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OPTIMAL ORDERING POLICIES WHEN ANTICIPATING PARAMETER CHANGES IN EOQ SYSTEM UNDER RANDOM INPUT

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ABSTRACT

The classical EOQ model requires all the parameters to be constant. Subsequent development have considered models in which just one or more of the cost or demand parameters change at a point of time and, the number of units received does not necessarily match with the number of units ordered but has a known mean and variance. In practice, price rises are often announced, in advance, and such changes may affect the demand rate. We determine the optimal ordering policy for such systems and present a simple algorithm for computing it. This is supported by a numerical example, which shows some of the interdependencies of the various parameters.

1. INTRODUCTION

The classical EOQ inventory model has several basic assumptions that gives the solution of ordering as $Q = \sqrt{\frac{2AR}{h}}$ where R, A and h stands for demand, ordering cost and holding cost respectively. The most basic assumption is that all the parameters are constant and system receives same order as requisitioned. Several models have been developed in which either the demand rate or the purchase price may vary with time; namely, Goyal (1975), Buzacott (1975), Naddor (1976), Bardosa and Friedman (1978), Rash *et al*. (1976) Sivazlian and Stanfel (1975) etc.

In all these papers, the parameter changes are continuous with time and only one parameter is permitted to change with the basic assumption that the quantity received matches with quantity ordered. But due to number of reasons, namely, inadequate raw material leading to a lower than the planned run quantity, exceptionally good production runs leading to a larger than planned quantity, due to shortage of man power, machines breakdown, failure of electricity, pilferage and / or damage in transit etc.

Silver (1976) formulated model to include the case where the quantity received from the supplier may not necessarily match the quantity ordered. He established that the optimum order quantity to be dependent on the mean and variance of the amount ordered. Noori and Keller (1986) developed a model under price change anticipation under random input which was extended by

Trivedi, *et al*. (1994) for deteriorating items. In all the above models, again only the unit cost was permitted to change.

In this paper, we consider EOQ model in which any or all of the parameter/s may change at some point of time and the quantity received does not match with the quantity ordered. The system, we examine, has advantages over the established models. The main and the foremost is that a change in any of the costs is likely to affect the demand rate and we allow this. The second advantage is, that often, the times when prices will rise, are well known by announcement or by previous experience.

In section 2, we develop a mathematical model and determine the necessary conditions for a policy to be optimal. In section 3, computational results for several sets of parameters with the help of hypothetical numerical illustration are presented. The concluding section 4 completers the article.

2. MATHEMATICAL MODEL

Let T be the finite time horizon which is partitioned into two disjoint time periods: the closed interval $[0, S]$ called period 1 and the open – closed interval $(S, T]$ called period 2.

The costs associated with period 1 are; unit cost C_1; a holding cost h_1, for all items brought into the inventory; and set-up cost A_1, charged per order. The quantity received is Y_1 – units with mean $E(Y_1) = bQ_1$ and variance $V(Y_1) = \sigma_0^2 + \sigma_1^2 Q_1^2$ where $b > 0$ is a bias factor, σ_0^2 and σ_1^2 are non-negative constants and Q_1 is the actual quantity ordered during period 1.

For the items brought into the stock during period 2, the unit cost, holding cost and set-up costs are C_2, h_2 and A_2 respectively. During this period retailer receives the quantity Y_2 – units with mean $E(Y_2) = bQ_2$ and variance $V(Y_2) = \sigma_0^2 + \sigma_1^2 Q_2^2$ where $b > 0$ is a bias factor, σ_0^2 and σ_1^2 are non-negative constants and Q_2 is the actual quantity ordered.

At S, a retailer receives a special order Y_a – units with mean $E(Y_a) = bQ_a$ and variance $V(Y_a) = \sigma_0^2 + \sigma_1^2 Q_a^2$ where $b > 0$ is a bias factor, σ_0^2 and σ_1^2 are non-negative constants and Q_a is the actual quantity ordered.

