}} j\left\{\frac{C_{r-1}^{(k+1)}}{(r-1)!}\int_{Q_1}^{P_1}\phi(x)[\bar F(x)]^{\gamma_r^{(k+1)}-1} f(x)\, g_m^{r-1}(F(x))\,dx\right\},$$

where $\gamma_r^{(k+1)} = (k+1) + (n-r)(m+1)$ and hence the Theorem.

To prove (2.10), note that

$$\frac{\bar F(x)}{f(x)} = -\frac{1}{p}\{(1-P)x^{1-p}e^{x^p} - x^{1-p}\}$$

and the result follows from (2.3).

Theorem 2.3: For the given Weibull distribution and $n \in N$, $m \in \Re$, $2 \le r \le n$.

$$E[X^j(r,n,m,k)] - E[X^j(r-1,n,m,k)]$$

$$= -P_2\, K^{**}\{E[X^j(r,n-1,m,k+m)] - E[X^j(r-1,n-1,m,k+m)]\}$$

$$+\frac{j}{p\gamma_r} E[X^{j-p}(r,n,m,k)], \tag{2.11}$$

where $K^{**} = \dfrac{C_{r-2}}{C_{r-2}^{(n-1,\,k+m)}} = \prod_{i=1}^{r-1}\left(\dfrac{\gamma_i}{\gamma_i^{(n-1)} + m}\right) = \prod_{i=1}^{r-1}\left(\dfrac{\gamma_i}{\gamma_i - 1}\right)$.

Proof: Proof follows on the lines of Theorem 2.1 using (1.16) and (2.1).

Remark 2.3: The recurrence relation for the non-truncated exponential distribution given by Pawlas and Syznal (2001 a) are obtained by setting $p=1$, $P=1$, $Q=0$ and $j=j+1$.

Theorem 2.4: For the given Weibull distribution and $n \in N$, $m \in \Re$, $2 \le r \le n$.

$$E[X^j(r-1,n,m,k)] - E[X^j(r-1,n-1,m,k)]$$

$$= P_2 \frac{(m+1)(r-1)K^{**}}{\gamma_1}$$

$$\{E[X^j(r,n-1,m,k+m)] - E[X^j(r-1,n-1,m,k+m)]\}$$

$$-\frac{(m+1)(r-1)}{p\gamma_r\,\gamma_1}\, j\, E[X^{j-p}(r,n,m,k)]. \tag{2.12}$$

Proof: Proof follows from (1.16) and (2.2).

Case II: $\gamma_i \ne \gamma_j$, $i,\ j = 1,2,\cdots,n-1$.

Theorem 2.5: For distribution given in (1.14) and $n \in N$, $m \in \Re$, $2 \le r \le n$, $k \ge 1$.

$$E[X^j(r,n,\tilde{m},k)] - E[X^j(r-1,n-1,\tilde{m},k)]$$

$$= \frac{j}{p\,\gamma_1}\{-(1-P)\,E[\phi(X(r,n,\tilde{m},k))] + E[X^{j-p}(r,n,\tilde{m},k)]\}. \tag{2.13}$$

$$= \frac{(P-Q)}{p\,\gamma_1} K^*\, j\, E[\phi\{X(r,n,\tilde{m},k+1)] \tag{2.14}$$

Proof: In view of Athar and Islam (2004), we have

$$E[X^j(r,n,\tilde{m},k)] - E[X^j(r-1,n-1,\tilde{m},k)]$$

$$= \frac{\gamma_r}{\gamma_1} C_{r-2}\, j \int_{Q_1}^{P_1} x^{j-1} \sum_{i=1}^{r} a_i(r)[\bar{F}(x)]^{\gamma_i}\, dx \tag{2.15}$$

On using equation (1.16), RHS of (2.15) becomes

$$= \frac{\gamma_r}{\gamma_1} C_{r-2}\, j \int_{Q_1}^{P_1} x^{j-1} \sum_{i=1}^{r} a_i(r)[\bar{F}(x)]^{\gamma_i - 1} \left\{ -P_2 + \frac{1}{p} x^{1-p} f(x) \right\} dx$$

and hence the required result.

Result (2.14) can be proved by using (1.15) in (2.15).

Remark 2.4: Theorem 2.2 can be deduced from Theorem 2.5 by replacing $\tilde{m}$ with m, $m \neq -1$. Remaining results for case II, $(\gamma_i \neq \gamma_j)$ can also be obtained by replacing m with $\tilde{m}$ in Theorem 2.3 and 2.4.

3. RECURRENCE RELATIONS FOR PRODUCT MOMENTS

Case I: $m_i = m_j = m$, $i, j = 1, 2, \cdots, n-1$.

Theorem 3.1: For the given Weibull distribution $1 \leq r < s \leq n-1$, $m \in \Re$, $n \geq 2$ and $k = 1, 2, \cdots$

$$E[X^i(r,n,m,k)\ X^j(s,n,m,k)] - E[X^i(r,n,m,k)\ X^j(s-1,n,m,k)]$$

$$= -P_2\, K_1 \{E[X^i(r,n-1,m,k+m)\ X^j(s,n-1,m,k+m)]$$

$$- E[X^i(r,n-1,m,k+m)\ X^j(s-1,n-1,m,k+m)]\}$$

$$+ \frac{j}{p\gamma_s} E[X^i(r,n,m,k)\ X^{j-p}(s,n,m,k)], \tag{3.1}$$

where $K_1 = \dfrac{C_{s-2}}{C_{s-2}^{(n-1,k+m)}} = \displaystyle\prod_{i=1}^{s-1} \left(\frac{\gamma_i}{\gamma_i^{(n-1)} + m} \right) = \frac{\gamma_1}{\gamma_s} \prod_{i=1}^{s-1} \left(\frac{\gamma_i^{(n-1)}}{\gamma_i^{(n-1)} + m} \right)$.