Let R_1 and R_2 be the demand rates during periods 1 and 2 respectively. A finite number of the orders are to be purchased to satisfy the demand.

We assume that the initial inventory level is zero and the delivery is instantaneous. Of course, if there is any lead-time, the results of this article hold, but the orders are to be placed earlier according to the duration of the lead-time.

Following Silver (1976), the total average expected cost $Z(Q)$ for a single order is given by

$$Z(Q) = A + C_1 Q + h_1 [\sigma_0^2 + (\sigma_1^2 + b^2) Q^2] / 2R$$

Similar to Lev and Weiss (1993), it can be seen that an optimal policy must have the property that all the orders placed and depleted in period 1 and period 2 are of the same size. Thus, two consecutive orders placed and depleted during the same period are of the same size in either one of these two periods.

Since the orders must be placed and depleted during the same period, crossing of two orders are not permitted. The inventory level is zero at S. Then the structure of the optimal policy is to place $m \geq 0$ orders of size $Q_1 = R_1 S / m$ during $[0, S)$, place an order of size Q_a, $0 \leq Q_a \leq R_2 (T - S)$ at S, and place $n > 0$ orders of size $Q_2 = [R_2 (T - S) - Q_a] / n$ during period 2.

The optimal number of orders to be placed for the finite time horizon inventory model with parameters R, h, A and T is given by Schwarz (1972) as the integer satisfying

$$n(n-1) \leq \frac{hRT^2}{2A} \leq n(n+1) \tag{2.1}$$

The right hand side inequality is $n^2 + n - \frac{hRT^2}{2A} \geq 0$, whose solution is

$$n \geq -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{hRT^2}{2A}}$$

The left inequality yields $n \leq \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{hRT^2}{2A}}$. Since n is a positive integer

$n = < \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{hRT^2}{2A}} >$ where $< x >$ denotes the least integer greater than or equal to x. Define an integer valued function as

$$N(R_1, h_1, A_1, T) = \sqrt{\frac{2A_1 R_1 + h_1 \sigma_0^2}{h_1 R_1^2 (\sigma_1^2 + b^2)}} \tag{2.2}$$

Hence, the optimal number of orders $m = m^*$ to be placed during $[0, S)$ is given by $m^* = N(R_1, h_1, A_1, S)$ and the optimal order size is $Q_1 = R_1 S / m^*$. The costs incurred in $(S, T]$ are given by

$$F(t_a, n) = A_1 + \frac{h_1[\sigma_0^2 + (\sigma_1^2 + b^2) R_2^2 t_a^2]}{2R_2} + C_1 b R_2 t_a$$

$$+ n\left[A_2 + \frac{h_2[\sigma_0^2 + (\sigma_1^2 + b^2) R_2^2 t_2^2]}{2R_2} + C_1 b R_2 t_2\right] \qquad (2.3)$$

where t_a is the length of time it takes to deplete the order placed at S and $t_2 = (T - S - t_a)/n$.

Put $x = T - S$. So $t_2 = (x - t_a)/n$. Then t_a can be obtained by setting

$\frac{\partial F(t_a, n)}{\partial t_a} = 0$ i.e.

$$t_a = \frac{C_1 b n (2R_2 - h_1) + 4(\sigma_1^2 + b^2) x R_2^2}{2R_2(\sigma_1^2 + b^2)(h_1 n + 2R_2)} \qquad (2.4)$$