Proof: In view of Athar and Islam (2004), we have

$$E[X^i(r,n,m,k)\ X^j(s,n,m,k)] - E[X^i(r,n,m,k)\ X^j(s-1,n,m,k)]$$

$$= \frac{C_{s-1}}{\gamma_s (r-1)!(s-r-1)!}\, j \int_{Q_1}^{P_1} \int_{x}^{P_1} x^i y^{j-1} [\bar{F}(x)]^m f(x) g_m^{r-1}(F(x))$$

$$[h_m(F(y)) - h_m(F(x))]^{s-r-1} [\bar{F}(y)]^{\gamma_s}\, dy\, dx \tag{3.2}$$

Now using (1.16) in (3.2), we get

$$= \frac{C_{s-1}}{\gamma_s (r-1)!(s-r-1)!} j \int_{Q_1}^{P_1} \int_x^{P_1} x^i y^{j-1} [\overline{F}(x)]^m f(x) g_m^{r-1}(F(x))$$

$$[h_m(F(y)) - h_m(F(x))]^{s-r-1} [\overline{F}(y)]^{\gamma_s - 1} \left\{ -P_2 + \frac{1}{p} y^{1-p} f(y) \right\} dy\, dx$$

$$= -P_2 \frac{C_{s-1}}{\gamma_s (r-1)!(s-r-1)!} j \int_{Q_1}^{P_1} \int_x^{P_1} x^i y^{j-1} [\overline{F}(x)]^m f(x)\, g_m^{r-1}(F(x))$$

$$[h_m(F(y)) - h_m(F(x))]^{s-r-1} [\overline{F}(y)]^{\gamma_s - 1}\, dy\, dx$$

$$+ \frac{C_{s-1}}{p \gamma_s (r-1)!(s-r-1)!} j \int_{Q_1}^{P_1} \int_x^{P_1} x^i y^{j-p} [\overline{F}(x)]^m g_m^{r-1}(F(x))$$

$$[h_m(F(y)) - h_m(F(x))]^{s-r-1} [\overline{F}(y)]^{\gamma_s - 1} f(x) f(y)\, dy\, dx$$

$$= -P_2 \frac{C_{s-2}}{(r-1)!(s-r-1)!} j \int_{Q_1}^{P_1} \int_x^{P_1} x^i y^{j-1} [\overline{F}(x)]^m f(x)\, g_m^{r-1}(F(x))$$

$$[h_m(F(y)) - h_m(F(x))]^{s-r-1} [\overline{F}(y)]^{\gamma_s^{(n-1,k+m)}}\, dy\, dx$$

$$+ \frac{j}{p \gamma_s} E[X^i(r,n,m,k)\ X^{j-p}(r,n,m,k)],$$

where $\gamma_s - 1 = \gamma_s^{(n-1,k+m)}$, $C_{s-1} = \gamma_s C_{s-2}$ and hence the result.

Remark 3.1: At $p = 1$, $Q = 0$, $P = 1$, $s = r + 1$ and $j = j + 1$, Theorem 3.1 reduces to

$$E[X^i(r,n,m,k)\ X^{j+1}(r+1,n,m,k)] - E[X^i(r,n,m,k)\ X^{j+1}(r,n,m,k)]$$

$$= \frac{j+1}{\gamma_{r+1}} E[X^i(r,n,m,k)\ X^j(r+1,n,m,k)]$$

or $\quad E[X^i(r,n,m,k)\ X^{j+1}(r+1,n,m,k)]$

$$= \frac{j+1}{\gamma_{r+1}} E[X^i(r,n,m,k)\ X^j(r+1,n,m,k)] + E[X^{i+j+1}(r,n,m,k)] \tag{3.3}$$

which is the result given by Pawlas and Syznal (2001 a) for the non-truncated exponential distribution.

Remark 3.2: Recurrence relations between product moments of order statistics $(m = 0,\ k = 1)$ is

$$E(X_{r,s:n}^{(i,j)}) - E(X_{r,s-1:n}^{(i,j)}) = -P_2 \frac{n}{n-s+1}\{E(X_{r,s:n-1}^{(i,j)}) - E(X_{r,s-1:n-1}^{(i,j)})\}$$

$$+ \frac{j}{p\,(n-s+1)} E(X_{r,s:n}^{(i,j-p)})\,. \qquad (3.4)$$

as $K_1 = \dfrac{n}{n-s+1}$ and $\gamma_s = n-s+1$ for $m = 0,\ k = 1$.

which is the relation obtained by Khan *et al*. (1983 b).

Case II: $\gamma_i \neq \gamma_j$

Results can be obtained by replacing m with $\tilde{m}$.

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Department of Statistics and Operations Research,
Aligarh Muslim University, Aligarh-202 002. India.
e-mail: aruke@rediffmail.com,
haseebathar@hotmail.com