Also note that, if t_a is given by equation (2.4) then $(x - t_a)$ is the time in which $n-$orders are placed. Thus $n^*(x - t_a)$ represent the optimal number of orders to be placed in the second period; where following Lev, *et al.* (1979), the optimal value of $n = n^*$ is given by

$$n^3 + n^2(2h_1h_2 + h_1^2) + n(h_2^2 + 2h_1h_2 - Z) + h_2^2 = 0 \qquad (2.5)$$

with

$$Z = \frac{R_2 h_2 [h_1 x - (C_2 - C_1)]^2}{2A_2}$$

Hence, optimum value of order quantity Q_a at S can be obtained and the total cost TC is given by

$$TC = F(t_a^*, n^*) + m^* A_1 + \frac{m^* h_1[\sigma_0^2 + (\sigma_1^2 + b^2) Q_1^2]}{2R_1} + R_1 C_1 t_1 b \qquad (2.6)$$

where

$$t_1 = \sqrt{\frac{2A_1R_1 + h_1\sigma_0^2}{h_1 R_1^2(\sigma_1^2 + b^2)}} \qquad (2.7)$$

and optimum $m = m^*$ can be found using equations (2.2) and (2.3).

3. COMPUTATIONAL ALGORITHM

The sequential steps are as follows:

1. Compute t_1 using (2.7)
2. Using (2.3), obtain optimum number of orders $m = m^*$.
3. Obtain optimum $n = n^*$ by (2.5).
4. Calculate t_a using (2.4).

For obtained set (t_1, m^*, n^*, t_a), compute total cost, TC using (2.6).

Computational Results

It is interesting to determine effect of varying the time horizon at which these parameters would have on the optimal policy. The effect of changes in b, σ_0^2 and σ_1^2 are studied on total cost and optimum procurement quantities. Consider the following parametric values in appropriate units:

$[R_1, R_2, A_1, A_2, C_1, C_2, h_1, h_2, S] = [5000, 10000, 25, 250, 50, 51, 12.50, 12.75, 1]$

Table 1: $\sigma_0^2 = 5$ $\sigma_1^2 = 0.1$

T \ b		0.75	0.80	0.85
0.192	Q_a	1027	1019	1012
	t_a	0.254	0.204	0.203
	TC	36504.65	38756.04	41011.97
0.231	Q_a	1235	1227	1224
	t_a	0.248	0.247	0.245
	TC	43208.67	45963.79	48728.92
0.269	Q_a	1487	1476	1465
	t_a	0.298	0.295	0.293
	TC	48188.66	51426.70	54686.64

Table 2: $b = 0.75 \;\; \sigma_1^2 = 0.1$

σ_0^2 / T		0	5	10
0.192	Q_a	1027	1027	1027
	t_a	0.205	0.205	0.205
	TC	35004.53	35004.58	35004.62
0.231	Q_a	1244	1244	1244
	t_a	0.248	0.248	0.248
	TC	42212.52	42212.57	42212.61
0.269	Q_a	1487	1487	1487
	t_a	0.298	0.298	0.298
	TC	49430.91	49430.95	49430.99

Table 3: $T = 10/52 \;\; \sigma_1^2 = 0.1$

σ_0^2 / b		0	5	10
0.75	Q_a	1027	1027	1027
	t_a	0.205	0.205	0.205
	TC	35004.53	35004.58	35004.62
0.80	Q_a	1019	1019	1019
	t_a	0.204	0.204	0.204
	TC	37270.66	37270.70	37270.75
0.85	Q_a	1012	1012	1012
	t_a	0.202	0.202	0.202
	TC	39543.41	39543.45	39543.49

Table 4: $b = 0.75 \quad \sigma_0^2 = 5$

σ_1^2 / T		0.50	0.55	0.60
0.192	Q_a	963	959	954
	t_a	0.192	0.191	0.190
	TC	35029.23	35095.52	35161.80
0.231	Q_a	1154	1150	1145
	t_a	0.230	0.230	0.229
	TC	42740.30	42806.26	42872.23
0.269	Q_a	1345	1340	1336
	t_a	0.269	0.268	0.267
	TC	49962.82	50029.30	50095.78

Table 5: $T = 10/52 \quad \sigma_0^2 = 5$

σ_1^2 / b		0.50	0.55	0.60
0.75	Q_a	963	959	954
	t_a	0.1927	0.1918	0.1910
	TC	35029.23	35095.52	35161.80
0.80	Q_a	963	958	954
	t_a	0.1926	0.1917	0.1910
	TC	37261.68	37237.97	37394.25
0.85	Q_a	962	958	954
	t_a	0.1924	0.1916	0.1910
	TC	39500.76	39567.76	39633.33

Table 6: $T = 10/52 \quad b = 0.75$

σ_0^2 \ σ_1^2		0.50	0.55	0.60
0.00	Q_a	963	959	954
	t_a	0.1927	0.1918	0.1909
	TC	41746.43	41812.70	41878.99
5.00	Q_a	963	958	954
	t_a	0.1926	0.1918	0.1909
	TC	41746.47	41812.75	41879.04
10.00	Q_a	963	959	954
	t_a	0.1927	0.1918	0.1902
	TC	41746.51	41812.80	41879.08

4. CONCLUSIONS

Using the hypothetical numerical example, we have the following results:

- From tables 1 and 2, we can conclude that with increase in time horizon (T), the total cost increases. Keeping all other parameters constant, increase in bias factor b results increase in total cost from tables 1 and 3.
- In table 2, we are trying to measure the effect of parameter σ_0^2 on total cost. It can be seen that σ_0^2 does not affect Q_a and there is a negligible effect on total cost. Thus, we can say that the optimal order quantity for period t_a as well as total cost are unaffected by the changes in the parameter σ_0^2.
- Table 3 helps in evaluating the influence of the parameter σ_1^2 on the optimal order quantity and total cost. From tables 4 and 5, it is observed that with increase in σ_1^2 there is increase in Q_a and total cost. Table 4 also highlights the facts that increase in σ_1^2 increases t_a, Q_a and total cost significantly.

- Table 6 depicts the effect of σ_0^2 and σ_1^2 on total cost. We see that total cost increases only when σ_1^2 changes while change in σ_0^2 does not bring significant change in total cost.

REFERENCES

Barbosa, L.C. and Friedman, H. (1978): Deterministic inventory lot-size models: General Root Law. *Manag. Sci.,* **24**, 819-826.

Buzacott, J.A. (1975): Economic order quantity with inflation. *Oper. Res.*, **26**, 3-10.

Goyal, S.K. (1975): An inventory model for a product for which purchase price fluctuates. *New Zealand Oper. Res.*, **3**, 34-40.

Lev, B.H., Weiss, J. and Soyster, A.L. (1979): Inventory models with finite horizons and price changes. *Oper. Res.*, **30**, 43-53.

Naddor, E. (1966): *Inventory Systems*. John Wiley, New York.

Noori, A.H. and Keller A.H. (1986): The lot-size reorder model with upstream – downstream uncertainty. *Decis. Sci.,* **17**, 285-291.

Resh, M., Friedman, M. and Barbosa, L.C. (1976): On a general solution of the deterministic lot-size problem with time proportional demand. *Oper. Res.*, **24**, 718-725.

Schwartz, L.B. (1972): Economic order quantities for products with finite demand horizons. *AIIE Trans.*, **4**, 234-237.

Silver, E.A., (1976): Establishing the order quantity when the amount received is uncertain. *INFOR*, **4**, 32-39.

Sivazlian, B.D. and Stanfel, L.E. (1975): *Analysis of Systems in Operations Research.* Prentice-Hall, Englewood Cliffs, New Jersey.

Trivedi, C.J., Shah, Nita H. and Shah, Y.K. (1994): EOQ model under temporary price discount when supply is random. *Econom. Comput. Econom. Cybernet. Stud. Res.*, **1-4**, 81 – 89.

Trivedi, C.J., Shah, Nita H. and Shah, Y.K. (1996): An EOQ model for exponentially decaying items under price change anticipation when supply is random. *Int. J. Manag. Syst.*, **12**, 225-234.

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